Optimal Control of the Spread of Dengue Fever by Controlling the Vectors Growth Affected by Climate Change and Treatment

Basuki Widodo, Nur Asiyah, Aulia Rahma, Kamiran, and Chairul Imron

***Abstract*—Dengue Hemorrhagic Fever (DHF) is an infectious disease caused by the dengue virus and is spread through the bite of an adult female *Aedes aegypti* mosquito, as a vector (disease-carrying animal), to humans. This disease is still a major health problem in tropical and subtropical regions. Indonesia is reported as the 2nd highest country among 30 other endemic countries. Warm temperatures during the rainy season are ideal conditions for mosquitoes to lay eggs optimally, increasing egg maturity, and shortening the virus incubation period. This has an impact on increasing the number of mosquitoes and the risk of disease transmission. In this study, control of DHF was carried out by controlling the growth of vectors in the egg and adult phases of mosquitoes, which were influenced by rainfall and air temperature, as well as the treatment of infected humans. Before carrying out the control, stability analysis around the equilibrium point is first conducted. Next, the numerical solution is obtained using the Runge-Kutta method of order 4 with the help of MATLAB software. The results of the analysis show that, based on the optimal control effect in the form of mosquito egg death (k_1), adult mosquito death (k_2), and human treatment (k_3), in the cities of Pekanbaru and Solok, there is not much difference between the two. However, there is a slight difference in the increase in the human population that is susceptible to disease.**

***Index Terms*—Climate Change, Dengue Fever, Optimal Control, Treatment, Vector Growth.**

I. Introduction

DENGUE Hemorrhagic Fever (DHF) is an infectious disease caused by a virus and spread by vectors (disease-carrying animals). Dengue fever is transmitted through the bite of an adult female *Aedes aegypti* mosquito infected with the dengue virus to humans. The habitat of the *Aedes aegypti* mosquito is generally in areas with high rainfall, hot temperatures, and humidity, which means that this disease is still a major health problem in tropical and subtropical regions. Indonesia is reported as the 2nd highest country among 30 other endemic countries [1]. The number of DHF cases in Indonesia fluctuates every year. DHF cases in Indonesia in 2020 recorded 108,303 cases and 747 deaths. In 2021 there were 73,518 cases of DHF with a total of 705 deaths, while in 2022 there were 131,265 dengue cases and 1,183 deaths [2].

The main factors triggering the rapid transmission or spread of DHF are climate (such as rainfall, temperature, and humidity), population density, and public awareness in maintaining environmental sanitation. Another factor is the intrinsic nature of the *Aedes aegypti* mosquito, which is very tough [3]. Extreme temperatures can kill vectors, but warm temperatures can increase their survival. Continuous rainfall causes the availability of Mosquito Breeding Sites (MBS) to increase, as well as warm temperatures during the rainy season, creating ideal conditions for mosquitoes to lay eggs optimally, increase egg maturity, and shorten the virus incubation period. This, in turn, increases the mosquito population and the risk of disease transmission [4].

As a disease that is not transmitted directly from human to human, DHF control can be carried out by breaking the chain of transmission of the virus by reducing the number of *Aedes aegypti* mosquitoes. Eradicating mosquitoes, which are the vectors for the spread of dengue fever, is the primary prevention method to control its spread. Research conducted by [6] shows that when control resources are limited, it is more effective to apply vector control and transmission control than vaccination. One method is administering larvicides to Mosquito Breeding Sites (MBS) to kill mosquito eggs and spraying insecticides in the home environment or areas where mosquitoes breed to reduce the adult mosquito population [5].

Based on the problems above, the research conducted optimal control of DHF by controlling the growth of the vector in the egg and adult phases of mosquitoes, which are affected by rainfall and air temperature. The controls implemented included applying larvicides to MBS to kill mosquito eggs, spraying insecticides in areas where mosquitoes develop, and managing the treatment of infected humans. Before applying the control measures, a stability analysis was performed around the equilibrium point. Furthermore, numerical solutions were computed using the Runge-Kutta method of order 4, with the assistance of MATLAB software.

II. Research Method

A. Study of literature

At this stage a literature study was carried out by identifying problems from the topics, namely regarding the spread of dengue fever, the effect of climate change on mosquito growth, the theory of optimal control with its completion using the Pontryagin Minimum Principle and the theory of numerical simulation using the Runge-Kutta Order 4 method. The studies used at this stage are from a number of books, research journals, articles, research, and another research.

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B. Model Formulation

At this stage, the development of a model of the spread of DHF disease is carried out, analysis of stability around the equilibrium point, linearization, provision of control, and determining the objective function and boundary conditions.

C. Numerical Simulation and Results Analysis

At this stage a numerical simulation was carried out using the Runge-Kutta method of order 4 with the help of Matlab software. Then the analysis of the results is carried out on the simulation results that have been carried out.

D. Conclusions and Suggestions

Furthermore, conclusions are drawn on the results of the discussion, simulation, and analysis that has been carried out and providing criticism and suggestions so that this research can be used as a reference for further research.

III. RESULTS AND DISCUSSION

A. Development of Dengue Fever Spread Model with Optimal Control

Model of the spread of dengue fever by adding the optimal control variable into the compartment diagram as shown in Figure 1. Mathematical Model of Dengue Epidemic

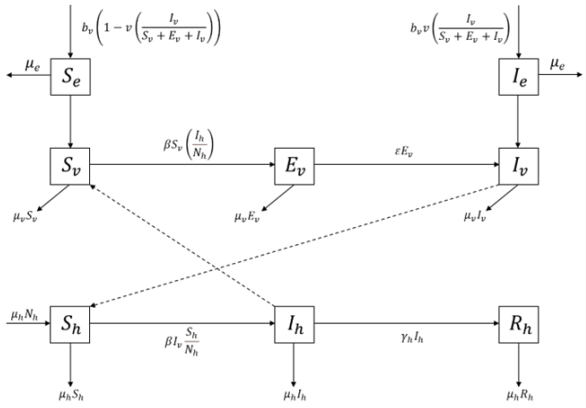

Fig. 1: Compartment Diagram of the Dengue Epidemic Model with the Effects of Climate and Treatment

with Effects of Climate and Treatment, as follow equations, i.e.Equation 1-8, respectively:

$$\frac{dS_e}{dt} = b_v\left(1-\nu\left(\frac{I_v}{S_v+E_v+I_v}\right)\right)-(1+k_1)\mu_e S_e-(1-k_1)\omega S_e,$$

$$\frac{dI_e}{dt} = b_v\nu\left(\frac{I_v}{S_v+E_v+I_v}\right)-(1+k_1)\mu_e I_e-(1-k_1)\omega I_e,$$

$$\frac{dS_v}{dt} = (1-k_1)\omega S_e-\beta\frac{I_h k_3}{N_h}S_v-(\mu_v+k_2)S_v,$$

$$\frac{dE_v}{dt} = \beta\frac{I_h k_3}{N_h}S_v-\varepsilon E_v-(\mu_v+k_2)E_v,$$

$$\frac{dI_v}{dt} = \varepsilon E_v+(1-k_1)\omega I_e-(\mu_v+k_2)I_v,$$

$$\frac{dI_h}{dt} = \beta\frac{S_h}{N_h}I_v-\gamma_h k_3,$$

$$\frac{dS_h}{dt} = \mu_h N_h-\beta\frac{S_h}{N_h}I_v-\mu_h S_h,$$

$$\frac{dR_h}{dt} = \gamma_h k_3-\mu_h R_h.$$

Notation and variable definition of each parameter are as follows.

S_e: Susceptible mosquito eggs,
I_e: Mosquito eggs infected with dengue virus,
S_v: Sub class of susceptible mosquito population,
E_v: Sub class of mosquito population exposed but not yet infected,
I_v: Sub class of mosquito population infected with dengue virus,
S_h: sub class of susceptible human population,
I_h: Sub class of human population infected with dengue virus,
R_h: Subclass of human population that recovered from dengue virus infection,
b_v: Oviposition rate of adult mosquitoes,
ω: Mosquito egg hatching rate,
μ_h: The natural birth or death rate of humans,
μ_e: Mosquito egg death rate,
β: Probability of spread of disease by the bite of an infected mosquito to susceptible humans,
ε: Dengue virus incubation period,
γ: The rate of migration of infected human populations to cured humans,
v: Proportion of vertical infection incidence of adult female mosquitoes to mosquito eggs,
k_1: Percentage of mosquito egg mortality due to insecticides,
k_2: Percentage of adult mosquito mortality due to fogging,
k_3: Percentage of treatment in humans,

B. Equilibrium Point Analysis

To analyze the stability of the dengue fever model, an equilibrium point will be sought in Equations (1) - (8).
In order to obtain a disease-free equilibrium point $E_0\left(S_e^*,I_e^*,S_v^*,E_v^*,I_v^*,S_h^*,I_h^*,R_h^*\right) = \left(\frac{b_v}{(1+k_1)\mu_e-(1-k_1)\omega},0,\frac{(1-k_1)\omega b_v}{(\mu_v+k_2)((1+k_1)\mu_e-(1-k_1)\omega)},0,0,N_h,0,0\right)$ and an endemic equilibrium point $E_1\left(S_e^{**},I_e^{**},S_v^{**},E_v^{**},I_v^{**},S_h^{**},I_h^{**},R_h^{**}\right)$ with,

$$S_e^{**} = \frac{b_v-I_e(A+B)}{A+B}$$

$$I_e^{**} = \frac{b_v v I_v}{(S_v+E_v+I_v)(A+B)}$$

$$S_v^{**} = \frac{BS_e-(\varepsilon+C)E_v}{C}$$

$$E_v^{**} = \frac{\beta(I_h k_3)}{N_h(\varepsilon+C)}S_v$$

$$S_h^{**} = \frac{\mu_h N_h^2}{\beta I_v+\mu_h N_h}$$

$$I_h^{**} = \frac{\mu_h(N_h-S_h)}{\mu_h k_3-\gamma_h k_3}$$

$$R_h^{**} = \frac{\gamma_h I_h k_3}{\mu_h}$$

C. Linearization

The model for the spread of dengue hemorrhagic fever is a non-linear model, so it is necessary to conduct linearization to analyze stability. The linearization of the model for the spread of dengue hemorrhagic fever uses the Taylor series expansion of Equations (1) - (8) to obtain the Jacobian matrix as follows.

$$J = \begin{bmatrix} j_{11} & 0 & j_{13} & 0 & j_{15} & 0 & 0 & 0 \\ 0 & j_{22} & j_{23} & j_{24} & j_{25} & 0 & 0 & 0 \\ j_{31} & 0 & j_{33} & j_{43} & 0 & 0 & j_{37} & 0 \\ 0 & 0 & j_{43} & j_{44} & j_{54} & 0 & j_{47} & 0 \\ 0 & 0 & 0 & \varepsilon & j_{55} & 0 & j_{65} & 0 \\ 0 & 0 & 0 & 0 & 0 & j_{66} & j_{76} & 0 \\ 0 & 0 & j_{37} & 0 & j_{75} & j_{76} & j_{77} & j_{87} \\ 0 & 0 & 0 & 0 & 0 & 0 & j_{87} & j_{88} \end{bmatrix}$$

with,

$$j_{11} = -(1+k_1)\mu_e - (1-k_1)\omega$$
$$j_{13} = \frac{b_v v I_v}{(S_v+E_v+I_v)^2}$$
$$j_{14} = b_v v \left(\frac{I_v}{(S_v+E_v+I_v)^2} \right)$$
$$j_{15} = b_v v \left(\frac{I_v}{(S_v+E_v+I_v)^2} \right)$$
$$j_{22} = -(1+k_1)\mu_e - (1-k_1)\omega$$
$$j_{23} = -b_v v \left(\frac{I_v}{(S_v+E_v+I_v)^2} \right)$$
$$j_{24} = -b_v v \left(\frac{I_v}{(S_v+E_v+I_v)^2} \right)$$
$$j_{25} = -b_v v \left(\frac{I_v}{(S_v+E_v+I_v)^2} \right)$$
$$j_{31} = (1-k_1)\omega$$
$$j_{33} = -\frac{\beta I_h k_3}{N_h} - (\mu_v + k_2)$$
$$j_{37} = -\frac{\beta k_3}{N_h} S_v$$
$$j_{43} = \frac{\beta I_h k_3}{N_h}$$
$$j_{44} = -\varepsilon - (\mu_v + k_2)$$
$$j_{47} = \frac{\beta k_3}{N_h} S_v$$
$$j_{52} = (1-k_1)\omega$$
$$j_{54} = \varepsilon$$
$$j_{55} = -(\mu_v + k_2)$$
$$j_{65} = -\frac{\beta S_h}{N_h}$$
$$j_{66} = -\frac{\beta I_v}{N_h} - \mu_h$$
$$j_{75} = \frac{\beta S_h}{N_h}$$
$$j_{76} = -\mu_h k_3 - \gamma_h k_3$$
$$j_{77} = -\mu_h k_3 - \gamma_h k_3$$
$$j_{87} = \gamma_h k_3$$
$$j_{88} = -\mu_h$$

D. Numerical Simulation and Results Analysis

In this section, the results of numerical simulations of the dengue hemorrhagic fever model are presented using the parameters of the equation to obtain stability, controllability, and comparison of the results of numerical simulations with control and without control.
In Tables 1 and 2 below, the initial values of the variables and parameters used in this numerical simulation are presented.

TABLE I: Initial Value of Model Variables of Dengue Fever Spread

Parameter	Pekanbaru	Solo	Source
b_v	0.3614758369	0.2840196049	[5]
ω	1.7891486	1.31076641	[5]
μ_h	0.0249	0.0199	[5]
μ_e	0.3644277688	0.3274149631	[5]
μ_v	0.006469390419	0.01743452765	[5]
β	0.02076941776	0.01255547746	[5]
ε	0.2955969909	0.3402025690	[5]
γ	0.0007690192	0.001424585	[5]
v	0.028	0.028	[5]
k_1	0.80	0.80	[5]
k_2	0.75	0.75	[5]
k_3	0.90	0.90	[5]

TABLE II: Variables Value of Dengue Model

Variables	Pekanbaru	Solok	Source
S_e	1,000,000	100,000	[5]
I_e	50,000	5,000	[5]
S_v	100,000	7,000	[5]
E_v	5,000	350	[5]
I_v	200	25	[5]
S_h	1,091,083	68,607	[5]
I_h	15	10	[5]
R_h	14	9	[5]

E. Stability Analysis of Disease-Free Equilibrium Points

Stability analysis was carried out by substituting the disease-free equilibrium point into the Jacobian matrix, in order to obtain the following equation:

$$\begin{aligned} &(-(1+k_1)\mu_e - (1-k_1)\omega - \lambda) \\ &(-(1+k_1)\mu_e - (1-k_1)\omega - \lambda) \\ &(-(\mu_v + k_2) - \lambda) \\ &(-\varepsilon - (\mu_v + k_2) - \lambda) \\ &(-(\mu_v + k_2) - \lambda) \\ &(-\mu_h - \lambda) \\ &(-\mu_h k_3 - \gamma_h k_3 - \lambda) \\ &(-\mu_h - \lambda) = 0 \end{aligned}$$

Thus, the eigenvalues are obtained:

$$\lambda_1 = -(1+k_1)\mu_e - (1-k_1)\omega$$
$$\lambda_2 = -(1+k_1)\mu_e - (1-k_1)\omega$$
$$\lambda_3 = -(\mu_v + k_2)$$

$$\lambda_4 = -\varepsilon - (\mu_v + k_2)$$
$$\lambda_5 = -(\mu_v + k_2)$$
$$\lambda_6 = -\mu_h$$
$$\lambda_7 = -\mu_h k_3 - \gamma_h k_3$$
$$\lambda_8 = -\mu_h$$

By substituting the existing parameters, we get:

$$\lambda_1 = -1.01379970384$$
$$\lambda_2 = -1.01379970384$$
$$\lambda_3 = -0.756469390419$$
$$\lambda_4 = -1.052066381319$$
$$\lambda_5 = -0.756469390419$$
$$\lambda_6 = -0.0249$$
$$\lambda_7 = -0.02310211728$$
$$\lambda_8 = -0.0249$$

Therefore, since all eigenvalues are negative, the system is said to be asymptotically stable at the disease-free equilibrium point.

Stability Analysis of Endemic Equilibrium Point

Furthermore, stability analysis is carried out by substituting endemic equilibrium points into the Jacobian matrix, so that the following equation is obtained:

$$\begin{aligned}
&(-(1+k_1)\mu_e - (1-k_1)\omega - \lambda) \\
&\quad (-(1+k_1)\mu_e - (1-k_1)\omega - \lambda) \\
&\quad \left(-\beta\left(\frac{\mu_h(N_h - S_h)}{N_h(\mu_h - \gamma_h)}\right) - (\mu_v + k_2) - \lambda\right) \\
&\quad (-\varepsilon - (\mu_v + k_2) - \lambda) \\
&\quad (-(\mu_v + k_2) - \lambda) \\
&\quad \left(-\beta\left(\frac{\varepsilon E_v + BI_e}{N_h C}\right) - \mu_h - \lambda\right) \\
&\quad (-\mu_h k_3 - \gamma_h k_3 - \lambda) \\
&\quad (-\mu_h - \lambda) = 0
\end{aligned}$$

So the eigenvalues are obtained as follows:

$$\lambda_1 = -(1+k_1)\mu_e - (1-k_1)\omega$$
$$\lambda_2 = -(1+k_1)\mu_e - (1-k_1)\omega$$
$$\lambda_3 = -\beta\left(\frac{\mu_h(N_h - S_h)}{N_h(\mu_h - \gamma_h)}\right) - (\mu_v + k_2)$$
$$\lambda_4 = -\varepsilon - (\mu_v + k_2)$$
$$\lambda_5 = -(\mu_v + k_2)$$
$$\lambda_6 = -\beta\left(\frac{\varepsilon E_v + BI_e}{N_h C}\right) - \mu_h$$
$$\lambda_7 = -\mu_h k_3 - \gamma_h k_3$$
$$\lambda_8 = -\mu_h$$

By entering the existing parameters, we get:

$$\lambda_1 = -1.01379970384$$
$$\lambda_2 = -1.01379970384$$
$$\lambda_3 = -0.00000057$$
$$\lambda_4 = -1.052066381319$$
$$\lambda_5 = -0.756469390419$$
$$\lambda_6 = -0.00028885$$
$$\lambda_7 = -0.02310211728$$
$$\lambda_8 = -0.0249$$

Thus, since all eigenvalues are negative, the system is said to be asymptotically stable at the endemic equilibrium point.

Stability Analysis of Endemic Equilibrium Point

Furthermore, stability analysis is carried out by substituting endemic equilibrium points into the Jacobian matrix, so that the following equation is obtained:

$$\begin{aligned}
&(-(1+k_1)\mu_e - (1-k_1)\omega - \lambda) \\
&\quad (-(1+k_1)\mu_e - (1-k_1)\omega - \lambda) \\
&\quad \left(-\beta\left(\frac{\mu_h(N_h - S_h)}{N_h(\mu_h - \gamma_h)}\right) - (\mu_v + k_2) - \lambda\right) \\
&\quad (-\varepsilon - (\mu_v + k_2) - \lambda) \\
&\quad (-(\mu_v + k_2) - \lambda) \\
&\quad \left(-\beta\left(\frac{\varepsilon E_v + BI_e}{N_h C}\right) - \mu_h - \lambda\right) \\
&\quad (-\mu_h k_3 - \gamma_h k_3 - \lambda) \\
&\quad (-\mu_h - \lambda) = 0
\end{aligned}$$

So the eigenvalues are obtained as follows:

$$\lambda_1 = -(1+k_1)\mu_e - (1-k_1)\omega$$
$$\lambda_2 = -(1+k_1)\mu_e - (1-k_1)\omega$$
$$\lambda_3 = -\beta\left(\frac{\mu_h(N_h - S_h)}{N_h(\mu_h - \gamma_h)}\right) - (\mu_v + k_2)$$
$$\lambda_4 = -\varepsilon - (\mu_v + k_2)$$
$$\lambda_5 = -(\mu_v + k_2)$$
$$\lambda_6 = -\beta\left(\frac{\varepsilon E_v + BI_e}{N_h C}\right) - \mu_h$$
$$\lambda_7 = -\mu_h k_3 - \gamma_h k_3$$
$$\lambda_8 = -\mu_h$$

By substituting the existing parameters, we get:

$$\lambda_1 = -1.01379970384$$
$$\lambda_2 = -1.01379970384$$
$$\lambda_3 = -0.00000057$$
$$\lambda_4 = -1.052066381319$$
$$\lambda_5 = -0.756469390419$$
$$\lambda_6 = -0.00028885$$
$$\lambda_7 = -0.02310211728$$
$$\lambda_8 = -0.0249$$

Therefore, since all eigenvalues are negative, the system is said to be asymptotically stable at the endemic equilibrium point.

CONTROLLABILITY

From Equations (1) to (8), if $S_e = x_1, I_e = x_2, S_v = x_3, E_v = x_4, I_v = x_5, S_h = x_6, I_h = x_7$, and $R_h = x_8$, the state space is obtained as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & a_{31} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & a_{52} & 0 & 0 & 0 \\ a_{13} & a_{23} & a_{33} & a_{43} & 0 & 0 & 0 & 0 \\ a_{14} & a_{24} & 0 & a_{44} & a_{54} & 0 & 0 & 0 \\ a_{15} & a_{25} & 0 & 0 & a_{55} & a_{65} & a_{75} & 0 \\ 0 & 0 & 0 & 0 & a_{66} & a_{76} & 0 & 0 \\ 0 & 0 & a_{37} & a_{47} & 0 & 0 & a_{77} & a_{78} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{88} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + B \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} \tag{1}$$

where

$$A = \begin{bmatrix} a_{11} & 0 & a_{31} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & a_{52} & 0 & 0 & 0 \\ a_{13} & a_{23} & a_{33} & a_{43} & 0 & 0 & 0 & 0 \\ a_{14} & a_{24} & 0 & a_{44} & a_{54} & 0 & 0 & 0 \\ a_{15} & a_{25} & 0 & 0 & a_{55} & a_{65} & a_{75} & 0 \\ 0 & 0 & 0 & 0 & a_{66} & a_{76} & 0 & 0 \\ 0 & 0 & a_{37} & a_{47} & 0 & 0 & a_{77} & a_{78} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{88} \end{bmatrix}$$

with

$$\begin{aligned}
a_{11} &= -(1+k_1)\mu_e - (1-k_1)\omega, \\
a_{13} &= b_v v \left(\frac{I_v}{(S_v+E_v+I_v)^2} \right), \\
a_{14} &= b_v v \left(\frac{I_v}{(S_v+E_v+I_v)^2} \right), \\
a_{15} &= b_v v \left(\frac{I_v}{(S_v+E_v+I_v)^2} \right), \\
a_{22} &= -(1+k_1)\mu_e - (1-k_1)\omega, \\
a_{23} &= -b_v v \left(\frac{I_v}{(S_v+E_v+I_v)^2} \right), \\
a_{24} &= -b_v v \left(\frac{I_v}{(S_v+E_v+I_v)^2} \right), \\
a_{25} &= b_v v \left(\frac{I_v}{(S_v+E_v+I_v)^2} \right), \\
a_{31} &= (1-k_1)\omega, \\
a_{33} &= -\beta \frac{I_h k_3}{N_h} - (\mu_v + k_2), \\
a_{37} &= -\beta \frac{k_3}{N_h} S_v, \\
a_{43} &= \beta \frac{I_h k_3}{N_h}, \\
a_{44} &= -\varepsilon - (\mu_v + k_2), \\
a_{47} &= \beta \frac{k_3}{N_h} S_v, \\
a_{52} &= (1-k_1)\omega, \\
a_{54} &= \varepsilon, \\
a_{55} &= -(\mu_v + k_2), \\
a_{65} &= -\beta \frac{S_h}{N_h}, \\
a_{66} &= -\beta \frac{I_v}{N_h} - \mu_h, \\
a_{75} &= \beta \frac{S_h}{N_h}, \\
a_{76} &= -\mu_h k_3 - \gamma_h k_3, \\
a_{77} &= -\mu_h k_3 - \gamma_h k_3, \\
a_{87} &= \gamma_h k_3, \\
a_{88} &= -\mu_h.
\end{aligned}$$

and matrix $B_{3\times 3}$ So that a controllable matrix M_c is obtained which has the same rank as the matrix A so that the system is said to be controlled.

BASIC REPRODUCTION NUMBER

The basic reproduction number R_0 represents the average number of infected individuals per unit of time. To calculate R_0, the next-generation matrix of the infected equations can be used as follows:

$$\begin{aligned}
\frac{dI_e}{dt} &= b_v v \left(\frac{I_v}{(S_v+E_v+I_v)} \right) - \mu_e I_e - \omega I_e, \\
\frac{dI_v}{dt} &= \varepsilon E_v + \omega I_e - \mu_v I_v, \\
\frac{dI_h}{dt} &= \beta \frac{S_h}{N_h} I_v - \mu_h I_h - \gamma_h I_h.
\end{aligned}$$

By inserting the disease-free equilibrium point, we obtain the Jacobian matrix:

$$J = \begin{bmatrix} -\omega + k_1\omega & 0 & 0 \\ (1-k_1)\omega & -\mu_v - k_2 & 0 \\ 0 & \beta & -\mu_h k_3 - \gamma_h k_3 \end{bmatrix}$$

Furthermore, the Jacobian matrix will be decomposed into a transmission matrix F and transition matrices V as follows:

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\mu_v - k_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} \omega - k_1\omega & 0 & 0 \\ 0 & \mu_v + k_2 & 0 \\ 0 & 0 & \mu_h k_3 + \gamma_h k_3 \end{bmatrix}$$

so the inverse of V

$$V^{-1} = \begin{bmatrix} \frac{1}{\omega - k_1\omega} & 0 & 0 \\ 0 & \frac{1}{\mu_v + k_2} & 0 \\ 0 & 0 & \frac{1}{\mu_h k_3 + \gamma_h k_3} \end{bmatrix}$$

and got

$$FV^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\mu_v - k_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\omega - k_1\omega} & 0 & 0 \\ 0 & \frac{1}{\mu_v + k_2} & 0 \\ 0 & 0 & \frac{1}{\mu_h k_3 + \gamma_h k_3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

then calculates the basic reproduction number where the basic reproduction number is $\rho(FV^{-1})$ namely:

$$\begin{aligned}
(FV^{-1} - \lambda I) &= \begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -1-\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} = 0, \\
-\lambda^2 - \lambda^3 &= 0, \\
\lambda^2(-1-\lambda) &= 0, \\
\lambda_1 = 0 \quad &\text{or} \quad \lambda_2 = -1
\end{aligned}$$

obtained

$$R_0 = \max\{\lambda_1, \lambda_2\} = 0$$

So that the basic reproduction number (R_0) setelah diberikan kontrol adalah 0 after being given control is 0 which indicates that on average each infected individual will infect less than one new individual, which means the disease will not spread.

IV. Optimal Control Design

The problem of optimal control in this study is solved using the Pontryagin Maximum Principle. The aim is to minimize the number of infected human populations through treatment, increase the death of adult mosquitoes through fumigation with insecticides, and extermination/control of mosquito eggs/larvae through the use of Abate larvicide. Mathematically, the objective of optimal control can be expressed in terms of the objective function as follows:

$$J(k_1,k_2,k_3) = \min \int_{t_0}^{t_f} \left(a_1 I_h + a_2 I_v + a_3 I_e + \left(\frac{m_1 k_1^2}{2} + \frac{m_2 k_2^2}{2} + \frac{m_3 k_3^2}{2} \right) \right) dt \quad (1)$$

where:

$a_1 =$ weight of infected human population (I_h)
$a_2 =$ weight of infected mosquito population (I_v)
$a_3 =$ weight of infected mosquito egg population (I_e)
$m_1 =$ percentage of dead mosquito eggs
$m_2 =$ percentage of dead adult mosquitoes
$m_3 =$ percentage of treatment in humans

A. Optimal Control Completion

Completion of optimal control can be solved using the Pontryagin Minimum Principle with the following steps. **Form the Hamiltonian function**

$$\begin{aligned} H = {} & a_1 I_h + a_2 I_v + a_3 I_e + \left(\frac{m_1 k_1^2}{2} + \frac{m_2 k_2^2}{2} + \frac{m_3 k_3^2}{2} \right) \\ & + \lambda_{S_e} \left(b_v \left(1 - v \left(\frac{I_v}{S_v + E_v + I_v} \right) \right) - (1+k_1) M_e S_e - (1-k_1) \omega S_e \right) \\ & + \lambda_{I_e} \left(b_v v \left(\frac{I_v}{S_v + E_v + I_v} \right) - (1+k_1) M_e I_e - (1-k_1) \omega I_e \right) \\ & + \lambda_{S_v} \left((1-k_1) \omega S_e - \frac{\beta I_h k_3}{N_h} S_v - (M_v + k_2) S_v \right) \\ & + \lambda_{E_v} \left(\frac{\beta I_h k_3}{N_h} S_v - \varepsilon E_v - (M_v + k_2) E_v \right) \\ & + \lambda_{I_v} (\varepsilon E_v + (1-k_1) \omega I_e - (M_v + k_2) I_v) \\ & + \lambda_{S_n} \left(M_h N_h - \frac{\beta S_h}{N_h} I_v - M_h S_h \right) \\ & + \lambda_{I_h} \left(\frac{\beta S_h}{N_h} I_v - M_h I_h k_3 - \gamma_h I_h k_3 \right) + \\ & + \lambda_{R_h} (\gamma_h I_h k_3 - M_h R_h) \end{aligned} \quad (2)$$

B. Define Stationary Condition

To get the optimal control equation, H is derived from k_1, k_2, k_3, so that we get:

a)

$$\begin{aligned} \frac{dH}{dk_1} &= m_1 k_1 + \lambda_{S_e} (M_e S_e - \omega S_e) + \lambda_{I_e} (-M_e I_e + \omega I_e) \\ &\quad + \lambda_{S_v} (-\omega S_e) + \lambda_{I_v} (-\omega I_e), \\ \frac{dH}{dk_1} &= 0, \\ k_1^* &= \frac{\lambda_{S_e} (M_e S_e - \omega S_e)}{m_1} \\ &\quad + \frac{\lambda_{I_e} (-M_e I_e + \omega I_e) + \lambda_{S_v} (-\omega S_e) + \lambda_{I_v} (-\omega I_e)}{m_1} \end{aligned}$$

b)

$$\begin{aligned} \frac{dH}{dk_2} &= m_2 k_2 + \lambda_{S_v} (-S_v) + \lambda_{E_v} (-E_v) + \lambda_{I_v} (-I_v), \\ \frac{dH}{dk_2} &= 0, \\ k_2^* &= \frac{\lambda_{S_v} (-S_v) + \lambda_{E_v} (-E_v) + \lambda_{I_v} (-I_v)}{m_2} \end{aligned}$$

c)

$$\begin{aligned} \frac{dH}{dk_3} &= m_3 k_3 + \lambda_{S_v} \left(-\frac{\beta I_h}{N_h} S_v \right) + \lambda_{E_v} \left(-\frac{\beta I_h}{N_h} S_v \right) \\ &\quad + \lambda_{I_h} (-M_h I_h) + \lambda_{R_h} (\gamma_h I_h), \\ \frac{dH}{dk_3} &= 0, \\ k_3^* &= \frac{(\lambda_{S_v} + \lambda_{E_v}) \left(-\frac{\beta I_h}{N_h} S_v \right) + \lambda_{I_h} (-M_h I_h) + \lambda_{R_h} (\gamma_h I_h)}{m_3}. \end{aligned}$$

C. Determine the Optimal H

The optimal control equations obtained from equations k_1^*, k_2^*, and k_3^* are substituted into the equation H, thus obtaining:

$$\begin{aligned} H^* = {} & a_1 I_h + a_2 I_v + a_3 I_e + \left(\frac{m_1 k_1^{*2}}{2} + \frac{m_2 k_2^{*2}}{2} + \frac{m_3 k_3^{*2}}{2} \right) \\ & + \lambda_{S_e} \left(b_v \left(1 - v \left(\frac{I_v}{S_v + E_v + I_v} \right) \right) - (1+k_1^*) \mu_e S_e - (1-k_1^*) \omega S_e \right) \\ & + \lambda_{I_e} \left(b_v v \left(\frac{I_v}{S_v + E_v + I_v} \right) - (1+k_1^*) \mu_e I_e - (1-k_1^*) \omega I_e \right) \\ & + \lambda_{S_v} \left((1-k_1^*) \omega S_e - \frac{\beta I_h k_3^*}{N_h} S_v - (\mu_v + k_2^*) S_v \right) \\ & + \lambda_{E_v} \left(\frac{\beta I_h k_3^*}{N_h} S_v - \varepsilon E_v - (\mu_v + k_2^*) E_v \right) \\ & + \lambda_{I_v} (\varepsilon E_v + (1-k_1^*) \omega I_e - (\mu_v + k_2^*) I_v) \\ & + \lambda_{S_h} \left(\mu_h N_h - \frac{\beta S_h}{N_h} I_v - \mu_h S_h \right) \\ & + \lambda_{I_h} \left(\frac{\beta S_h}{N_h} I_v - M_h I_h k_3^* - \gamma_h I_h k_3^* \right) \\ & + \lambda_{R_h} (\gamma_h I_h k_3^* - \mu_h R_h). \end{aligned}$$

D. Solving the State and Co-state Equations to Obtain an Optimal System

State Equations

$$\begin{aligned} S_e^* &= b_v \left(1 - v \left(\frac{I_v}{S_v + E_v + I_v} \right) \right) - (1+k_1^*) \mu_e S_e - (1-k_1^*) \omega S_e \\ I_e^* &= b_v v \left(\frac{I_v}{S_v + E_v + I_v} \right) - (1+k_1^*) \mu_e I_e - (1-k_1^*) \omega I_e \\ S_v^* &= (1-k_1^*) \omega S_e - \frac{\beta I_h k_3^*}{N_h} S_v - (\mu_v + k_2^*) S_v \\ E_v^* &= \frac{\beta I_h k_3^*}{N_h} S_v - \varepsilon E_v - (\mu_v + k_2^*) E_v \end{aligned}$$

$$I_v^* = \varepsilon E_v + (1-k_1^*)\omega I_e - (\mu_v + k_2^*) I_v$$
$$S_h^* = \mu_h N_h - \frac{\beta S_h}{N_h} I_v - \mu_h S_h$$
$$I_h^* = \frac{\beta S_h}{N_h} I_v - M_h I_h k_3^* - \gamma_h I_h k_3^*$$
$$R_h^* = \gamma_h I_h k_3^* - \mu_h R_h$$

Co-state Equations

$$x^{*'}(t) = -\left(\frac{dH^*}{dx}\right)$$
$$\lambda S_e^{'*} = -\left(\lambda_{S_e}\left(-(1+k_1^*)\mu_e - (1-k_1^*)\omega\right) + \lambda_{S_v}\left((1+k_1^*)\omega\right)\right)$$
$$\lambda I_e^{'*} = -\left(\lambda_{I_e}\left(-(1+k_1^*)\mu_e - (1-k_1^*)\omega\right) + \lambda_{I_v}\left((1+k_1^*)\omega\right)\right)$$
$$\lambda S_v^{'*} = -\left(\lambda_{S_v}\left(-\frac{\beta I_h k_3^*}{N_h} - \mu_v - k_2^*\right) + \lambda_{E_v}\left(\frac{\beta I_h k_3^*}{N_h}\right)\right)$$
$$\lambda E_v^{'*} = -\left(\lambda_{E_v}\left(\varepsilon - M_v - k_2^*\right) + \lambda_{I_v}\left(\varepsilon\right)\right)$$
$$\lambda I_v^{'*} = -\left(\lambda_{S_e}(v) + \lambda_{I_e}(b_v V) + \lambda_{I_v}\left(-M_v - k_2^*\right) + \lambda_{S_h}\left(-\frac{\beta S_h}{N_h}\right) + \lambda_{I_h}\left(\frac{\beta S_h}{N_h}\right)\right)$$
$$\lambda S_h^{'*} = -\left(\lambda_{S_h}\left(-\frac{\beta I_v}{N_h} - M_h\right) + \lambda_{I_h}\left(\frac{\beta I_v}{N_h}\right)\right)$$
$$\lambda I_h^{'*} = -\left(\lambda_{S_v}\left(-\frac{\beta k_3^*}{N_h} S_v\right) + \lambda_{E_v}\left(\frac{\beta k_3^*}{N_h} S_v\right) + \lambda_{I_h}\left(M_h k_3^* - \gamma_h k_3^*\right) + \lambda_{R_h}\left(\gamma_h k_3^*\right)\right)$$
$$\lambda R_h^{'*} = -\left(\lambda_{R_h}\left(-M_h\right)\right)$$

E. Results and Analysis of Simulation Graphs

Comparison of the results of numerical simulations with control and without control within $t = 12$ months by entering parameter values in each population using the Runge-Kutta Order-4 method.

1) Case in Pekanbaru City

a) Suspectible Mosquito Eggs(S_e) Figure 2 shows that

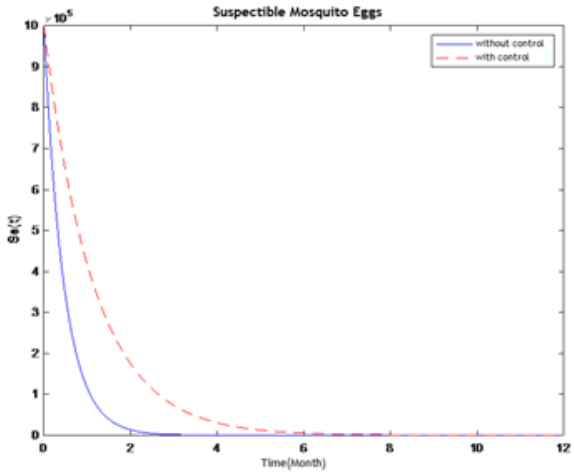

Fig. 2: Graph of Changes in Suspectible Mosquito Egg Population Comparison (S_e) before and after being given control

there are differences in the susceptible mosquito eggs before and after being given control in the initial conditions $S_e(0) = 1.091.083$. Without control, susceptible mosquito eggs will decrease more drastically than with control. The difference is not given control and given control will be seen with a distance between 0-8.

b) Infected Mosquito Eggs (I_e)

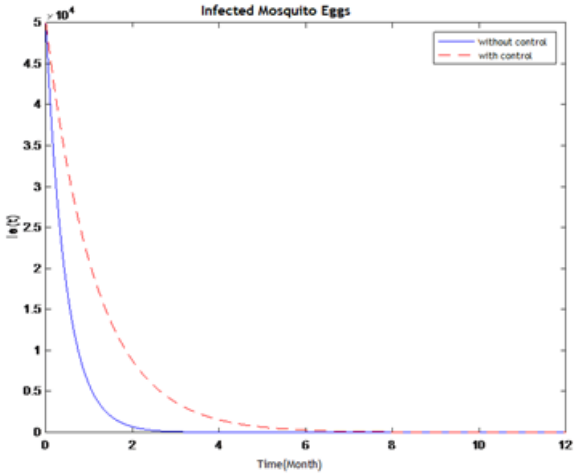

Fig. 3: Graph of Changes in Comparison of Infected Mosquito Egg Populations (I_e)before and after being given control.

Figure 3 shows that there are differences in infected mosquito eggs before and after being given control with the initial conditions $I_e(0) = 50.000$. Without control, infected mosquito eggs will decrease more drastically than with control. The difference is not given control and given control will be seen with a distance between 0-8 months.

c) Suspectible Adult Mosquitoes (S_v)

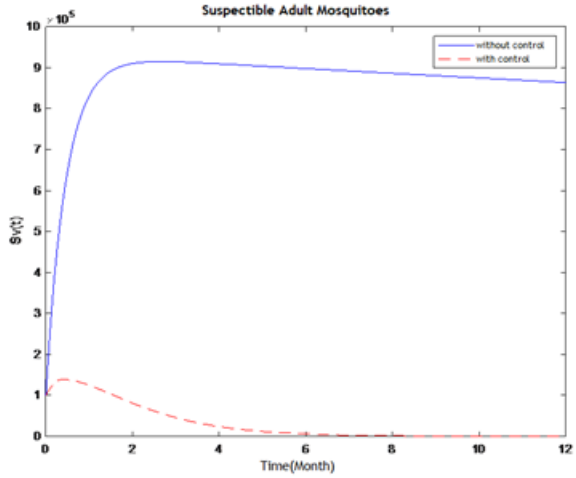

Fig. 4: Graph of Changes in the Comparison of Suspectible (S_v) Adult Mosquito Populations before and after being given control.

In Figure 4 it can be seen that the administration of control will reduce the susceptible mosquito population and will even have a value of 0 in the 8th month onwards. Meanwhile, without providing control, it will increase the adult mosquito population which is susceptible to disease, even the population will stagnate in the next few months.

d) Adult Mosquitoes Exposed (E_v)

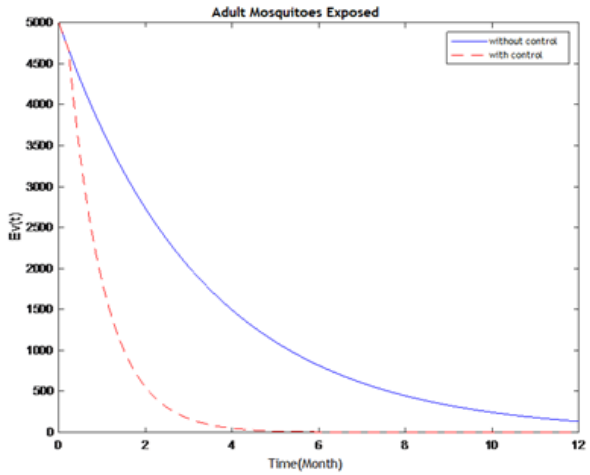

Fig. 5: Graph of Changes in Comparison of Exposed Adult Mosquito Populations(E_v) before and after being given control.

In Figure 5 it can be seen that the administration of control will drastically reduce the mosquito population exposed to it in the 2nd to 6th months. Meanwhile, without control, the exposed mosquito population also decreased steadily after the 12th month.

e) Infected Adult Mosquitos (I_v)

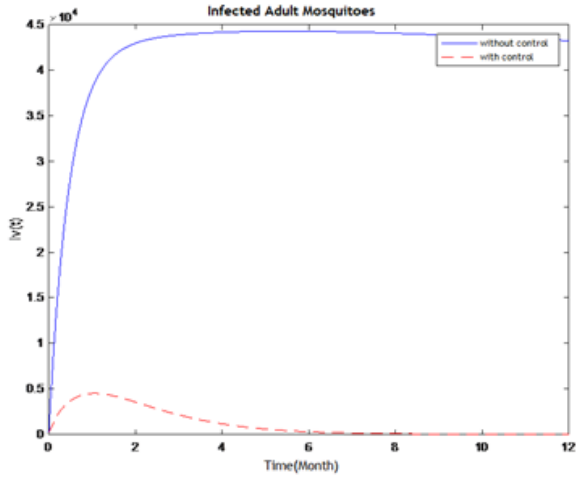

Fig. 6: Graph of Changes in Comparison of Exposed Adult Mosquito Populations(E_v) before and after being given control.

In Figure 6 it can be seen that giving control will increase the infected mosquito population until the first month, then after that it will decrease until the 8th month. Meanwhile, without providing control, it will increase the adult mosquito population which is susceptible to disease, even the population will stagnate in the next few months.

f) Suspectable Humans (S_h)

In Figure 7 without giving control the susceptible human population will decrease. Meanwhile, by giving control, the susceptible human population will decrease, then it will increase but not significantly

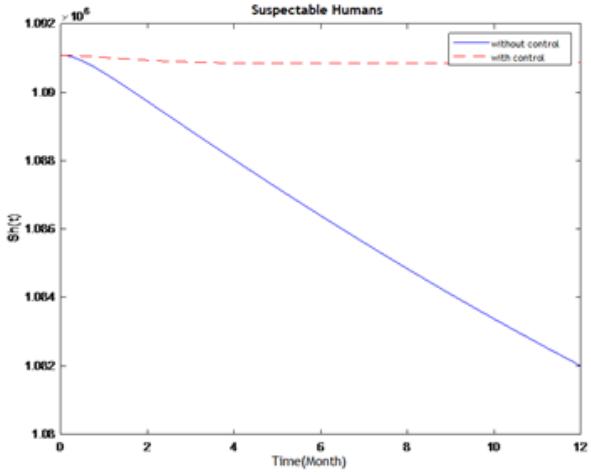

Fig. 7: Graph of Changes in Suspectible Human Population Comparison(S_h) before and after being given control.

g) Infected Humans (I_h)

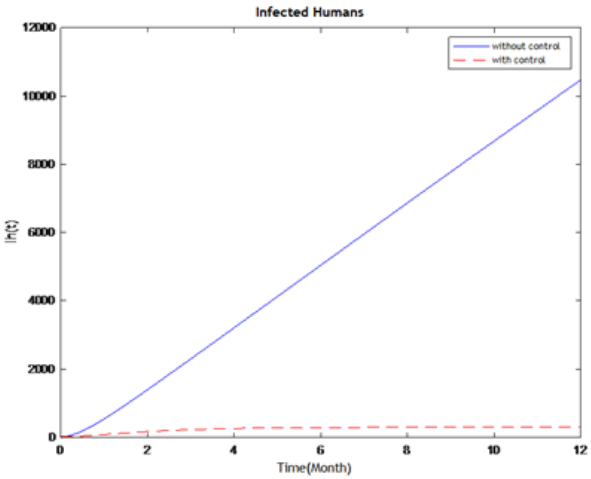

Fig. 8: Graph of Changes in the Comparison of Infected Human Populations before and after being given control(I_h).

In Figure 9 the infected human population without control will increase drastically. Whereas with control, the infected human population will experience a non-significant increase and will have a constant value after the 6th month.

h) Recovery Human (R_h)

In Figure 9 it can be seen that there is no difference between giving control and without giving control in the human population that has recovered from infection.

2) Case in Solok City

a) Suspectible Mosquito Eggs (S_e)

Figure 10 shows that there are differences in the susceptible mosquito eggs before and after being given control with the initial conditions $S_e(0) = 68.608$. Without control, susceptible mosquito eggs will decrease more drastically than with control. The differ-

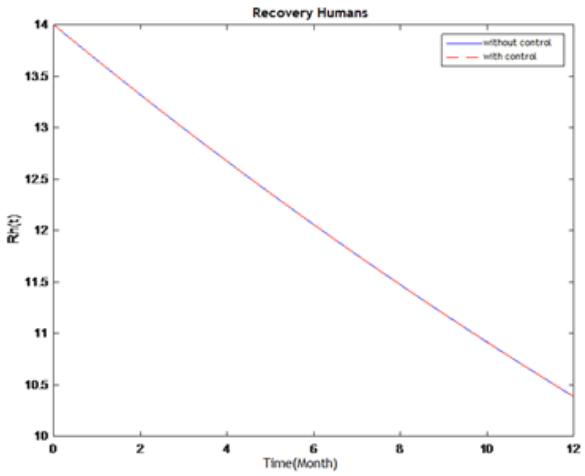

Fig. 9: Graph of Changes in Comparison of Human Population Recovery(R_h) before and after being given control.

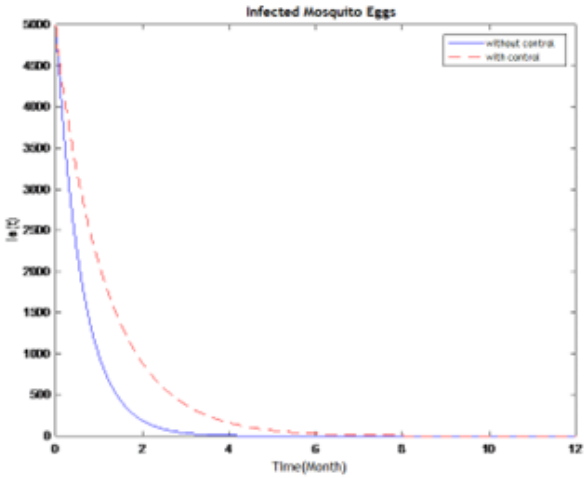

Fig. 11: Graph of Changes in Comparison of Infected Mosquito Egg Populations (I_e) before and after being given control.

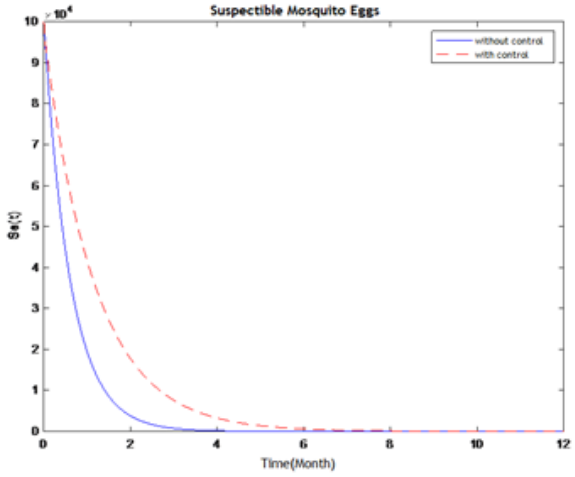

Fig. 10: Graph of Changes in Comparison of Suspectible (S_e) Mosquito Egg Populations before and after being given control.

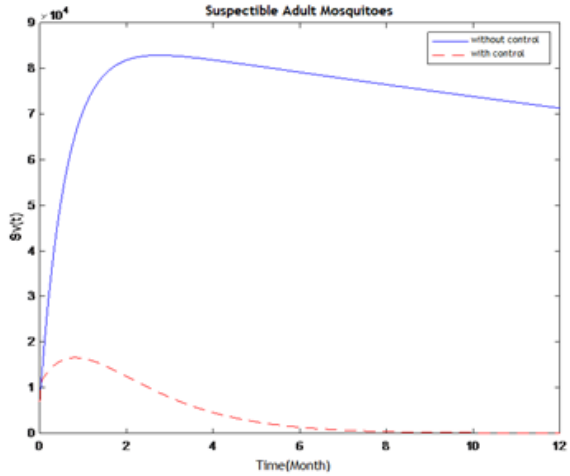

Fig. 12: Graph of Changes in the Comparison of Suspectible (S_v) Adult Mosquito Populations before and after being given control.

ence is not given control and given control will be seen with a distance between 0-8 months.

b) Infected Mosquito Eggs (I_e)
Figure 11 shows that there are differences in infected mosquito eggs before and after being given control with the initial conditions $I_e(0) = 5.000$. Without control, infected mosquito eggs will decrease more drastically than with control. The difference is not given control and given control will be seen with a distance between 0-8 months.

c) Suspectible Adult Mosquitoes (S_v)
In Figure 12 it can be seen that the administration of control will reduce the susceptible mosquito population and will even have a value of 0 in the 8th month onwards. Meanwhile, without giving control, it will increase the adult mosquito population which is susceptible to disease, even the population will stagnate in the next few months.

d) Adult Mosquitoes Exposed (E_v)
In Figure 13 it can be seen that the administration of control will drastically reduce the mosquito population exposed in the 2nd to 6th months. Meanwhile, without control, the exposed mosquito population also decreased steadily after the 12th month.

e) Infected Adult Mosquitoes (I_v)
In Figure 14 it can be seen that giving control will increase the infected mosquito population until the first month, then after that it will decrease until the 8th month. Meanwhile, without providing control it will increase the adult mosquito population which is susceptible to disease, even the population will stagnate in the next few months.

f) Suspectable Human (S_h)

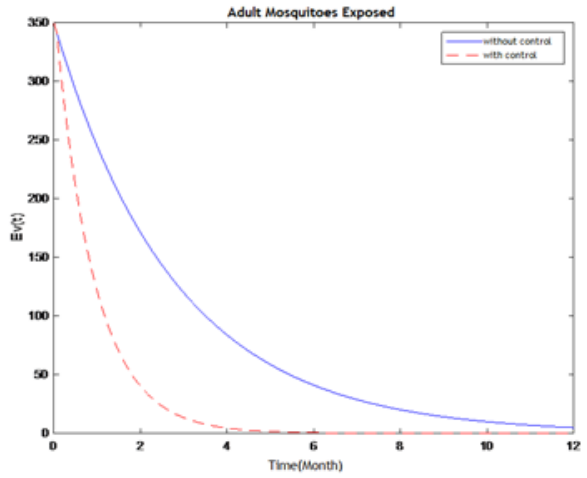

Fig. 13: Graph of Changes in the Comparison of Exposed (E_v) Adult Mosquito Populations before and after being given control.

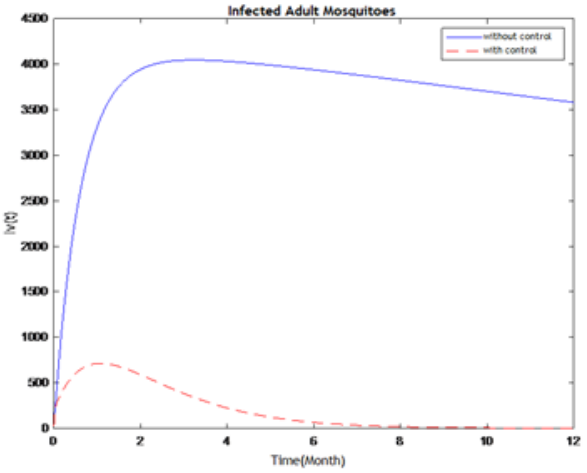

Fig. 14: Graph of Comparison of Changes in Infected (I_v) Adult Mosquito Populations before and after being given control.

In Figure 15 without giving control the susceptible human population will decrease. Meanwhile, by giving control, the susceptible human population will decrease, then it will increase but not significantly.

g) Infected Humans (I_h)
In Figure 16 the infected human population without control will experience a drastic increase. Whereas with control, the infected human population will experience a non-significant increase and will have a constant value after the 6th month.

h) Recovery Human (R_h)
In Figure 17 it can be seen that there is no difference between administration of controls and without administration of controls in the human population that has recovered from infection.

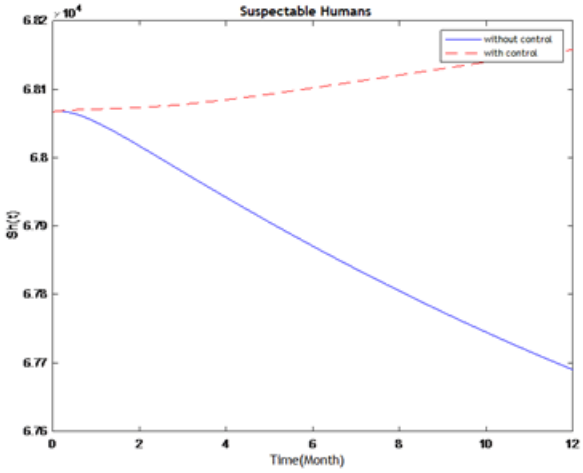

Fig. 15: Graph of Changes in the Comparison of Suspectible (S_h) Human Populations before and after being given control.

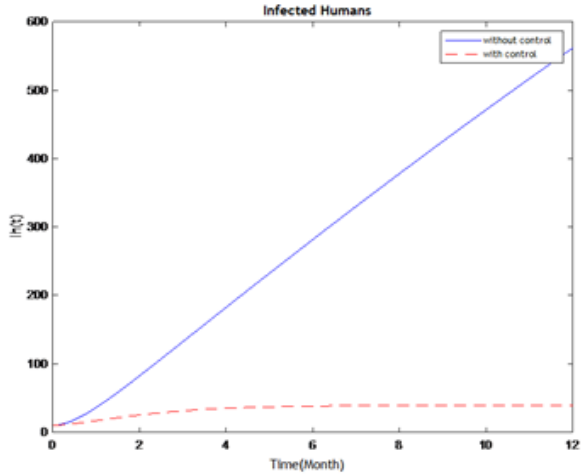

Fig. 16: Graph of Changes in the Comparison of Infected (I_h) Human Populations before and after being given control.

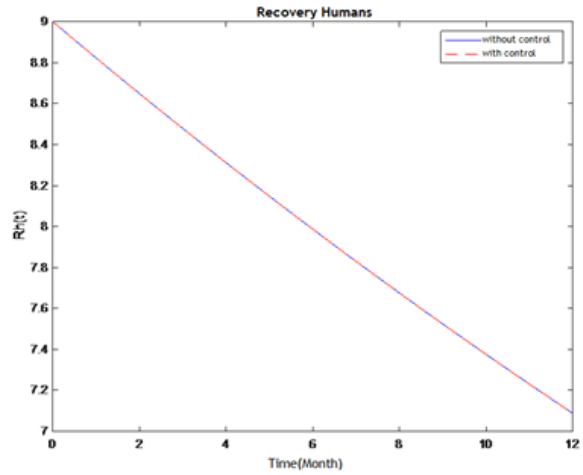

Fig. 17: Graph of Changes Comparison of Human Population Recovery (R_h) before and after being given control.

V. CONCLUSION

VI. CONCLUSIONS

Based on the analysis and discussion given in the previous chapter, several conclusions were obtained, namely:

1) Model of the spread of dengue fever with the influence of climate and treatment is as follows.

$$\begin{aligned}
\frac{dS_e}{dt} &= b_v\left(1-v\left(\frac{I_v}{S_v+E_v+I_v}\right)\right)-(1+k_1)\mu_e S_e-(1-k_1)\omega S_e,\\
\frac{dI_e}{dt} &= b_v v\left(\frac{I_v}{S_v+E_v+I_v}\right)-(1+k_1)\mu_e I_e-(1-k_1)\omega I_e,\\
\frac{dS_v}{dt} &= (1-k_1)\omega S_e-\beta\left(\frac{I_h k_3}{N_h}\right)S_v-(\mu_v+k_2)S_v,\\
\frac{dE_v}{dt} &= \beta\left(\frac{I_h k_3}{N_h}\right)S_v-\varepsilon E_v-(\mu_v+k_2)E_v,\\
\frac{dI_v}{dt} &= \varepsilon E_v+(1-k_1)\omega I_e-(\mu_v+k_2)I_v,\\
\frac{dS_h}{dt} &= \mu_h N_h-\beta\left(\frac{S_h}{N_h}\right)I_v-\mu_h S_h,\\
\frac{dI_h}{dt} &= \beta\left(\frac{S_h}{N_h}\right)I_v-\mu_h I_h k_3-\gamma_h I_h k_3,\\
\frac{dR_h}{dt} &= \gamma_h I_h k_3-\mu_h R_h.
\end{aligned}$$

2) Optimal influence control is obtained in the form of mosquito egg death (k_1), adult mosquito death (k_2), and human treatment (k_3) will be optimal if,

$$\begin{aligned}
k_1^* &= \frac{\lambda_{S_e}(\mu_e S_e-\omega S_e)+\lambda_{I_e}(-\mu_e I_e+\omega I_e)+\lambda_{S_v}(-\omega S_e)+\lambda_{I_v}(-\omega I_e)}{m_1},\\
k_2^* &= \frac{\lambda_{S_v}(-S_v)+\lambda_{E_v}(-E_v)+\lambda_{I_v}(-I_v)}{m_2},\\
k_3^* &= \frac{(\lambda_{S_v}+\lambda_{E_v})\left(-\frac{\beta I_h}{N_h}S_v\right)+\lambda_{I_h}(-\mu_h I_h)+\lambda_{R_h}(\gamma_h I_h)}{m_3}.
\end{aligned}$$

Based on the effect of the optimal control in the form of mosquito egg death (k_1), adult mosquito death (k_2), and human treatment (k_3) in the cities of Pekanbaru and Solokk, there is not much difference between the two. However, there is little difference in the increase in the human population that is susceptible to disease.

ACKNOWLEDGMENTS

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