The Aplication of Max-Plus Algebra to Determine The Optimal Time of Ikat Kupang Woven Production

Lusiana Prastiwi and Yuni Listiana

Abstract—The problem of scheduling is a problem that becomes part of project management that aims to plan the implementation of activities in a project in a structured manner with a clear time limit. Kupang ikat woven is an Indonesian handicraft that the process of making it through the stages of structured activities with a long time limit. Thus, in this study we intend to estimate the optimal time required to manufacture the Kupang ikat woven by applying the Max-Plus algebra method. In this case, the linkages between activities within a project can be transformed into a matrix form which can be analyzed using Max-Plus algebra method. This matrix will be applied in the calculation to get the solution needed in project scheduling, in this case is the optimum time required of making Kupang ikat woven.

Index Terms—Max-Plus Algebra, Scheduling Projects, Optimum time, Kupang Ikat Woven.

I. INTRODUCTION

IKAT Kupang woven is one kind of Indonesian handicraft which made form cloth that comes from East Nusa Tenggara (NTT) province. Generally, Ikat Kupang woven handicraft has great potential to be developed in order to increase the potential of the regional economy and the community of weaving craftsmen in particular. However, the problems that arise in the making of traditional Ikat Kupang woven is no time calculations and fixed scheduling. So far, the calculation time of making Ikat Kupang woven only based on estimates without any indicator of the right time [1]. The process of making Ikat Kupang woven itself has several stages, namely: arrangement of yarn on the tool, binding motif and decoration, coloring, and weaving [2]. So because there is no optimal time in the process of making, as a result the process of making Ikat Kupang woven from one stage to another stage has a delay time which less of productive.

In 2016, Lusiana P. conducted a study on the calculation time of making ikat kupang woven using the critical path method (CPM) method. From the result of the research, it was found that to produce one handicraft ikat kupang woven took 20 days of making process. Another method that can be used to get optimal time is Algebra Max-Plus.

The Max-Plus algebra is an algebraic structure consisting of real numbers sets joined with \(-\infty\), where the standard operations of addition, \(\oplus\), defined as operation of taking a maximum and the standard operations of multiplication, \(\otimes\), defined as operation of standard addition on real number sets. Related to this problem, Max-Plus algebra can be used to optimize the manufacturing time on Kupang bundling making process, so that the making time can be used efficiently and effectively. So with reference to the above background then in this study will be discussed on how to determine the optimal time for making ikat kupang woven using Algebra Max-Plus.

II. LITERATURE STUDY

A. Ikat Kupang Woven

One of the traditional weaving craft is relatively well known by the people of East Nusa Tenggara, namely ikat kupang woven. Named “weaving” because before being colored, the threads to be woven are tied with raffia ropes on certain parts, then dyed into the dyeing fluid. The section tied to the raffia, once opened, remains white, while the unattached parts of the raffia become colored in color to the liquid. The color composition of the threads is colored and there are parts that remain white. At the time of woven will form patterns of decoration with certain colors. The yarn used for weaving is made of cotton or silk, which is specially used for ikat weaving [2]. The following will describe the process of making fabrics ikat kupang woven that has several stages:

a. Provision of raw materials
Raw material is the initial capital of the crafters because without any raw materials, the production process of woven fabric can not work.

b. Structuring the yarn on the tool
Structuring the yarn is done by inserting the feed yarn (yarn in transverse position) repeatedly and alternating on the thread lungsi (yarn in longitudinal position).

c. Binding motifs and decorations
The part of the thread to be left is white tied with a raffia rope, while the unbound part will be colored.

d. Coloring
The yarn is dipped into the color liquid obtained from the concoction of leaves and vegetation. The coloring process takes a long time to allow the dye to completely seep into the yarn.

e. Drying
After dyed, the colored yarn is drained and dried by air-aerated.
f. Unlock the motive bond
   After drying, the raffia straps that bind the yarn are
   opened and the result of the threads has a combination
   of white and dyed colors.

g. Weaving
   After the staining process is complete and the thread has
   completely dried, then the thread is mounted on a loom
   or ATBM (non-machine loom).

In this case, we assumed that both of cost and quantities of
order is neglected. So that, based on above explanation, both
of activities and time durations on the Kupang Ikat Woven
production is described on Table I and network illustration of
that is described on Figure 1.

### Table I

<table>
<thead>
<tr>
<th>Activities</th>
<th>Description</th>
<th>Following Activities</th>
<th>Time (day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Provision of raw materials</td>
<td>B</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>Structuring the yarn on the tool</td>
<td>C,D</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>Binding motifs and decorations</td>
<td>F</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>Coloring</td>
<td>E</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>Drying</td>
<td>F</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>Unlock the motive bond</td>
<td>G</td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>Weaving</td>
<td>I</td>
<td>10</td>
</tr>
</tbody>
</table>

Fig. 1. Network of production.

### B. Max-Plus Algebra

The Max-Plus algebra is an algebraic structure consisting
of real numbers where the standard operations of addition
and multiplication are replaced by the operation of taking a
maximum and the operation of standard addition, respectively.
More precisely, let \( \mathbb{R}_{\max} \) denote the set \( \mathbb{R} \cup \{-\infty\} \), let \( \oplus \) be a
binary operator on \( \mathbb{R}_{\max} \) with \( x \oplus y = \max(x, y) \), and let \( \otimes \) be a
binary operator on \( \mathbb{R}_{\max} \) with \( x \otimes y = x + y \). Then the Max-
Plus Algebra is the algebraic structure consisting of \( \mathbb{R}_{\max} \)
and the binary operations \( \oplus \) and \( \otimes \), denoted by \( (\mathbb{R}_{\max}, \oplus, \otimes) \) [3].

The Max-Plus Algebra can be extended to matrices. Operations
on matrix elements are required to perform calculations
on scheduling project. Let matrix set \( n \times n \) in Max-Plus
algebra denoted by \( \mathbb{R}_{\max}^{n \times n} \). For \( n \in \mathbb{N} \), where \( n \neq 0 \), defined
that \( n \not\equiv 0 \). Let \( A \in \mathbb{R}_{\max}^{n \times m} \) where the \( i \)th row and the \( j \)th column
denoted by \( a_{i,j} \) for \( i \in n \) and \( j \in m \). In this case matrix A stated as:

\[
A = \begin{bmatrix}
a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\
a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n,1} & a_{n,2} & \cdots & a_{n,m}
\end{bmatrix}
\]

Occasionally, element \( a_{i,j} \) also denoted as:

\[
[A]_{i,j}, i \in n, j \in m
\]

An addition matrix \( A, B, \in \mathbb{R}_{\max}^{n \times m} \) denoted as \( A \oplus B \) defined by:

\[
[A \oplus B] = a_{i,j} + b_{i,j} = \max\{a_{i,j}, b_{i,j}\}
\]

for \( i \in n \) and \( j \in m \). While, for \( A, B, \in \mathbb{R}_{\max}^{n \times p} \) and \( B, \in \mathbb{R}_{\max}^{p \times m} \),
a multiplication of matrix \( A \otimes B \), defined as : with \( i \in n \) and

\[
[A \otimes B] = a_{i,k} \otimes b_{k,j} = \max_{k=1}^{p} \{a_{i,k} + b_{k,j}\}
\]

\( j \in m \). The multipication of this matrix is similar with the
multipication of general matrix algebra, where \( + \) and \( \times \)
operation replaced by \( \max \) and \( + \), respectively. Then, for \( A, \in \mathbb{R}_{\max}^{n \times n} \), the \( k \)th rank of A denoted by \( A^{\otimes k} \) defined as follow:

\[
A^{\otimes k} = A \otimes A \otimes \cdots \otimes A
\]

for \( k \in \mathbb{N} \) and \( k \neq 0 \). The following are the steps to find project
scheduling solution using Max-Plus algebra :

1) Formulate the existing data into diagram. Give additional
activities \((\alpha, \omega)\) on diagram. Each activities \( i \) that has
no predecessor activity, give direction from \( \alpha \) to \( i \) with
weight \( 0 \). Each activities \( j \) that has no activity after
it, give direction from \( j \) to \( \omega \) with the weight of the
completion time of activity \( j \). Give direction on activity
\( i \) toward activity \( j \) that are related to the same weight
with the length of activity process of \( i \).

2) Create a Max-Plus Matrix \( X \), where element \( x_{i,j} \) is
weight from activity \( i \) to weight \( j \). If \( i \) and \( j \) are not
related, then element \( x_{i,j} \) has value \(-\infty\).

3) Calculate \( X^* = X \oplus X^2 \oplus \cdots \oplus X^{n+1} \), where \( n \) is frequency
of activities before added by \( \alpha, \omega \).

4) The optimum completing time is \( X_{\alpha \omega}^* \).

5) Find a value of the furthest route vector \((V)\) and vector
slack \((S)\). Both of that vector has size \( n \times 1 \) without
contain \( \alpha, \omega \).

\[
v_i = x_{\alpha i} \otimes x_{i \omega}
\]

\[
s_i = x_{\alpha \omega} - v_i
\]

The critical path is a element on vector slack that has
value \((s_i = 0)\).
III. RESULT AND DISCUSSION

On the scheduling project, the first resolving step to get the optimum time using Max-Plus Algebra Methode is formulate all activities Ikat Kupang Woven production as had explained on Table I in a diagram form as Figure 1.

In a process of making diagram follow basic principle of AON (Activity on Node), that is a rule that activity represented with a circle (node) symbol. Then, put additional activity $\alpha$ and $\omega$ on the connection activity of diagram that we have created. For every single of activity $i$ that has no predecessor activity, we can give direction from $\alpha$ to $i$ where its weight is 0. Then, for every single activity $j$ that has no activity after it, let give it direction from $k$ to $\omega$ with the weight of the completion time of activity $j$. While, for the other activities, weight of activity $i$ to $j$ is equal with the length of activity process of $i$. Result of adding activity $\alpha$ and $\omega$ along with its weight shown on Figure 2. The next step is create matrix Max-

![Diag](image.png)

Fig. 2. Activity and weight.

Plus X. Matrix Max-Plus X has element that symbolized with $x_{i,j}$, where $x_{i,j}$ is weight of activity $i$ to activity $j$. If activity $i$ and activity $j$ are unrelvant, then $x_{i,j} = -\infty$. So, lets see the matrix.

and

$$X^2 = \begin{pmatrix} -\infty & -\infty & 4 & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & 5 & 5 & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & 2 & 5 & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 5 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \end{pmatrix}$$

$$X^3 = \begin{pmatrix} -\infty & -\infty & -\infty & 6 & 9 & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & 4 & 6 & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 15 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \end{pmatrix}$$

$$X^4 = \begin{pmatrix} -\infty & -\infty & -\infty & -\infty & 8 & 10 & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 16 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \end{pmatrix}$$

$$X^5 = \begin{pmatrix} -\infty & -\infty & -\infty & -\infty & -\infty & 9 & 20 & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 15 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \end{pmatrix}$$

$$X^6 = \begin{pmatrix} -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 9 & 20 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 19 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \end{pmatrix}$$

$$X^7 = \begin{pmatrix} -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 19 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \end{pmatrix}$$

$$X^8 = \begin{pmatrix} -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \end{pmatrix}$$
To get the optimum time, we should to find the matrix $X^*$ where is:

$$X^* = X \oplus X^2 \oplus ... \oplus X^8$$

where

$$
\begin{pmatrix}
\infty & 0 & 4 & 5 & 5 & 9 & 10 & 20 \\
\infty & \infty & 4 & 5 & 6 & 9 & 10 & 20 \\
\infty & \infty & \infty & 1 & 2 & 5 & 6 & 16 \\
\infty & \infty & \infty & \infty & \infty & \infty & 4 & 5 & 15 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & 1 & 3 & 4 & 14 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 2 & 3 & 13 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1 & 11 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 10 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\end{pmatrix}
$$

The optimum time for Ikat Kupang Woven production is $X_{\omega \omega} = 20$. So, the optimum time to produce Ikat Kupang Woven is 20 days.

The next steps is find critical path. Searching process of critical path has advantageous to know whichever activity that can't delayed its implementation. Critical path obtained by geting vector $V$ where the $ith (v_i)$ element of it obtained from :

$$v_i = x_{\omega i} \oplus x_{i \omega}$$

and the $jth$ element of vector $S$ stated as:

$$s_j = x_{\omega j} \oplus v_j - v_i$$

where $i = A, B, C, D, E, F, G$. sehingga

$$V = \begin{pmatrix}
v_A \\
v_B \\
v_C \\
v_D \\
v_E \\
v_F \\
v_G \\
\end{pmatrix} = \begin{pmatrix}
x_{\omega A} \oplus x_{A \omega} \\
x_{\omega B} \oplus x_{B \omega} \\
x_{\omega C} \oplus x_{C \omega} \\
x_{\omega D} \oplus x_{D \omega} \\
x_{\omega E} \oplus x_{E \omega} \\
x_{\omega F} \oplus x_{F \omega} \\
x_{\omega G} \oplus x_{G \omega} \\
\end{pmatrix} = \begin{pmatrix}
0 \otimes 20 \\
4 \otimes 16 \\
5 \otimes 15 \\
5 \otimes 14 \\
6 \otimes 13 \\
9 \otimes 11 \\
10 \otimes 10 \\
\end{pmatrix} = \begin{pmatrix}
20 \\
20 \\
20 \\
19 \\
19 \\
20 \\
20 \\
\end{pmatrix}$$

and

$$S = \begin{pmatrix}
s_A \\
s_B \\
s_C \\
s_D \\
s_E \\
s_F \\
s_G \\
\end{pmatrix} = \begin{pmatrix}
x_{\omega A} - v_A \\
x_{\omega B} - v_B \\
x_{\omega C} - v_C \\
x_{\omega D} - v_D \\
x_{\omega E} - v_E \\
x_{\omega F} - v_F \\
x_{\omega G} - v_G \\
\end{pmatrix} = \begin{pmatrix}
20 - 20 \\
20 - 20 \\
20 - 20 \\
20 - 19 \\
20 - 19 \\
20 - 20 \\
20 - 20 \\
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
0 \\
1 \\
1 \\
0 \\
0 \\
\end{pmatrix}$$

Critical path is an element on vector $S$ that has value 0. While $s_i \neq 0$ show time duration for activity that can’t be delayed. So, activity that can’t be delayed are activity A, B, C, F,dan G. While, activity E and G can be delayed only for one day. This result is suitable with Prastiwi (2016).

IV. CONCLUSIONS

To find the optimum time on Ikat Kupang Woven production, we can use Max-Plus Algebra method. From the result and discussion, we can conclude that we need 20 days to produce a Ikat Kupang Woven.

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