SPACE INTERSECTION BY COLLINEARITY

Hepi Hapsari Handayani
Program Studi Teknik Geomatika FTSP-ITS, Kampus ITS Sukolilo Surabaya

Abstrak

Intersection refers to the determination of a point's position in object space by intersecting the image rays from two or more images. The standard method is application of the collinearity equations, with two equations for each image of the point. Approximate coordinates of the point, calculated by collinearity equations. EOPs are obtained by using space resection.

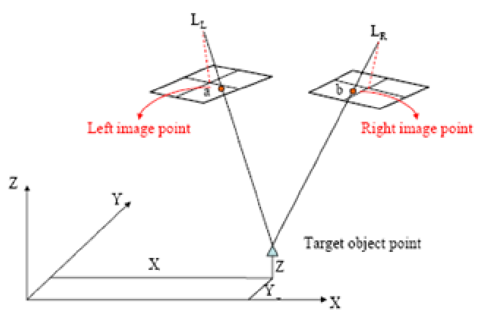
Initial approximations are required for ground coordinate. In this experiment, we use several data types on flat, rugged, and incline terrain. The data has random and systematic error. We create a simulated data of ground coordinate points then we compute the image points using collinearity equations.

Finally, we can conclude that the data with systematic error achieves the best precise than data with random error. This is occurred in all of terrain types. The RMS error in data with systematic error achieves constantly after the limitation number 100 control points. Therefore, data with systematic radial lens error can be used in intersection case. The RMSx is more than RMSy in almost all of data types.

Introduction

If space resection is used to determine the elements of the exterior orientation for both photographs of a stereopair, then the object point coordinates for points that lie in the stereo overlap area can be calculated. Therefore, corresponding rays to the same object point from the two photographs must intersect at the point. If the images are available, a total of four equations containing three unknowns, the object space coordinates of the point, are obtained. There is one degree of freedom, and the linearized set of equations can be solved by least squares methods. Adding more images increases the number of degrees of freedom and therefore improves the solution.

Space Intersection



To calculate the ground point A (X,Y,Z) by space intersection, collinearity equations of the linearized should be defined. In the space intersection, however, since that the six elements of exterior orientation are known, the only remaining unknowns in these equations are dX_A, dY_A, and dZ_A. These are corrections to be applied to initial approximations for object space coordinates X_A, Y_A, and Z_A, respectively for ground point G. The linearized forms of the space intersection equations for point A are

$$b_{14}dX_A + b_{15}dY_A + b_{16}dZ_A = J + v_{x_A} \quad (1)$$
$$b_{24}dX_A + b_{25}dY_A + b_{26}dZ_A = K + v_{y_A} \quad (2)$$

Hence, two equations of this form can be written for point A in the left image, and two more in the right image. So there are four equations result, and three unknowns dX_A, dY_A, and dZ_A can be computed by least squares solution. These corrections are added to the initial approximations to obtain revised values for X_A, Y_A, and Z_A. The solution is then repeated until the magnitude of the corrections become neglible. Therefore, because the equations have been linearized using Taylor's theorem, initial approximations are required for each point whose object space coordinates are to be computed.

Least-squares solution
Linearization

Since the collinearity equations are nonlinear, and have been linearized using Taylor's theorem. In linearizing them, equation (1) and (2) are written as follows:

$$F = x_0 - f\frac{r}{q} = x_A \qquad (3)$$

$$G = y_0 - f\frac{s}{q} = y_A \qquad (4)$$

Where

$$q = m_{31}(X_G - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)$$
$$r = m_{11}(X_G - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)$$
$$s = m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)$$

According to Taylor's theorem, equation (3) and (4) may be expressed in linearized form by taking partial derivatives with respect to the unknowns:

$$F_0 + \left(\frac{\partial F}{\partial \omega}\right)_0 d\omega + \left(\frac{\partial F}{\partial \phi}\right)_0 d\phi + \left(\frac{\partial F}{\partial \kappa}\right)_0 d\kappa + \left(\frac{\partial F}{\partial X_L}\right)_0 dX_L + \left(\frac{\partial F}{\partial Y_L}\right)_0 dY_L + \left(\frac{\partial F}{\partial Z_L}\right)_0 dZ_L + \left(\frac{\partial F}{\partial X_A}\right)_0 dX_A + \left(\frac{\partial F}{\partial Y_A}\right)_0 dY_A + \left(\frac{\partial F}{\partial Z_A}\right)_0 dZ_A = x_a \qquad (5)$$

$$G_0 + \left(\frac{\partial G}{\partial \omega}\right)_0 d\omega + \left(\frac{\partial G}{\partial \phi}\right)_0 d\phi + \left(\frac{\partial G}{\partial \kappa}\right)_0 d\kappa + \left(\frac{\partial G}{\partial X_L}\right)_0 dX_L + \left(\frac{\partial G}{\partial Y_L}\right)_0 dY_L + \left(\frac{\partial G}{\partial Z_L}\right)_0 dZ_L + \left(\frac{\partial G}{\partial X_A}\right)_0 dX_A + \left(\frac{\partial G}{\partial Y_A}\right)_0 dY_A + \left(\frac{\partial G}{\partial Z_A}\right)_0 dZ_A = y_a \qquad (6)$$

In equation (5) and (6), F_0 and G_0 are functions F and G of equation (3) and (4) evaluated at the initial approximations for the nine unknowns. The terms $\left(\frac{\partial F}{\partial \omega}\right)_0, \left(\frac{\partial G}{\partial \omega}\right)_0, \left(\frac{\partial F}{\partial \phi}\right)_0, \left(\frac{\partial G}{\partial \phi}\right)_0$, etc, are partial derivatives of functions F and G with respect to the indicated unknowns evaluated at the initial approximations; and $d\omega, d\phi, d\kappa$, etc., are unknown corrections to be applied to the initial approximations. The units of $d\omega, d\phi,$ and $d\kappa$ are radians. Since the photo coordinates x_a and y_a are measured values, if the equations are to be used in a least squares solution, residual terms must be included to make the equations consistent. The following simplified forms of the linearized collinearity equations include these residuals.

$$b_{11}d\omega + b_{12}d\phi + b_{13}d\kappa - b_{14}dX_L - b_{15}dY_L - b_{16}dZ_L + b_{14}dX_A + b_{15}dY_A + b_{16}dZ_A = J + v_{x_a} \qquad (7)$$

$$b_{21}d\omega + b_{22}d\phi + b_{23}d\kappa - b_{24}dX_L - b_{25}dY_L - b_{26}dZ_L + b_{24}dX_A + b_{25}dY_A + b_{26}dZ_A = K + v_{y_a} \qquad (8)$$

In equation (7) and (8), J and K are equal to $x_a - F_0$ and $y_a - G_0$ respectively. The b's are coefficients equal to the partial derivatives. In these coefficients ΔX, ΔY, and ΔZ are equal to X_A- X_L, Y_A- Y_L, and Z_A- Z_L, respectively. Numerical values for these coefficient terms are obtained by using initial approximations for the unknowns.

$$b_{14}dX_A + b_{15}dY_A + b_{16}dZ_A = J + v_{x_A}$$
$$b_{24}dX_A + b_{25}dY_A + b_{26}dZ_A = K + v_{y_A}$$

Where

$$b_{14} = \frac{f}{q^2}(rm_{31} - qm_{11}) \qquad b_{15} = \frac{f}{q^2}(rm_{32} - qm_{12})$$
$$b_{16} = \frac{f}{q^2}(rm_{33} - qm_{13}) \qquad J = x_a - x_0 + f\frac{r}{q}$$
$$b_{24} = \frac{f}{q^2}(sm_{31} - qm_{21}) \qquad b_{25} = \frac{f}{q^2}(sm_{32} - qm_{22})$$
$$b_{26} = \frac{f}{q^2}(sm_{33} - qm_{23}) \qquad K = y_a - y_0 + f\frac{s}{q}$$

The observation error equations can be then formed as

V = AX – L

Then, we can determine X. That is

$$X = (A^T A)^{-1} A^T L$$

$$\begin{bmatrix} b_{11_1} & b_{12_1} & b_{13_1} & -b_{14_1} & -b_{15_1} & -b_{16_1} \\ b_{21_1} & b_{22_1} & b_{23_1} & -b_{24_1} & -b_{25_1} & -b_{26_1} \\ b_{11_2} & b_{12_2} & b_{13_2} & -b_{14_2} & -b_{15_2} & -b_{16_1} \\ b_{21_2} & b_{22_2} & b_{23_2} & -b_{24_2} & -b_{25_2} & -b_{26_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{11_n} & b_{12_n} & b_{13_n} & -b_{14_n} & -b_{15_n} & -b_{16_n} \\ b_{21_n} & b_{22_n} & b_{23_n} & -b_{24_n} & -b_{25_n} & -b_{26_n} \end{bmatrix};$$

$$L = \begin{bmatrix} J_1 \\ K_1 \\ J_2 \\ K_2 \\ \vdots \\ J_2 \\ K_2 \end{bmatrix}; \; V = \begin{bmatrix} V_{x_1} \\ V_{y_1} \\ V_{x_2} \\ V_{y_2} \\ \vdots \\ V_{x_n} \\ V_{y_n} \end{bmatrix}; \; X = \begin{bmatrix} d\omega \\ d\phi \\ d\kappa \\ dX_L \\ dY_L \\ dZ_L \end{bmatrix}$$

Iterative Least-Square

Two equations are formed for each control point, which gives four equations if the minimum of two control points is used. In the case a unique solution results for the four unknowns, and the residual terms on the right sides of equation (1) and (2) will be zero. If three or more control points are used, more than four equations can be performed, allowing a least squares solution.

Initial approximations are required for ground coordinate. In this experiment, I use several data types on flat, rugged, and incline terrain. The data has random and systematic error. We assume the ground point A based on first experiment. In the first experiment, we create ground coordinate points then we compute the image points using collinearity equations. Therefore, the ground point A that is intersection point is calculated in several iterations. The iteration will stop until the difference become insignificant.

Methodology

Data, We obtain EOPs using space resection. Then, the initial ground coordinate have been determined based on simulation using collinearity equations. The simulation has created on 3 type's terrain: flat, rugged, and incline with random and systematic error.

Solution, The procedure of solution for determining ground coordinate intersection is

1. Collect a simulation data of ground coordinate points and exterior orientation parameters. Ground coordinate points is taken from flat, incline and rugged terrain. Firstly, we assume that the coordinates is free error. After calculating the error free data using collinearity equations, we add random error and systematic error. Then, we compute it to get image coordinate points. Systematic error used to this data is radial lens distortion. The form of the polynomial, based on lens design theory, is
   $$\Delta r = k_1 r^1 + k_2 r^3 + k_3 r^5 + k_4 r^7 \qquad (9)$$
   Where, Δr is the amount of radial lens distortion, r is the radial distance from the principal point, and k_1, k_2, k_3, and k_4 are coefficients of the polynomial.
2. Compute length of baseline for checking the simulation data using equation (10)
   B = (1- μ). p.sf *(10)*
   Where,
   B = length of baseline
   μ = overlapping = 0.6 (in this experiment)
   p = length of image = 23 cm
   sf = scale factor of map = 15000
3. Check the control points in image coordinate either left or right, using baseline. The equation for checking is equation (11)
   $$B = \sqrt{(x_{L_1} - x_{L_1})^2 + (y_{L_1} - y_{L_1})^2} \qquad (11)$$
4. Check the ground coordinate intersection based on two images in the simulation data.
5. EOPs from previous as defined. Image points either in left and right, the approximations ground coordinate as input for initial value.
6. Calculate the ground coordinate based on least squares adjustment as in section 3.

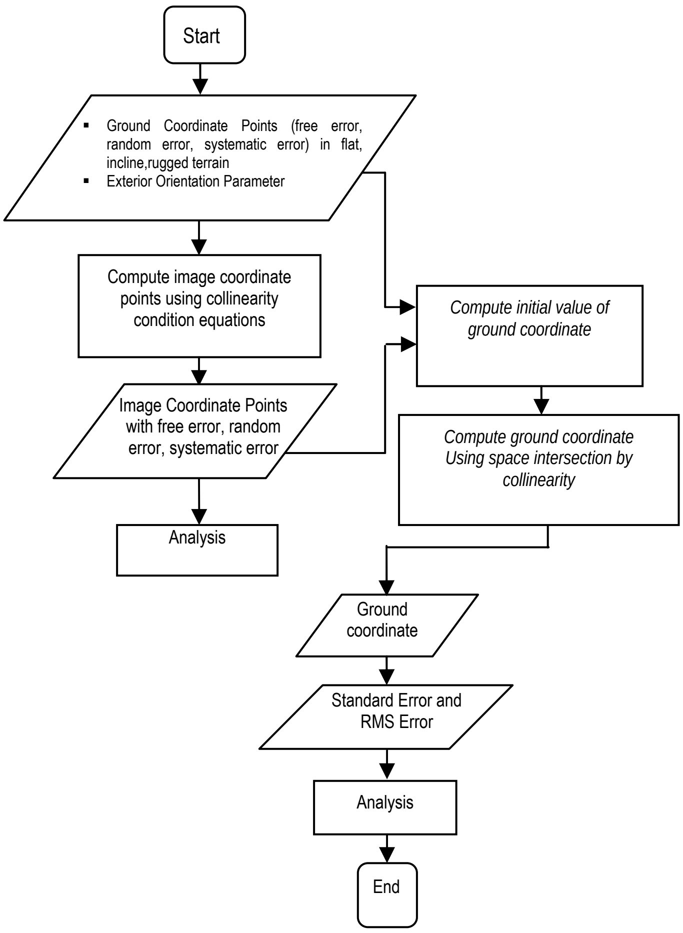


Figure 2. Workflow of this experiment

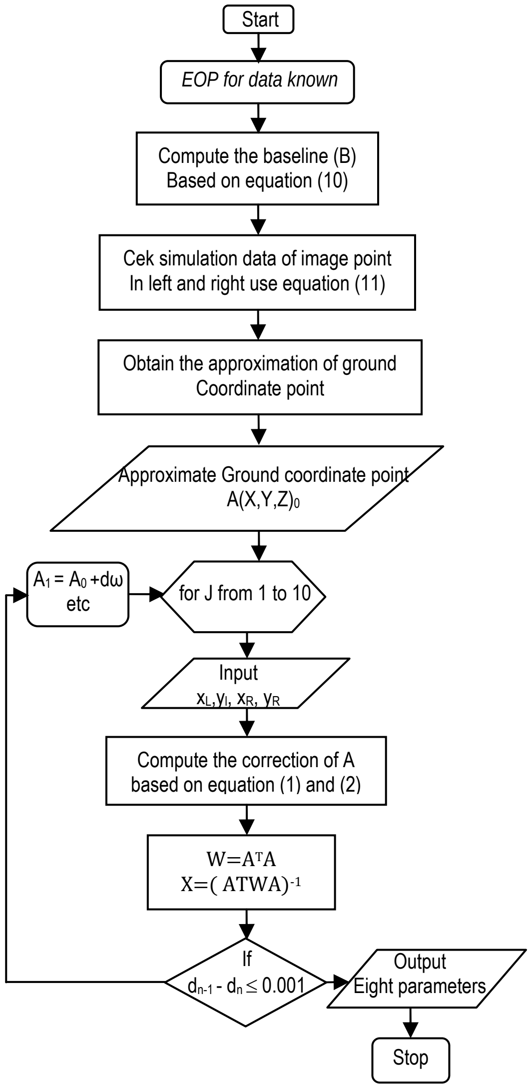


Figure 3. Flowchart of the computational program

Experimental Results

RMS Error

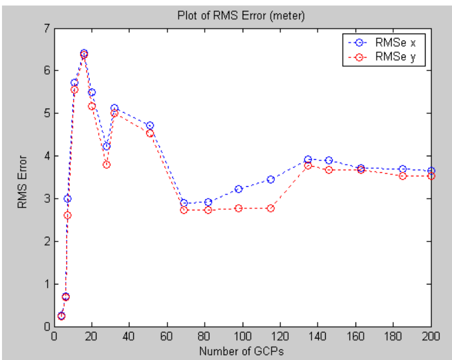


Figure 4. RMSe (m) of free error data on flat terrain

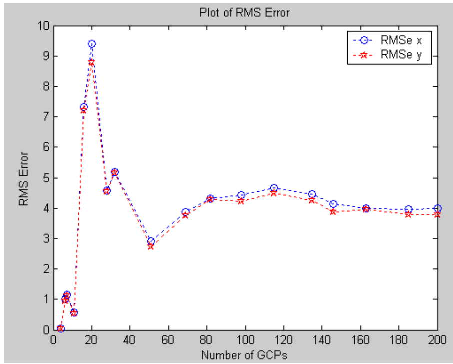


Figure 5. RMSe (m) of random error data on flat terrain

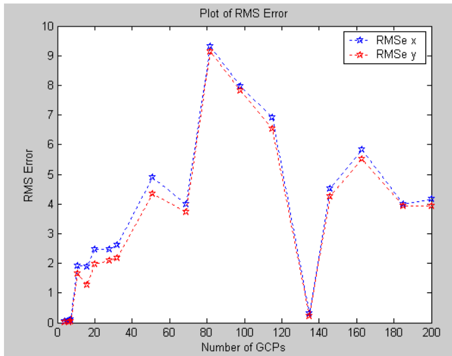


Figure 6. RMSe (m) of systematic error data on flat terrain

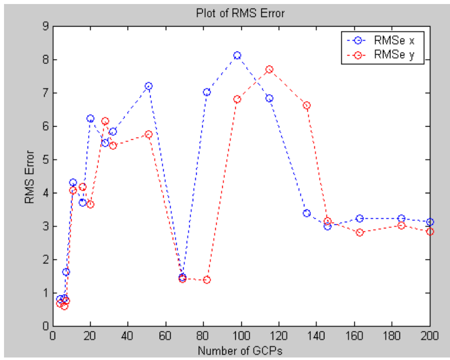


Figure 7. RMSe (m) of free error data on rugged terrain

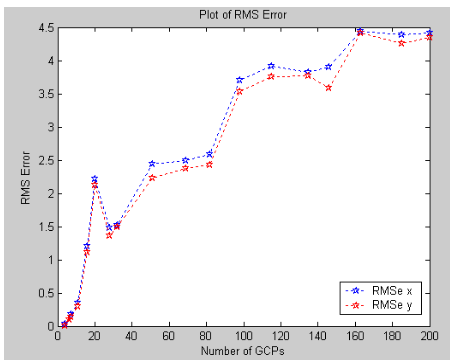


Figure 8. RMSe (m) of random error data on rugged terrain

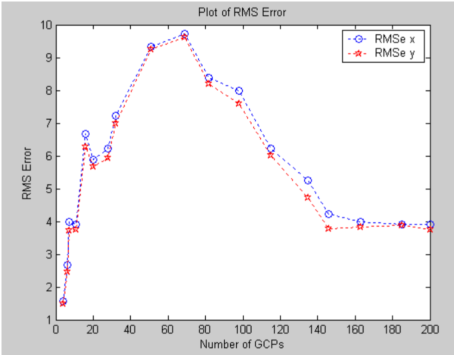


Figure 9. RMSe (m) of systematic error data on rugged terrain

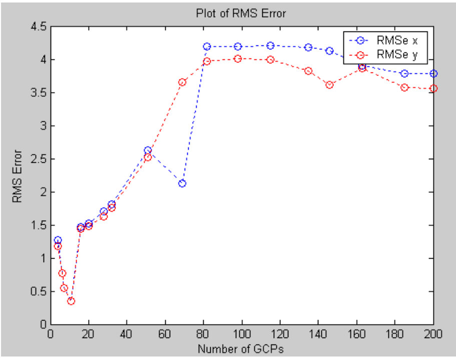


Figure 10. RMSe (m) of free error data on incline terrain

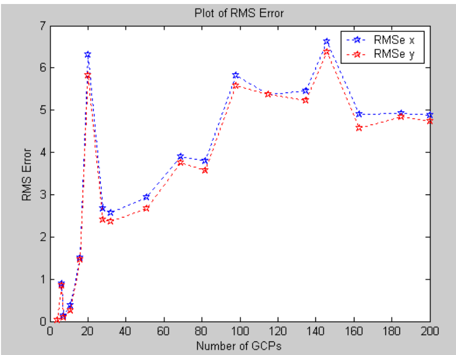


Figure 11. RMSe (m) of random error data on incline terrain

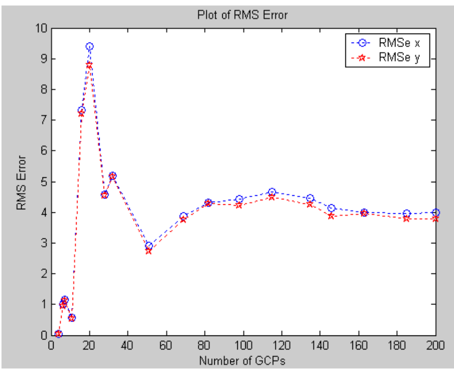


Figure 12. RMSe (m) of systematic error data on incline terrain

Conclusions

Based on the figure 4 to 12, we can conclude that the data with systematic error achieves the best precise than data with random error. This is occurred in all of terrain types. So that, the result is the same conclusion with the one in space resection experiment. The RMS error in data with systematic error achieves constantly after the limitation number 100 control points. Therefore, data with systematic radial lens error can be used either in either space resection or intersection. The RMSx is more than RMSy in almost all of data types. However, in data with systematic error on rugged terrain create the largest range. They create the worst result by comparing with the others.

The limitation of this experiment is about computing the conjugate points between right and left images. The computation is based on the baseline. If we create the incorrect simulation, the result will be poor. Space intersection will fully depend on the conjugate points. The geometry of data simulation is important in this experiment.

References

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