Restricted Size Ramsey Number for Matching versus Tree and Triangle Unicyclic Graphs of Order Six

Elda Safitri, Peter John, and Denny Riama Silaban

***Abstract*—Let F, G, and H be simple graphs. The graph F arrows (G, H) if for any red-blue coloring on the edge of F, we find either a red-colored graph G or a blue-colored graph H in F. The Ramsey number $r(G, H)$ is the smallest positive integer r such that a complete graph K_r arrows (G, H). The restricted size Ramsey number $r^*(G, H)$ is the smallest positive integer r^* such that there is a graph F, of order $r(G, H)$ and with the size r^*, satisfying F arrows (G, H). In this paper we give the restricted size Ramsey number for a matching of two edges versus tree and triangle unicyclic graphs of order six.**

***Index Terms*—Restricted size Ramsey number, matching graph, tree graph, triangle unicyclic graph.**

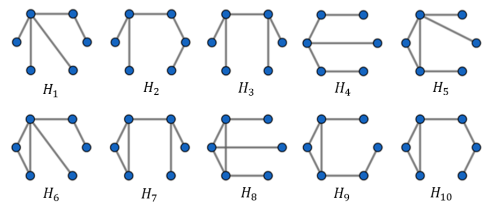

Fig. 1. Lists of all tree graphs and triangle unicyclic graphs of order six.

I. Introduction

GIVEN simple graphs F, G and H. We say $F \to (G, H)$, if for any red-blue coloring on the edge of F, we found either a red-colored graph G or a blue-colored graph H. A red-blue coloring of F is called $(G, H) - good$ if in the coloring neither a red-colored graph G nor a blue-colored graph H is found. If there is at least one $(G, H) - good\ coloring$ in F, then we write $F \not\to (G, H)$.

For a graph $F = (V, E)$, we denote the order and the size of F as $v(F)$ and $e(F)$ respectively. A degree of a vertex v, denoted by $d(v)$, is the number of edges incident to vertex v. The minimum degree of a graph F, denoted by $\delta(F)$, is the smallest degree of the vertices in F. The maximum degree of a graph F, denoted by $\Delta(F)$, is the greatest degree of the vertices in F. A complete graph K_n is graph of order n that any pair of vertices is adjacent. A matching graph tK_2 is a t disjoint union of K_2. A cycle C_n is a connected graph of order n that all vertices have degree 2. A tree graph is a connected graph that do not contain any cycle. A triangle unicyclic graph is a connected graph that contain exactly one triangle C_3.

Let F and H be graphs. If H is a subgraph of F, then $F - H$ is the resulting graph by removing all edges of H from F. For a vertex $v \in V(F)$, $F - v$ is the resulting graph by removing v and all its incident edges from F.

Given graph G and H with no isolates. The Ramsey number of G and H, $r(G, H)$, is the smallest positive integer r such that a complete graph K_r satisfies $K_r \to (G, H)$. The size Ramsey number of G and H, $\hat{r}(G, H)$, is the smallest positive integer $\hat{r}$ such that a graph F with the size of $\hat{r}$ satisfies $F \to (G, H)$. The restricted size Ramsey number of G and H, $r^*(G, H)$, is the smallest positive integer r^* such that a graph F with order of $r(G, H)$ and size of r^* satisfies $F \to (G, H)$.

The (restricted) size Ramsey numbers for small graphs initially found by Harary and Miller in [1]. The research is continued by Faudree and Sheehan [2]. They give the (restricted) size Ramsey number for any pair of graphs of order at most four. In 1998, Lortz and Mengersen [3] gave the size Ramsey number and the restricted size Ramsey number for a pair of forest graphs of order at most five. Until now, there are many studies regarding the size Ramsey and restricted size Ramsey numbers of graphs. Some results on the restricted size Ramsey numbers of graph can be found in [4], [5], [6].

Silaban et al. [7] have given the restricted size Ramsey number for $2K_2$ versus dense connected graphs of order six. The restricted size Ramsey number for $2K_2$ versus disconnected graphs of order six are also given in [8]. We continue this research by investigating the restricted size Ramsey number for $2K_2$ versus tree and triangle unicyclic graphs of order six.

II. Preliminaries

There are 112 non-isomorphic connected graphs of order six. In Figure 1 we give all tree graphs and triangle unicyclic graphs of order six.

Theorem 1 by Chvatal and Harary [9] gives the Ramsey number for a pair of $2K_2$ versus any graph with no isolates. This Ramsey number gives the order of a graph F that satisfies $F \to (2K_2, H)$, where H is any tree graphs or triangle unicyclic graphs of order six.

Theorem 1 ([9]): For any graphs H with no isolates,

$$r(2K_2, H) = \begin{cases} v(H) + 2, & H \text{ is complete} \\ v(H) + 1, & \text{otherwise} \end{cases}$$

Silaban, Baskoro, and Uttunggadewa in [10], [11] gave some Lemmas as a tool to prove if a graph F satisfies

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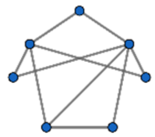

Fig. 2. Graph F with $v(F) = 7$ and $e(F) = 10$ such that $F \to (2K_2, H_1)$.

$F \to (2K_2, H)$. Lemma 1 gives the conditions when $F \to (2K_2, H)$ for any simple graph H. Lemma 2 gives the property of F that satisfies $F \to (2K_2, H)$ for any simple graphs H with no isolates. Lemma 3 gives the property of F that satisfies $F \to (2K_2, H)$ for H graphs that contains a cycle.

Lemma 1 ([10]): Let H be a graph. $F \to (2K_2, H)$ holds if and only if the following conditions are satisfied:

1) $H \subseteq F - v$ for every v in F and
2) $H \subseteq F - C_3$ for every C_3 in F.

Lemma 2 ([11]): Let H be a graph with no isolates. If $F \to (2K_2, H)$ and $v(F) = r(2K_2, H)$, then $\delta(F) \geq 2$.

Lemma 3 ([11]): For $n \geq 4$, let H be a graph with $v(H) = n$ and H contains a cycle of length t, C_t, for $3 \leq t \leq n$. If $F \to (2K_2, H)$, then F contains at least two C_t which do not share a vertex nor incident to a C_3.

III. Main Results

Observe that H_1, H_2, H_3, and H_4 are all tree graphs of order six. We give the restricted size Ramsey numbers for $2K_2$ versus each of these graphs in separate theorems as follows.

Theorem 2: $r^*(2K_2, H_1) = 10$.

Proof: By Theorem 1, we know that $r(2K_2, H_1) = 7$. To show the upper bound, let F be a graph with $v(F) = 7$ and $e(F) = 10$ as shown in Figure 2. We can see that for vertex v_1 in F with $d(v_1) = 5$ and vertex v_2 in F with $d(v_2) = 4$, satisfy $H_1 \subseteq F - v_1 \subseteq F - v_2$, and for vertex v_3 in F with $d(v_3) = 3$ and vertex v_4 in F with $d(v_4) = 2$, satisfy $H_1 \subseteq F - v_3 \subseteq F - v_2$. Graph F only have one triangle C_3 that is induced by vertices of degree 2, 3, and 5, and we can show that $H_1 \subseteq F - C_3$. So, by Lemma 1, $F \to (2K_2, H_1)$, thus $r^*(2K_2, H_1) \leq 10$.

Next, we show the lower bound. Let F all graphs with $v(F) = 7$, $e(F) = 9$, and because there is a $K_{1,4}$ in H_{11}, F must have at least two vertices of degree 4 that are not adjacent and do not incident to a C_3. Since there is no graph satisfying this condition, we have $r^*(2K_2, H_1) \geq 10$. ■

Theorem 3: $r^*(2K_2, H_2) = 9$.

Proof: By Theorem 1, $r(2K_2, H_2) = 7$. Let a graph F with $v(F) = 7$ and $e(F) = 9$ as shown in Figure 3. We see that the vertices in F are of degree 2 and 3. For vertex v_1 with $d(v_1) = 3$, we can show that $H_2 \subseteq F - v_1$ and for vertex v_2 with $d(v_2) = 2$, we see that $H_2 \subseteq F - v_2$. Thus, we have $H_2 \subseteq F - v$ for all vertex v in F. Graph F contains a pair of isomorphic triangles C_3 and it shows that $H_2 \subseteq F - C_3$. So, by Lemma 1 we have $F \to (2K_2, H_2)$. Thus, $r^*(2K_2, H_2) \leq 9$.

Next we show the lower bound for $r^*(2K_2, H_2)$. Let F a graph with $v(F) = 7$ and $e(F) = 8$. Graph F must satisfy $2 \leq \delta(F) \leq \Delta(F) \leq 3$, since $e(H_2) = 5$ and if there is one vertex v in F where $d(v) \geq 4$, then $e(F - v) \leq 4$ so we will not find H_2 in $F - v$, or $H_2 \not\subseteq F - v$. Observe that there is one vertex in H_2 of degree 3, then there has to be at least two vertices in F of degree 3 that are not adjacent. Figure 4 shows all graph F satisfying the condition above with $(2K_2, H_2)$-good coloring. By Lemma 1, we have $F \not\to (2K_2, H_2)$ for all F. Thus, $r^*(2K_2, H_2) \geq 9$. ■

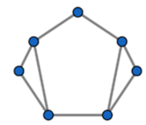

Fig. 3. Graph F with $v(F) = 7$ and $e(F) = 9$ such that $F \to (2K_2, H_2)$.

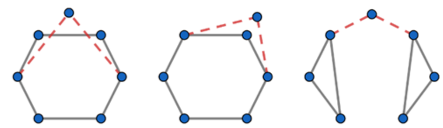

Fig. 4. $(2K_2, H_2)$-good coloring of F.

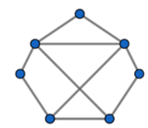

Fig. 5. Graph F with $v(F) = 7$ and $e(F) = 10$ that satisfy $F \to (2K_2, H_3)$.

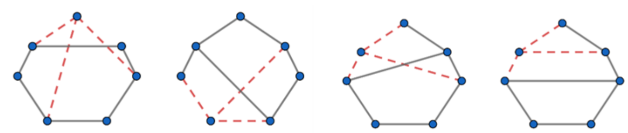

Fig. 6. $(2K_2, H_3)$-good coloring of F.

Theorem 4: $r^*(2K_2, H_3) = 10$.

Proof: By Theorem 1 we know that $r(2K_2, H_3) = 7$, then to show the upper bound let F a graph with $v(F) = 7$ and $e(F) = 10$ as shown in Figure 5. For v_1, v_2, v_3 in F with $d(v_1) = 4$, $d(v_2) = 3$, and $d(v_3) = 2$, we have $H_3 \subseteq F - v_1 \subseteq F - v_2 \subseteq F - v_3$. Graph F contains one triangle C_3 induced by one vertex of degree 2 and two vertices of degree 4. We can show that $H_3 \subseteq F - C_3$. Therefore, by Lemma 1, we have $F \to (2K_2, H_3)$ and $r^*(2K_2, H_3) \leq 10$.

To show the lower bound, let F all graph with $v(F) = 7$ and $e(F) = 9$. We know that $e(H_3) = 5$, then F must satisfy $2 \leq \delta(F) \leq \Delta(F) \leq 4$, because if there is one vertex v in F with $d(v) \geq 5$, then $e(F - v) \leq 4$ hence H_3 will not be found. We see that H_3 contains two vertices of degree 3, then F must contain at least three vertices of degree 3. Figure 6 shows all F satisfying this condition with $(2K_2, H_3)$-good coloring. From Lemma 1, we have $F \not\to (2K_2, H_3)$ and $r^*(2K_2, H_3) \geq 10$. ■

Theorem 5: $r^*(2K_2, H_4) = 9$.

Proof: According to Theorem 1, let F a graph with $v(F) = r(2K_2, H_4) = 7$ and $e(F) = 9$ as shown in Figure 7. We can see that all vertices in F is of degree 2 and 3

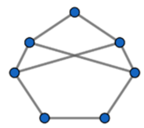

Fig. 7. Graph F with $v(F) = 7$ and $e(F) = 9$ sehingga $F \to (2K_2, H_4)$.

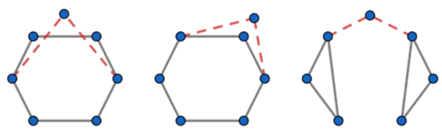

Fig. 8. $(2K_2, H_{14})$-good coloring of F.

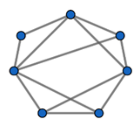

Fig. 9. Graph F with $v(F) = 7$ and $e(F) = 12$ such that $F \to (2K_2, H)$.

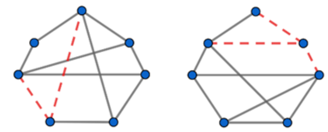

Fig. 10. $(2K_2, H)$-good coloring of F.

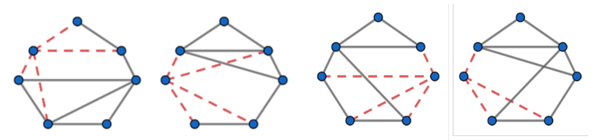

Fig. 11. $(2K_2, H_7)$-good coloring of F.

and F does not contain a triangle C_3. For a vertex v_1 in F with $d(v_1) = 3$, $H_4 \subseteq F - v_1$ and for a vertex v_2 in F with $d(v_2) = 2$, $H_4 \subseteq F - v_2$. So we have, $H_4 \subseteq F - v$ for all vertex v in F. Then, by Lemma 1, $F \to (2K_2, H_4)$ and $r^*(2K_2, H_4) \leq 9$.

Next, we will show the lower bound for $r^*(2K_2, H_4)$. Let F all graphs with $v(F) = 7$, $e(F) = 8$, and $2 \leq \delta(F) \leq \Delta(F) \leq 3$ because $e(H_4) = 5$ and if there is a vertex v in F with $d(v) \geq 4$, then $e(F - v) \leq 4$ hence $H_4 \not\subseteq F - v$. Since H_4 contains one vertex of degree 3, then there must be at least two vertices of degree 3 in F that are not adjacent. In Figure 8 we have all graphs F that satisfy this condition with $(2K_2, H_4)$-good coloring. Thus, according to Lemma 1 we have, $F \not\to (2K_2, H_4)$ and $r^*(2K_2, H_4) \geq 10$. ■

Now, we will give the restricted size Ramsey numbers for a pair of $2K_2$ with H_5, H_6, H_7, H_8, H_9, and H_{10} which are triangle unicyclic graphs of order six. First, we give Lemma 4 that shows the upper bound for $r^*(2K_2, H)$ where H are graphs H_5, H_6, and H_7.

Lemma 4: Let H be graph H_5, H_6, and H_7, then we have $r^*(2K_2, H) \leq 12$.

Proof: Graph H is of order six and based on Theorem 1, we choose F with $v(F) = 7$ and $e(F) = 12$ as shown in Figure 9. The vertices on graph F are of degree 2, 3, 4, and 5. For vertices v_1, v_2, v_3 in F with $d(v_1) = 5$, $d(v_2) = 3$, and $d(v_3) = 2$, we have $F - v_1 \subseteq F - v_2 \subseteq F - v_3$ and $H \subseteq F - v_1$, so it is clear that $H \subseteq F - v_1 \subseteq F - v_2 \subseteq F - v_3$. We can see that for vertex v_4 in F with $d(v_4) = 4$, $H \subseteq F - v_4$. Therefore, we have $H \subseteq F - v$, for all v in F.

Next, notice all triangles C_3 in F. Graph F contains five non-isomorphic C_3, namely C_3 induced by vertices of degree 2, 4, and 5 (denoted by C_3^1), C_3 induced by one vertex of degree 3 and two vertices of degree 4 (denoted by C_3^2), C_3 induced by two vertices of degree 3 and one vertex of degree 4 (denoted by C_3^3), C_3 induced by vertices of degree 3, 4, and 5 (denoted by C_3^4), and C_3 induced by two vertices of degree 3 and one vertex of degree 5 (denoted by C_3^5). We can see that $H \subseteq F - C_3^1$, $H \subseteq F - C_3^2$, $H \subseteq F - C_3^3$, $H \subseteq F - C_3^4$, and $H \subseteq F - C_3^5$. Therefore we have $H \subseteq F - C_3$, for all C_3 in F. Based on Lemma 1, since $H \subseteq F - v$, for all v in F and $H \subseteq F - C_3$, for all C_3 in F, then we have $F \to (2K_2, H)$ and $r^*(2K_2, H) \leq 12$. ■

Theorem 6: For H graph H_5, H_6, $r^*(2K_2, H) = 12$.

Proof: Based on Lemma 4, we have the upper bound for $2K_2$ versus H_5 and H_6, that is $r^*(2K_2, H) \leq 12$. Now we will show the lower bound. Let F all graphs with $v(F) = 7$, $e(F) = 11$, and by Lemma 2, graph F satisfies $\delta(F) \geq 2$. The graph H has one vertex of degree 4, then F must contain at least two vertices of degree 4 that are not adjacent. By Lemma 3, since H consist of one triangle C_3, then F must have at least two triangles C_3 that do not have the same vertex. In Figure 10 we give all graphs F that satisfy this condition with $(2K_2, H)$-good coloring. We can see that there is one vertex v on each F such that $H \not\subseteq F - v$, for all F. Thus, according to Lemma 1, it is found that $F \not\to (2K_2, H)$ and we have the lower bound $r^*(2K_2, H) \geq 12$. ■

Theorem 7: $r^*(2K_2, H_7) = 12$.

Proof: From Lemma 4 we have the upper bound $r^*(2K_2, H_7) \leq 12$. Next, we will show the lower bound. Let F be all graphs with $v(F) = 7$ and $e(F) = 11$. Since $e(H_7) = 6$ and according to Lemma 2, then $2 \leq \delta(F) \leq \Delta(F) \leq 5$. Graph H_7 contains one triangle C_3, then by Lemma 3, graph F must have at least two triangles C_3 which do not share a vertex. Graph H_7 has two vertices of degree 3, then graph F must have at least three vertices of degree 3. Figure 11 shows all graphs F that satisfy this condition with $(2K_2, H_7)$-good coloring. We can see that for every F, there exists one vertex v such that $H_7 \not\subseteq F - v$. Then, according to Lemma 1, we have $F \not\to (2K_2, H)$. Therefore, $r^*(2K_2, H_7) \geq 12$. ■

Theorem 8: $r^*(2K_2, H_8) = 12$.

Proof: From Theorem 1 we know that $r(2K_2, H_8) = 7$. Let F be a graph with $v(F) = 7$ and $e(F) = 12$ as shown in Figure 12. We can see that all vertices in F are of degree 2, 3, 4, and 5 and for vertices v_1, v_2, v_3 in F with $d(v_1) = 5$, $d(v_2) = 3$, and $d(v_3) = 2$, $H_8 \subseteq F - v_1 \subseteq F - v_2 \subseteq F - v_3$. Then, for vertex v_4 in F with $d(v_4) = 4$, $H_8 \subseteq F - v_4$. Therefore, we have $H_8 \subseteq F - v$, for every v in F.

Observe that the graph F contains four non-isomorphic triangles C_3, namely C_3 induced by two vertices of degree

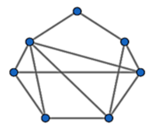

Fig. 12. Graph F with $v(F) = 7$ and $e(F) = 12$ such that $F \rightarrow (2K_2, H_18)$.

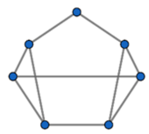

Fig. 13. Graph F with $v(F) = 7$ and $e(F) = 10$ such that $F \rightarrow (2K_2, H_9)$.

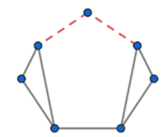

Fig. 14. $(2K_2, H_9)$-good coloring of F.

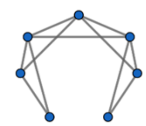

Fig. 15. Graph F with $v(F) = 7$ and $e(F) = 11$ such that $F \rightarrow (2K_2, H_10)$.

3 and one vertex of degree 5 (denoted by C_3^1), C_3 induced by one vertex of degree 3 and two vertices of degree 4 (denoted by C_3^2), C_3 induced by vertices of degree 3, 4, and 5 (denoted by C_3^3), and C_3 induced by two vertices of degree 4 and one vertex of degree 5 (denoted by C_3^4). We can see that $H_8 \subseteq F - C_3^1$, $H_8 \subseteq F - C_3^2$, $H_8 \subseteq F - C_3^3$, and $H_8 \subseteq F - C_3^4$. In other words, $H_8 \subseteq F - C_3$ for every C_3 in F. Based on Lemma 1, it is shown that $F \rightarrow (2K_2, H_8)$. Therefore, $r^*(2K_2, H_8) \leq 12$.

To show the lower bound, let F be all graphs with $v(F) = 7$, $e(F) = 11$ and by Lemma 2, $\delta(F) \geq 2$. Observe that the graph H_8 has size 6, then F must satisfy $2 \leq \delta(F) \leq \Delta(F) \leq 5$. Graph H_8 contains one triangle C_3 and three vertices of degree 3. By Lemma 3, graph F must consist of at least two triangles C_3 which do not share a vertex and at least four vertices of degree 3. Considering there is no graph F satisfying this condition, then we have $r^*(2K_2, H_8) \geq 12$. ■

Theorem 9: $r^*(2K_2, H_9) = 10$.

Proof: First, we will show the upper bound for $r^*(2K_2, H_9)$. By Theorem 1, let F be a graph of order 7 with size 10 as shown in Figure 13. Observe for vertices v_1, v_2 in F with $d(v_1) = 3$ and $d(v_2) = 2$, $H_9 \subseteq F - v_1$ and $H_9 \subseteq F - v_2$. The graph F contains two isomorphic triangles C_3, namely C_3 induced by three vertices of degree 3. We have that $H_9 \subseteq F - C_3$. Based on Lemma 1, since we have $H_9 \subseteq F - v$ for every v in F and $H_9 \subseteq F - C_3$ for every C_3 in F, then $F \rightarrow (2K_2, H_9)$. Therefore, $r^*(2K_2, H_9) \leq 10$.

Next, we will show the lower bound of $r^*(2K_2, H_9)$. Let F be a graph with $v(F) = 7$ and $e(F) = 9$. According to Lemma 2, the graph F must satisfy $\delta(F) \geq 2$. Observe that H_9 has size 6 then the F must have maximum degree of 3, or $\Delta(F) \leq 3$ because if there exists one vertex v in F with $d(v) \geq 4$, then $e(F - v) \leq 5$ so H_9 will not be found in $F - v$. Graph H_9 has two vertices of degree 3 and contains one triangle C_3, then F must have at least three vertices of degree 3 and, based on Lemma 3, contains at least two triangles C_3 that do not share the same vertex. We have the graph F as shown in Figure 14. We can see that there is one vertex v in F such that $H_9 \nsubseteq F - v$. According to Lemma 1, $F \nrightarrow (2K_2, H_9)$. Therefore, $r^*(2K_2, H_9) \geq 10$. ■

Theorem 10: $r^*(2K_2, H_10) = 11$.

Proof: To show the upper bound, by Theorem 1, we choose a graph F with $v(F) = 7$. Figure 15 shows the graph F with $v(F) = 7$ and $e(F) = 11$ that satisfy $F \rightarrow (2K_2, H_10)$. Observe that all vertices in F are of degree 2, 3, and 4. For the vertex v_1 in F with $d(v_1) = 4$, $H_{10} \subseteq F - v_1$. For vertices v_2, v_3 in F with $d(v_2) = 3$ and $d(v_3) = 2$, $H_{10} \subseteq F - v_2 \subseteq F - v_3$. So we have that $H_{10} \subseteq F - v$ for every vertex $v \in F$. Moreover, the graph F contains three non-isomorphic triangles C_3, namely C_3 induced by three vertices of degree 4 (denoted by C_3^1), C_3 induced by two vertices of degree 4 and one vertex of degree 3 (denoted by C_3^2), and C_3 induced by vertices of degree 2, 3, and 4 (denoted by C_3^3). We can show that $H_{10} \subseteq F - C_3^1$, $H_{10} \subseteq F - C_3^2$, and $H_{10} \subseteq F - C_3^3$. Then, we have $H_{10} \subseteq F - C_3$ for every $C_3 \in F$. Thus, by Lemma 1, $F \rightarrow (2K_2, H_10)$ and $r^*(2K_2, H_10) \leq 11$.

To show the lower bound, let F be all graphs with $v(F) = 7$ and $e(F) = 10$. Graph H_{10} has size 6, then F must satisfy $\Delta(F) \leq 4$ and by Lemma 2, $\delta(F) \geq 2$, or $2 \leq \delta(F) \leq \Delta(F) \leq 4$. Observe that graph H_{10} has one vertex of degree 3 and contains one triangle C_3. Then F must have at least two vertices of degree 3 that do not adjacent and based on Lemma 3 contains at least two triangles C_3 that do not share a vertex. Since there is no graph F with $v(F) = 7$ and $e(F) = 10$ that satisfy the conditions above, we have that $r^*(2K_2, H_10) \geq 11$. ■

IV. Conclusion

Out of 112 non-isomorphic connected graphs of order six, there are 10 tree and triangle unicyclic graphs. In this paper we give the restricted size Ramsey numbers for $2K_2$ versus these 10 graphs of order six. For further research, the restricted size Ramsey numbers for $2K_2$ versus the remaining non-isomorphic connected graphs of order six that have not yet been found in [5] can be investigated.

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