Forecasting of Indonesian Crude Prices using ARIMA and Hybrid TSR-ARIMA

Etik Zukhronah, Winita Sulandari, Sri Subanti, Isnandar Slamet, Sugiyanto, and Irwan Susanto

***Abstract*—Forecasting of Indonesian crude prices (ICP) is crucial for the government and policymakers. It helps them develop appropriate economic policies, budget allocations, and energy strategies. Forecasting methods that can be used are Time Series Regression (TSR) and Autoregressive Integrated Moving Average (ARIMA). This study aims to forecast ICP using ARIMA and hybrid TSR-ARIMA models. The data used in this study is the ICP per month, from January 2017 to November 2022. The data is divided into two groups, the data from January 2017 to December 2020 is used as training data, and the data from January 2021 to November 2022 is used as testing data. The MAPE values for the testing data of the TSR-ARIMA(2,1,0) and ARIMA(2,1,0) models are 8.24% and 17.37% respectively. Based on this, it can be concluded that the TSR-ARIMA(2,1,0) model is better than the ARIMA(2,1,0) model for forecasting ICP.**

Keywords: ARIMA, hybrid TSR-ARIMA, ICP.

I. Introduction

FORECASTING Indonesian crude prices (ICP) is importance for the government and policymakers as it allows them to formulate effective economic policies, allocate budgets appropriately, and devise energy strategies. By having reliable price forecasts, the government can estimate the revenue generated from oil exports, establish suitable tax rates, and evaluate the impact of oil price fluctuations on the overall economy. Moreover, accurate forecasts play a crucial role in planning and executing energy-related infrastructure projects. Techniques used for modeling ICP include ARIMA, and hybrid of time series regression with ARIMA models.

ARIMA models are a popular and powerful tool for forecasting time series data, such as sales, prices, or weather. ARIMA stands for AutoRegressive Integrated Moving Average, and it captures the patterns, trends, and seasonality of the data using a combination of past values, differences, and errors. ARIMA is a model that combines autoregressive (AR), differencing (I), and moving average (MA) components to capture the inherent patterns and dependencies within a time series [1]. The AR component represents the linear dependence between the current observation and its past values, while the MA component models the relationship between the current observation and the residual errors from previous observations. The I component deals with differencing operations to achieve stationarity. Many researchers have worked in their region using ARIMA techniques [2] [3] [4] [5].

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Time series regression is a statistical modeling technique used to analyze and forecast future values of a dependent variable based on its historical patterns and the relationship with one or more independent variables. It is particularly useful in analyzing data that is collected over time and exhibits a sequential dependence. Time series regression models incorporate both the time component and the regression component, allowing for the identification of trends, seasonality, and other underlying patterns in the data. A hybrid time series regression and ARIMA models called TSR-ARIMA can improve the forecasting accuracy [6] [7] [8]. ARIMA models, on the other hand, typically only consider the historical values of the time series itself. Hybrid models can integrate regression techniques to handle non-stationary components and capture the underlying trends and patterns effectively. Hybrid models can reduce forecasting errors compared to models that work independently [9] [10] [11]. Especially in [7] [8], hybrid TSR-ARIMA model demonstrated excellent performance by yielding small errors. The data patterns in [7] [8] are similar to the data patterns used in this paper for ICP. Based on this, we utilize the hybrid TSR-ARIMA model for forecasting ICP data. Previous research has utilized ICP data with different models [12] [13], not employing the hybrid TSR-ARIMA model. As a comparison to the hybrid TSR-ARIMA model, this paper uses the ARIMA model.

II. Methodology

This paper discusses an ARIMA and hybrid TSR-ARIMA used for predicting the ICP. The ICP is taken from website https://databoks.katadata.co.id/datapublish/2022 /04/08 /tertinggi-sejak-2013-icp-maret-2022-capai-us1135-per-bareldataboks.katadata.co.id/1 and https://databoks.katadata.co.id/datapublish/2022/12/05/harga-minyak-indonesia-turun-ke-us875-per-barel-pada-november-2022/2. The data is divided into two groups: one data group from 2017–2020 as training data and the second group from 2021–November 2022 as testing data.

The steps for ARIMA model are as follows:

a) Plot the training data of ICP to assess the stationarity of the data. If the data is not stationary, then differencing may be required.
b) Plot autocorrelation function (ACF) and partial autocorrelation function (PACF).
c) Define the order of AR and MA models by using ACF and PACF plots.

d) Test the significance of the parameters in the ARIMA model.
e) Check the diagnostics of the ARIMA model, i.e., whether the residuals follow a normal distribution using the Kolmogorov-Smirnov test and whether they exhibit a white noise process using the Ljung-Box test.
f) Calculate the error of each model using Mean Absolute Percentage Error (MAPE) and choose the model with the lowest MAPE.

The steps for hybrid TSR-ARIMA model are as follows:

a) Modeling the training data of ICP using time series regression and calculate the residue.
b) Modeling the residue of TSR model using the steps of ARIMA model a-e.
c) The forecast of the hybrid TSR-ARIMA model is obtained by summing the forecasts from the TSR model and the ARIMA model.
d) Calculate the MAPE of each model and choose the model with the lowest MAPE.

Calculating the MAPE values for testing data of ARIMA and hybrid TSR-ARIMA models. A good model is a model that has a smaller MAPE value.

A. ARIMA Model

ARIMA models are a type of linear models that can effectively capture both stationary and nonstationary time series. Autoregressive models, on the other hand, are a specific category within ARIMA models, primarily used for modeling stationary time series. Unlike other models, ARIMA models do not incorporate independent variables during their construction. Instead, they heavily rely on the autocorrelation patterns present in the data. The ARIMA(p,d,q) model, as described by Wei [14], can be expressed as follows

$$\phi_p(B)(1-B)^d Z_t = \theta_q(B)\varepsilon_t \tag{1}$$

where ϕ_p is a Moving Average parameter of order p, θ_q is an Autoregressive parameter of order q, B is a backshift operator, d is an order of differencing, Z_t is an actual data at time t, and ε_t is an error at time t that assumed to be normally distributed and independent with mean 0 and variance σ_ε^2. The order of an ARMA(p,q) model can be determined by examining the autocorrelation and partial autocorrelation patterns of the autoregressive-moving average model. Hanke and Wichern [15] provide a summary of these patterns in TableACFand-PACF, which can be used to identify the appropriate values of p and q for the model.

TABLE I: The characteristics of ACF and PACF

Model	ACF	PACF
MA(q)	Cut off after the order q of the process	Die out
AR(p)	Die out	Cut off after the order p of the process
ARMA(p,q)	Die out	Die out

The parameters ϕ_p and θ_q of the ARIMA (p,d,q) model, as described in eq1, are estimated using the Least Squares method. It is important to ensure that the residuals of the ARIMA model follow a normal distribution and exhibit a white noise process. To test the normality distribution of the residuals, the Kolmogorov-Smirnov test can be utilized [16].

The null hypothesis assumes that the residuals are normally distributed. The null hypothesis is rejected if the test statistic D, which represents the maximum absolute difference between the empirical cumulative distribution function $(S(x))$ of the sample residuals and the cumulative distribution function of a normal distribution $(F_0(x))$, exceeds a critical value $D_{\alpha,n}$. The critical value is determined based on the desired significance level α and the sample size (n). If the test statistic D exceeds the critical value $D_{\alpha,n}$, the null hypothesis of normality is rejected, indicating that the residuals do not follow a normal distribution.
The Ljung-Box Q-statistics is a method used to assess whether the residuals from an ARIMA (p,d,q) model exhibit characteristics of a white noise process [17] [18]. Ljung-Box Q-statistic, given by:

$$Q = n(n+2)\sum_{n=1}^{K}\frac{r_k^2}{n-k},$$

where n represents the number of data, K is the degrees of freedom representing the maximum lags considered, and r_k is the sample autocorrelation function at lag k.

Under the null hypothesis that all autocorrelation values (r_k) are equal to zero, the Q-statistic is compared to critical values derived from the chi-square distribution with K degrees of freedom. The degrees of freedom correspond to the maximum lags considered (K). If the model is correctly specified, the residuals should be uncorrelated, resulting in a small Q value and a large probability value. If the calculated Q value exceeds the critical value from the chi-square distribution, the null hypothesis of uncorrelated residuals can be rejected.

The MAPE is a metric used to assess the accuracy of forecasts. A lower MAPE value indicates a more accurate forecast. In general, the MAPE can be calculated using the following formula:

$$MAPE = \frac{\sum_{t=1}^{n}\left|\frac{Y_t-\hat{Y}_t}{Y_t}\right|}{n}\times 100\%.$$

B. Hybrid TSR-ARIMA Model

The time series regression model, according to Bowerman and O'Connell [19], can be expressed by eq2

$$L_t = T_t + S_t + \varepsilon_t \tag{2}$$

where L_t is the response variable at time t, T_t is the trend component at time t, S_t is the seasonal component at time t, ε_t is the error at time t assumed to be normally distributed and independent with mean 0 and variance of σ_ε^2. If the data contains a linear trend, eq2 can be written as follows:

$$L_t = T_t + \varepsilon_t$$

The hybrid TSR-ARIMA model combines the TSR and ARIMA models to forecast data. Zhang [1] presents the hybrid model formulation as follows:

$$y_t = L_t + Z_t$$

where y_t represents observation at time t, L_t represents the TSR component, Z_t represents the ARIMA component. At first, modelling the data using TSR and the corresponding forecast $\hat{L}_t$ at time t is obtained. Then, the residual at time t is given by $e_t = y_t - \hat{L}_t$. Next, modelling the residual using ARIMA model and the corresponding forecast $\hat{Z}_t$ at time t is obtained. The forecasting using the hybrid TSR-ARIMA model can be expressed as:

$$\hat{y}_t = \hat{L}_t + \hat{Z}_t.$$

III. Result and Discussion

The plot of training data for ICP from January 2017 to December 2020 is shown in figure1 figure1 shows that the training data follows an upward trend. In January 2020, a decline begins, which is associated with the outbreak of the COVID-19 pandemic. Starting from May 2020, there was an increase.

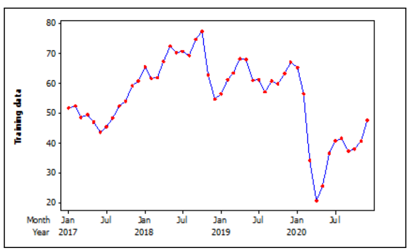

Fig. 1: Plot of training data

A. Modeling of ARIMA

The ARIMA modeling begins with testing the stationarity of the data. figure1 shows that the data exhibits an upward trend, which is further supported by the ACF plot in figure2.

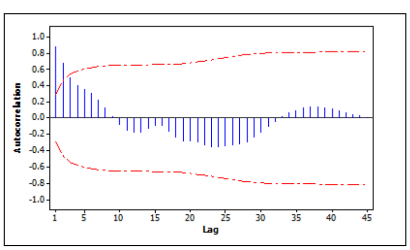

Fig. 2: Plot ACF of training data

Next, differencing of order 1 is performed, and the plot of differenced data is shown in figure3, while its corresponding ACF plot is displayed in figure4 and PACF plot is shown in figure5.

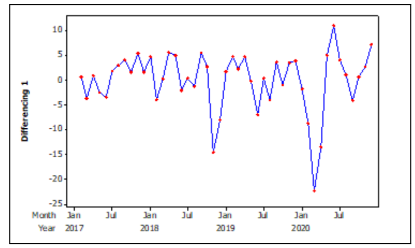

Fig. 3: Plot diff 1 of training data

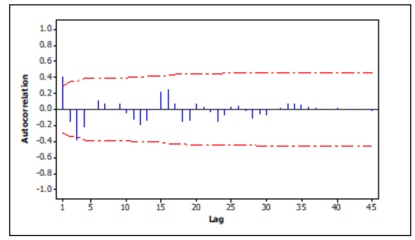

Fig. 4: Plot ACF of diff 1

figure3 shows that the data has become stationary. This is further supported by the ACF plot in figure4, which indicates a stationary pattern. figure4 shows that lag one is outside the confidence interval, suggesting an order of $q = 1$. Additionally, figure5 indicates that lag one and lag two are outside the confidence interval, indicating a possible order of $p = 2$. Based on these observations, the possible ARIMA models are ARIMA(0,1,1), ARIMA(1,1,0), ARIMA(1,1,1), ARIMA(2,1,0), and ARIMA(2,1,1). Next, parameter estimation for these models is performed, and the results are presented in Table2. Table2 shows that parameter estimation of ARIMA(0,1,1), ARIMA(1,1,0) and ARIMA(2,1,0) are significant because all parameters have p-value less than $\alpha = 0.05$. Meanwhile, ARIMA(1,1,1) and ARIMA(2,1,1) are

TABLE II: Estimation of parameters

Model	Parameters	Coeficient	T-value	p-value
ARIMA(0,1,1)	MA 1	-0.505	-3.88	0.000
ARIMA(1,1,0)	AR 1	0.423	3.11	0.003
ARIMA(1,1,1)	AR 1	0.151	0.53	0.599
	MA 1	-0.407	-1.55	0.129
ARIMA(2,1,0)	AR 1	0.589	4.25	0.000
	AR 2	-0.401	-2.89	0.006
ARIMA(2,1,1)	AR 1	-0.562	-3.87	0.000
	AR 2	0.275	1.90	0.064
	MA 1	-1.022	-293.24	0.000

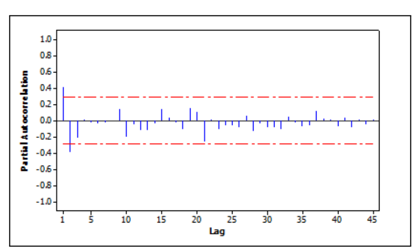

Fig. 5: Plot PACF of diff 1

not significant because the parameter has p-value more than $\alpha = 0.05$. The ARIMA model's estimated parameter values are significant, subsequently, a goodness-of-fit test is conducted to determine whether the model's residuals follow a normal distribution and white noise using Kolmogorov-Smirnov and Ljung-Box tests. The result showed that the residual of three models are normally distributed and white noise. Next, the MAPE values for the training data are calculated for the three models, and the results are presented in table3.

TABLE III: MAPE value for training data of ARIMA

Model	MAPE
ARIMA(0,1,1)	8.18
ARIMA(1,1,0)	8.55
ARIMA(2,1,0)	7.52

table3 indicates that the ARIMA(2,1,0) model has the smallest MAPE value, thus leading to the conclusion that it is the best model. The model of ARIMA(2,1,0) can be expressed as follows

$$\hat{z}_t = 1.589z_{t-1} - 0.99z_{t-2} + 0.401z_{t-3} \quad (3)$$

B. 3.2 Modeling of hybrid TSR-ARIMA

The hybrid TSR-ARIMA modeling begins with conducting a time series regression analysis. figure1 shows that the data exhibits a trend pattern, thus a dummy variable in the form of time (t) is included. The results of the time series regression analysis are presented in eq4.

$$\hat{L}_t = 62.34 - 0.285t \quad (4)$$

Next, the residuals of the TSR model are modeled using an ARIMA model. The time series plot of the TSR residuals is shown in figure6, while the plot of its ACF is presented in figure7.

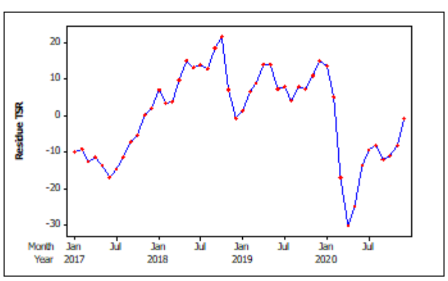

Fig. 6: Plot time series of residue TSR

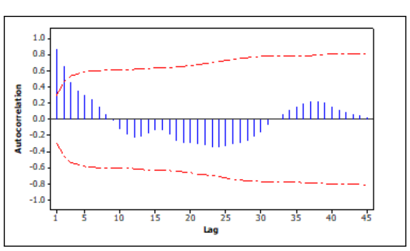

Fig. 7: Plot ACF of residue TSR

figure6 and figure7 show that the pattern of residue TSR is a trend. Based on that, it is necessary to perform differencing. The ACF and PACF plots of the differenced residue are shown in figure8 and figure9, respectively.

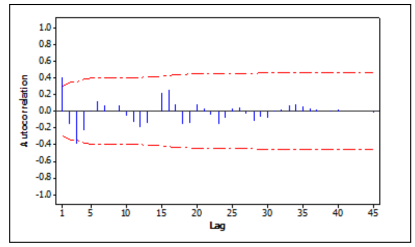

Fig. 8: Plot ACF of diff. residue TSR

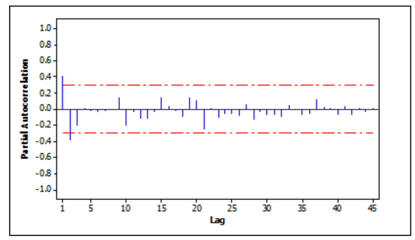

Fig. 9: Plot PACF of diff. residue TSR

Based on the figure8 and figure9, the possible model for hybrid TSR-ARIMA is TSR-ARIMA(0,1,1), TSR-ARIMA(1,1,0), TSR-ARIMA(1,1,1), TSR-ARIMA(2,1,0) and TSR-ARIMA(2,1,1). Next, a significance test is conducted on the five models, and the results show that the TSR-ARIMA(1,1,1) model's parameters are not significant. The Kolmogorov-Smirnov and Ljung-Box tests of the residuals for the significant parameter models showed that the residuals are normally distributed and white noise. The MAPE value for training data of four TSR-ARIMA models are presented in table4.

TABLE IV: MAPE value for training data of hybrid TSR-ARIMA

Model	MAPE
ARIMA(0,1,1)	3.89
ARIMA(1,1,0)	4.54
ARIMA(2,1,0)	3.03
ARIMA(2,1,1)	10.26

table4 indicates that the hybrid TSR-ARIMA(2,1,0) model has the smallest MAPE value, it means that it is the best model. Next, the MAPE values for the testing data are calculated for the ARIMA(2,1,0) and the hybrid TSR-ARIMA(2,1,0) models, and the results are presented in table5. Plot of testing data, forecasting data of ARIMA and hybrid TSR-ARIMA can be found in figure10.

The MAPE value for the ARIMA(2,1,0) model in table5 is 17.37. This indicates that the forecasting performance of the ARIMA(2,1,0) model is not very good. As shown in figure10, the forecasted values from the ARIMA(2,1,0) model are significantly below the testing data, resulting in a large MAPE value.

TABLE V: MAPE value for testing data

Model	MAPE
ARIMA(2,1,0)	17.37
TSR-ARIMA(2,1,0)	8.24

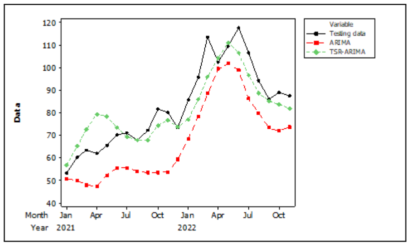

Fig. 10: Comparison of testing data and forecasting

The TSR-ARIMA(2,1,0) model has MAPE value less than ARIMA(2,1,0) model. figure10 shows that forecasting data of hybrid TSR-ARIMA(2,1,0) model is closer to the testing data compared to the forecasting data of ARIMA(2,1,0) model. This means that the TSR-ARIMA(2,1,0) model is better than ARIMA(2,1,0) model for forecasting ICP. The hybrid TSR-ARIMA(2,1,0) forecasting model is the summation of the TSR model forecast in eq3 and the ARIMA model forecast in eq4, the result of which is as follows

$$\hat{y}_t = 62.34 - 0.285t + 1.589y_{t-1} - 0.99y_{t-2} + 0.401y_{t-3}.$$

IV. Conclusion

The analysis of ICP was conducted using the ARIMA and hybrid TSR-ARIMA models. Based on the behavior of the ACF and PACF plots of the training data, the ARIMA(2,1,0) model yielded the smallest MAPE. Similarly, for the hybrid TSR-ARIMA model, the hybrid TSR-ARIMA(2,1,0) model had the smallest MAPE. The residuals of both models followed a normal distribution and exhibited white noise characteristics. When applied to the testing data, the hybrid TSR-ARIMA(2,1,0) model outperformed the ARIMA(2,1,0) model, resulting in a smaller MAPE. Therefore, the hybrid TSR-ARIMA(2,1,0) model is more suitable for forecasting ICP.

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References

[1] G. Zhang, "Time series forecasting using a hybrid arima and neural network model," *Neurocomputing*, vol. 50, pp. 159–175, 2003.

[2] S. Jere and E. Moyo, "Modeling epidemiological data using box-jenkins procedure," *Open Journal of Statistics*, vol. 6, pp. 295–302, 2016.

[3] Shivani, K. Sandhu, and A. Nair, "A comparative study of arima and rnn for short term wind speed forecasting," in *in Proceedings of 10th International Conference on Computing, Communication and Networking Technologies (ICCCNT)*, (Kanpur, India), IEEE, 2019.

[4] B. Siregar, E. Nababan, A. Yap, U. Andayani, and Fahmi, "Forecasting of raw material needed for plastic products based in income data using arima method," in *In Proceedings of 5th International Conference on Electrical, Electronics and Information Engineering (ICEEIE)*, IEEE, 2017.

[5] A. Rahman and M. Hasan, "Modeling and forecasting of carbon dioxide emissions in bangladesh using autoregressive integrated moving average (arima) models," *Open Journal of Statistics*, vol. 7, pp. 560–566, 2017.

[6] M. Arumsari and A. Dani, "Peramalan data runtun waktu menggunakan model hybrid time series regression – autoregressive integrated moving average," *Jurnal Siger Matematika*, vol. 2, no. 1, 2021.

[7] K. Ramadani, S. Wahyuningsih, and M. Hayati, "Peramalan harga saham pt. telkom menggunakan model hybrid time series regression linier – autoregressive integrated moving average," *Jurnal Matematika, Statistika komputasi*, vol. 18, no. 2, pp. 293–307, 2022.

[8] S. Pradhina, E. Zukhronah, and Y. Susanti, "Perbandingan akurasi peramalan wisatawan mancanegara di provinsi bali menggunakan model hybrid time series regression-autoregressive integrated moving average dan model autoregressive integrated moving average," *Prosiding Pendidikan Matematika, Matematika dan Statistika*, vol. 7, 2023.

[9] W. Sulandari, Subanar, Suhartono, and H. Utami, "Forecasting electricity load demand using hybrid exponential smoothing-artificial neural network model," *International Journal of Advances in Intelligent Informatics*, vol. 2, no. 3, pp. 131–139, 2016.

[10] W. Sulandari, Subanar, Suhartono, H. Utami, and M. Lee, "Ssa-based hybrid forecasting models and applications," *Bulletin of Electrical Engineering and Informatics*, vol. 9, no. 5, pp. 2178–2188, 2020.

[11] I. Khandelwal, R. Adhikari, and G. Verma, "Time series forecasting using hybrid arima and ann models based on dwt decomposition," *Procedia Computer Science*, vol. 48, pp. 173–179, 2015.

[12] D. Suryani, M. Fadhilla, and A. Labellapansa, "Indonesian crude oil price (icp) prediction using multiple linear regression algorithm," *Jurnal RESTI (Rekayasa Sistem dan Teknologi Informasi)*, vol. 6, no. 6, pp. 1057–1063, 2022.

[13] Sukono, E. Suryamah, and F. Novinta, "Application of arima issn: 2527-3426 -garch model for prediction of indonesian crude oil prices," *Operations Research: International Conference Series*, vol. 1, no. 1, pp. 25–32, 2020.

[14] W. Wei, "Time series analysis: Univariate and multivariate methods, 2^{nd} ed.," *Pearson Education, Inc.*, 2006.

[15] J. Hanke and D. Wichern, "Business forecasting, 9^{th} ed," *Pearson Prentice Hall*, 2009.

[16] Z. Drezner, O. Turel, and D. Zerom, "A modified kolmogorov–smirnov test for normality," *Communications in Statistics - Simulation and Computation*, vol. 39, no. 4, pp. 693–704, 2010.

[17] W. Enders, "Applied econometric time series, 3^{rd} ed.," *John Wiley Sons*, 2010.

[18] G. Ljung and G. Box, "On a measure of lack of fit in time series models," *Biometrika*, vol. 65, no. 2, pp. 297–303, 1978.

[19] B. Bowerman and D. O'Connel, "Forecasting and time series: An applied approach, 3^{rd} ed.," *Duxbury Press*, 1993.