# An Inventory Model for Deteriorated Item with Time- and Inventory-dependent Demand and Backorder

Dharma Lesmono, Taufik Limansyah, Ignatius A. Sandy

***Abstract*—The paper focuses on developing an inventory model for deteriorated item when demand is time- and inventory-dependent. Deteriorated items can usually be found in items such as vegetables, fruit, milk, chemical product, pharmaceutical and it needs special attention in managing this kind of inventory. We model the inventory control for these items by a mathematical model involving time- and inventory-dependent demand and considering the backorder policy in handling shortages. The developed model aims to find the optimal time between replenishment and when inventory drops to zero, which minimises the total inventory cost. The total inventory cost consists of the purchase cost, the order cost, the holding cost, and the shortage cost. Sensitivity analysis is performed to analyse the effect of changing the parameters' values to the time between replenishment, when inventory drops to zero, the order quantity, and the total inventory cost. The finding shows that changing the parameters' values of deterioration rate, demand, unit holding cost, unit holding cost and unit shortage cost will have an impact on the time between replenishment, time when inventory drops to zero, order quantity, and inventory cost.**

***Index Terms*—deteriorated items, time- and inventory-dependent demand, time between replenishment, order quantity.**

## I. Introduction

A RETAILER'S inventory becomes a critical asset that needs to be appropriately managed. Good inventory management requires knowledge of internal and external factors that affect inventory availability. Fulfilling the customers' demands is an important external factor for retailer which is related to a so-called service level. Demand as an external factor has been mathematically modelled in different forms in many papers over the last decades. The mathematical models that have been developed are trying to find the optimal order quantity or time between replenishment that minimises the total inventory cost.

Many papers have used to model demands as time-dependent, price-dependent, inventory-dependent or combination between them (see for some recent papers in [1], [2], [3], [4], [5], [6]). Other papers, for instance [7] and [8] modelled demand as stochactic and fluctuating. When demands are modelled as a combination of time-dependent, price-dependent, inventory-dependent or a combination with other factors, the approach in modelling uses an ordinary differential equation to describe the inventory dynamics at each time. However, other approaches do exist, such as game theory (see [9]), and semi-Markov (see [7]).

Another factor that the inventory manager must consider is the product itself which may deteriorate with time, such as fruit, vegetable, milk, meat, pharmaceutical products. Managing deteriorated items needs special attention compared to regular items with no deterioration, since the order quantity, the time between replenishment and other decision variables are affected by this deterioration feature in the model. In this paper, we develop an inventory model for deteriorating items where demands are assumed to be time- and inventory-dependent and shortages are handled with backorder policy. There are not many papers that use a combination of time- and inventory-dependent demand in their models, see for instance [10] and [6]. However, the use of time- and inventory-dependent demand is suitable for deteriorated or perishable goods when the price is not so senstive. As mentioned by [11] and [6], that demand for physical goods is time-dependent and the availability of perishable goods on shelves in the supermarket and the shelf life affect the demand. Most of the papers use a combination of price- and inventory-depedent demand or time- and price-dependent demand, or even time-, price- and inventory-dependent demand (see [5], [12], [13], [14], [15], [16], [17], [18], [19]).

Deterioration is also becoming one important factor that needs to be carefully considered by the inventory manager. In general, deteriorating inventory models are characterized by the lifetime of the products and demand (see [20]). There are also many papers deal with deteriorating inventory models combined with other factors for the last few decades (see [1], [2], [3], [4], [5], [7], [8], [10], [18], [19], [21], [22]). A comprehensive review of deteriorating inventory models can be found in [23] and [20].

The model we developed in this paper is a modification of the model developed in [22] by adding time when inventory drops to zero as one of decision variables. A numerical example is provided to give an illustration and a better understanding of the researchers' model. Sensitivity analysis is also performed to analyse the effect of changing one parameter's value on the decision variables: the time between replenishment and when inventory drops to zero. Moreover, it effects order quantity and total inventory cost as well.

In the next section, we introduce the model's construction along with the optimality condition. Numerical example and sensitivity analysis are given in the subsequent section. Finally, the conclusion and further research directions are provided in

D. Lesmono, T. Limansyah, and I. A. Sandy are with the Universitas Katolik Parahyangan, Bandung, 40141 Indonesia e-mail: jdharma@unpar.ac.id
Manuscript received July 18, 2023; accepted September 14, 2023.

the last section.

## II. The Model

### A. Notations

The notations we use in this paper are as follows:

| | |
|---|---|
| $TC$ | : total inventory cost |
| $PC$ | : purchasing cost |
| $OC$ | : ordering cost |
| $HC$ | : holding cost |
| $SC$ | : shortage cost |
| $T$ | : time between replenishment |
| $t_1$ | : time when inventory drops to zero |
| $D(t) = a + bI(t)$ | : demand at time $t$ |
| $b$ | : inventory-dependent demand factor |
| $a$ | : autonomous demand |
| $I(t)$ | : inventory level at time $t$ |
| $K$ | : number of inventory at interval $[0, t_1]$ |
| $M$ | : number of shortage at interval $[t_1, T]$ |
| $P$ | : unit purchasing cost |
| $A$ | : ordering cost per order |
| $h$ | : unit holding cost per year |
| $\pi$ | : unit shortage/backorder cost |
| $\theta$ | : constant deteriorating rate |
| $Q$ | : order quantity |

### B. Model Development

The dynamic of the inventory is represented in Figure 1. At the beginning, there are $Q-M$ inventory available for the consumers. The inventory later depletes due to the time- and inventory-dependent demand and deterioration. At $t = t_1$, no inventory is available, and demand arrives after $t_1$ will be fully back ordered, until it reaches its maximum of $M$.

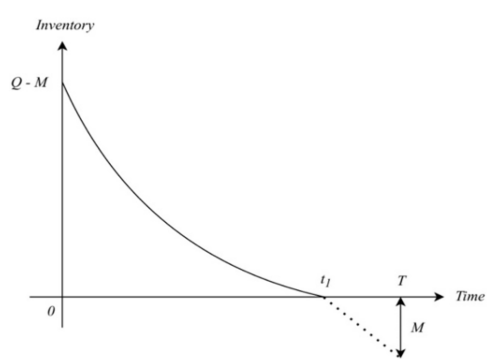


Fig. 1: Inventory dynamics

In developing our model, the basic assumption used is time- and inventory-dependent demand as given by the following equation. In the equation, the demand is depending not only on time $t$, but also on the current inventory $I(t)$, during period of $0 \le t \le t_1$ and then constant after $t = t_1$.

$$D(t) = \begin{cases} a + bI(t), & 0 \le t \le t_1 \\ a, & t_1, \le t \le T. \end{cases}$$

The number of inventories in the interval $[0, T]$ will decrease due to demand and deterioration that can be described mathematically using the following first order differential equation:

$$\frac{dI(t)}{dt} = \begin{cases} -\theta I(t) - (a + bI(t)), & 0 \le t \le t_1 \\ -a, & t_1 \le t \le T \end{cases}$$

When $t = t_1$, there is no inventory on hand, thus ones have $I(t_1) = 0$. Therefore, we have the solution for $I(t)$, when $0 \le t \le t_1$ as follows.

$$I(t) = \frac{-a + ae^{(\theta+b)(t_1-t)}}{(\theta+b)}$$

In the interval $t_1 \le t \le T$, ones have

$$\frac{dI(t)}{dt} = -a$$
$$I(t) = \int_{t_1}^{t} -a dx = -a(t - t_1).$$

Then, for $t \in [0, \ T]$, the dynamics of the inventory can be described as

$$I(t) = \begin{cases} \frac{-a+ae^{(\theta+b)(t_1-t)}}{(\theta+b)}, & 0 \le t \le t_1 \\ -a(t - t_1), & t_1 \le t \le T \end{cases}$$

Using the initial condition that at the beginning there are $Q - M$ inventory on hand, ones have $I(0) = Q - M = Q - a(T - t_1)$, and the following connection applies.

$$Q - a(T - t_1) = \frac{-a + ae^{(\theta+b)t_1}}{(\theta+b)}.$$

The inventory number during $[0, t_1]$ can be calculated as follows.

$$\begin{aligned} \alpha &= \int_0^{t_1} I(t)\,dt \\ &= \frac{a}{(\theta+b)} \int_0^{t_1} \left[e^{(\theta+\beta)(t_1-t)} - 1\right] dt \\ &= \frac{a}{(\theta+b)^2}\left[e^{(\theta+b)t_1} - 1\right] - \frac{a}{(\theta+b)} t_1. \end{aligned}$$

The number of shortages during $[t_1, T]$ is given by

$$K = \int_{t_1}^{T} a(t - t_1)\,dt = \frac{1}{2} a\left(T^2 + t_1^2\right) - a t_1 T.$$

### C. Model Formulation

In the model we developed, the objective is to find the optimal time between replenishment and the optimal time when inventory drops to zero that minimizes the total inventory cost. The total inventory cost (TC) consists of the purchasing cost (PC), the ordering cost (OC), the annual holding cost (HC), and the backorder/shortage cost (SC).

The purchasing cost is calculated based on the purchase price $P$ times the $Q$ order quantity divided by the time between replenishment $T$.

$$PC = \frac{PQ}{T} = \frac{P}{T}\left(\frac{-a + ae^{(\theta+b)t_1}}{(\theta+b)} + a(T - t_1)\right).$$

The ordering cost is obtained by multiplying the ordering cost per order and the frequency of ordering, which is given by $\frac{1}{T}$. Therefore, the ordering cost is given by

$$OC = \frac{A}{T}.$$

The unit holding cost times the inventory number during $[0,t_1]$, divided by $T$ to compose the holding cost.

$$HC = \frac{h}{T}\alpha = \frac{h}{T}\left(\frac{a}{(\theta+b)^2}\left[e^{(\theta+b)t_1}-1\right]-\frac{a}{(\theta+b)}t_1\right).$$

The shortage cost is calculated from the unit shortage cost times the number of shortages during $[t_1,T]$, divided by $T$.

$$SC = \frac{\pi}{T}K \ = \frac{\pi}{T}\left(\frac{1}{2}a\left(T^2+t_1^2\right)-at_1T\right).$$

Adding up those costs, ones have the total inventory cost as follows.

$$\begin{aligned} TC = & \frac{P}{T}\left(\frac{-a+ae^{(\theta+b)t_1}}{(\theta+b)}+a(T-t_1)\right)+\frac{A}{T} \\ & +\frac{h}{T}\left(\frac{a}{(\theta+b)^2}\left[e^{(\theta+b)t_1}-1\right]-\frac{a}{(\theta+b)}t_1\right) \\ & +\frac{\pi}{T}\left(\frac{1}{2}a\left(T^2+t_1^2\right)-at_1T\right). \end{aligned}$$

The decision variables in the formulation of the total cost above are $T$ and $t_1$. The necessary conditions for the optimal $T$ and $t_1$ that minimize the total cost are

$$\frac{\partial TC}{\partial T} = 0 \text{ and } \frac{\partial TC}{\partial t_1} = 0.$$

$$\begin{aligned} \frac{\partial TC}{\partial T} = & -\frac{1}{T^2}\left\{P\left(\frac{-a+ae^{(\theta+b)t_1}}{(\theta+b)}-at_1\right)+A+\frac{\pi a t_1^2}{2}\right. \\ & \left.+h\left(\frac{a}{(\theta+b)^2}\left[e^{(\theta+b)t_1}-1\right]-\frac{a}{(\theta+b)}t_1\right)\right\}+\frac{\pi a}{2} \\ = & \ 0. \end{aligned}$$

$$\begin{aligned} T^2 = & \frac{2}{\pi a}\left\{P\left(\frac{-a+ae^{(\theta+b)t_1}}{(\theta+b)}-at_1\right)+A+\frac{\pi a t_1^2}{2}\right. \\ & \left.+h\left(\frac{a}{(\theta+b)^2}\left[e^{(\theta+b)t_1}-1\right]-\frac{a}{(\theta+b)}t_1\right)\right\}. \end{aligned}$$

$$\begin{aligned} \frac{\partial TC}{\partial t_1} = & \frac{P}{T}\left(ae^{(\theta+b)t_1}-a\right)+\frac{\pi a t_1}{T}-\pi a \\ & +\frac{h}{T}\left(\frac{a}{(\theta+b)}e^{(\theta+b)t_1}-\frac{a}{(\theta+b)}\right)=0. \end{aligned}$$

The sufficient condition for minimum TC regarding the conditions below should also be satisfied.

$$\frac{\partial^2 TC}{\partial T^2}\frac{\partial^2 TC}{\partial t_1^2}-\left(\frac{\partial^2 TC}{\partial T \partial t_1}\right)^2 \text{ and } \frac{\partial^2 TC}{\partial T^2} > 0.$$

## III. Numerical Experiments and Sensitivity Analysis

To give a better understanding of our model, a numerical example is provided with the following hypothetical parameters' value:

$$\begin{aligned} & a = 200 \text{ units}, \ b = 0.1, \ \theta = 0.1, \ A = .800, \\ & h = 400, \ \pi = 100, \ P = 12{,}000, \end{aligned}$$

The optimal time between replenishment $T$ and the optimal time when inventory drops to zero $t_1$ that minimize the total inventory cost are $T = 0.2878$ and $t_1 = 0.0099$, with the order quantity $Q = 58$ and the total inventory cost $TC = 2{,}405{,}558$. Therefore, from this numerical experiment, optimal time between replenishment is 0.2878 year (around 3.45 months) and the time when inventory drops to zero is 0.0099 year (around 0.12 months). This is due to higher holding cost *(h)* compared to backorder cost $(\pi)$ that makes $t_1$ a lot shorter that $T$.

We also perform a sensitivity analysis to analyse the effect of changing one of the aforementioned parameters' values on the time between replenishment, optimal time when inventory drops to zero, order quantity and total inventory cost. Each parameter's values are increased or decreased from $\pm 5\%$, $\pm 10\%$ and $\pm 20\%$, while other parameters' values are kept fixed from the above numerical example. Results are presented in Table I.

From Table I, we can conclude the following.

- When $\theta$ increases, inventory depletes faster (due to demand and deterioration) making $t_1$ and time between replenishment shorter. Consequently, the total inventory cost increases, although there is no significant effect on the order quantity.
- As the autonomous demand, $a$ increases, inventory reaches zero in a shorter time and the time between replenishment also becomes shorter. The order quantity increases due to the higher demand and obviously the total inventory cost increases. This condition also happens when the inventory-dependent demand factor $b$ increases, although the effect on the order quantity is relatively small compared to the autonomous demand, $a$.
- When the ordering cost A is higher, the order quantity increases, making $t_1$ and time between replenishment longer. However, higher ordering cost, in this case, making the total inventory cost lower. The company will order more, and with the specific parameters' values, making the total inventory cost slightly lower.
- When holding cost per unit per year, $h$, increases, theoretically, the order quantity decreases, making $t_1$ and time between replenishment shorter. From the managerial point of view, the company avoids storing goods and prefer to apply the backorder policy. However, in this sensitivity analysis, the effect of the increase of holding cost per unit per year on order quantity, $t_1$ and time between replenishment is not quite significant. Moreover, effect on the total inventory cost is not significant as well.
- The increase of the shortage cost per unit will make the length of $T - t_1$ shorter as the company tries to avoid shortages and prefer to store goods as indicated by the

TABLE I: Effect on changing parameter's values of $\theta$, $a$, $b$, $A$, $h$ and $V$ on $T$, $t_1$, $Q$ and $TC$

| Parameter | Percentage | $T$ | $t_1$ | $Q$ | $TC$ |
|---|---|---|---|---|---|
| | -20% | 0,2883 | 0,0108 | 58 | 2.405.549 |
| | -10% | 0,2880 | 0,0103 | 58 | 2.405.554 |
| | -5% | 0,2880 | 0,0101 | 58 | 2.405.556 |
| $\theta$ | 0 | 0,2878 | 0,0099 | 58 | 2.405.558 |
| | 5% | 0,2877 | 0,0097 | 58 | 2.405.560 |
| | 10% | 0,2876 | 0,0095 | 58 | 2.405.562 |
| | 20% | 0,2874 | 0,0091 | 57 | 2.405.566 |
| | -20% | 0,3218 | 0,0110 | 51 | 1.924.971 |
| | -10% | 0,3034 | 0,0104 | 55 | 2.165.273 |
| | -5% | 0,2953 | 0,0101 | 56 | 2.285.417 |
| $a$ | 0 | 0,2878 | 0,0099 | 58 | 2.405.558 |
| | 5% | 0,2809 | 0,0096 | 59 | 2.525.695 |
| | 10% | 0,2744 | 0,0094 | 60 | 2.645.829 |
| | 20% | 0,2627 | 0,0090 | 63 | 2.886.089 |
| | -20% | 0,2883 | 0,0108 | 58 | 2.405.549 |
| | -10% | 0,2880 | 0,0103 | 58 | 2.405.554 |
| | -5% | 0,2879 | 0,0101 | 58 | 2.405.556 |
| $b$ | 0 | 0,2878 | 0,0099 | 58 | 2.405.558 |
| | 5% | 0,2877 | 0,0097 | 58 | 2.405.560 |
| | 10% | 0,2876 | 0,0095 | 58 | 2.405.562 |
| | 20% | 0,2874 | 0,0091 | 57 | 2.405.566 |
| | -20% | 0,2574 | 0,0088 | 51 | 2.404.971 |
| | -10% | 0,2730 | 0,0094 | 55 | 2.405.273 |
| | -5% | 0,2805 | 0,0096 | 56 | 2.405.417 |
| $A$ | 0 | 0,2878 | 0,0099 | 58 | 2.405.558 |
| | 5% | 0,2949 | 0,0101 | 59 | 2.405.695 |
| | 10% | 0,3018 | 0,0103 | 60 | 2.405.829 |
| | 20% | 0,3153 | 0,0108 | 63 | 2.406.089 |
| | -20% | 0,2879 | 0,0102 | 58 | 2.405.555 |
| | -10% | 0,2879 | 0,0100 | 58 | 2.405.557 |
| | -5% | 0,2878 | 0,0099 | 58 | 2.405.557 |
| $h$ | 0 | 0,2878 | 0,0099 | 58 | 2.405.558 |
| | 5% | 0,2878 | 0,0098 | 58 | 2.405.559 |
| | 10% | 0,2877 | 0,0097 | 58 | 2.405.559 |
| | 20% | 0,2877 | 0,0096 | 58 | 2.405.561 |
| | -20% | 0,3207 | 0,0089 | 64 | 2.404.988 |
| | -10% | 0,3028 | 0,0094 | 61 | 2.405.282 |
| | -5% | 0,2950 | 0,0096 | 59 | 2.405.422 |
| $\pi$ | 0 | 0,2878 | 0,0099 | 58 | 2.405.558 |
| | 5% | 0,2811 | 0,0101 | 56 | 2.405.690 |
| | 10% | 0,2749 | 0,0103 | 55 | 2.405.819 |
| | 20% | 0,2636 | 0,0108 | 53 | 2.406.068 |

longer $t_1$. The time between replenishment tends to be shorter although the $t_1$ a bit longer, but still making $T - t_1$ shorter.

From the results presented in Table I, we will now analyze the effect of changing multiple parameters that has interaction effects on the decision variables. In particular, the effect of changing the values of the inventory-dependent demand factor $b$ and deterioration rate $\theta$, together on the values of $T$, $t_1$, $Q$ and $TC$ is analyzed. Based on the results presented in Table II below, one can see these following points.

- For specific values of $b$, as the deterioration factor $\theta$ is getting larger, the time between replenishment and the time the inventory drops to zero become shorter. This fact is due to the increase of deteriorated items as $\theta$ gets larger. Consequently, the order quantity is slightly less and the total inventory cost increasing, although it is not quite substantial.
- On the other hand, when deterioration rate is kept at certain value and the inventory-dependent demand factor gets larger, $T$ and $t_1$ are also getting shorter. This is due to the increasing demand as $b$ gets larger. The order quantity becomes smaller and the total cost increases slightly.
- Larger values of $b$ and $\theta$ also make $T$ and $t_1$ become shorter, making the order quantity slightly lower and the total inventory cost slightly increases. In this situation the inventory drops to zero faster and with some specific parameters' values, the total inventory cost becomes slightly higher.

TABLE II: Effect of $b$ and $\theta$ on $T$, $t_1$, $Q$ and $TC$

| | $b \backslash \theta$ | 0.1 | 0.3 | 0.5 |
|---|---|---|---|---|
| $T$ | | 0,2878 | 0,2855 | 0,2846 |
| $t_1$ | | 0,0099 | 0,0053 | 0,0036 |
| $Q$ | 0.1 | 58 | 57 | 57 |
| $TC$ | | 2.405.558 | 2.405.603 | 2.405.620 |
| $T$ | | 0,2855 | 0,2846 | 0,2842 |
| $t_1$ | | 0,0053 | 0,0036 | 0,0028 |
| $Q$ | 0.3 | 57 | 57 | 57 |
| $TC$ | | 2.405.603 | 2.405.620 | 2.405.628 |
| $T$ | | 0,2846 | 0,2842 | 0,2839 |
| $t_1$ | | 0,0036 | 0,0028 | 0,0022 |
| $Q$ | 0.5 | 57 | 57 | 57 |
| $TC$ | | 2.405.620 | 2.405.628 | 2.405.634 |

## IV. CONCLUSION AND FURTHER RESEARCH

A mathematical model for deteriorated item with time- and inventory-dependent demand and backorder has been developed in this paper. Numerical example and sensitivity analysis were performed to give illustration for better understanding of the model. We found that the time between replenishment and when inventory drops to zero are affected by demand, deterioration rate, unit holding cost, unit order cost, and unit shortage cost. Consequently, the order quantity and the total inventory cost are also affected, although the changes are not quite significant for some parameters.

Despite the model we have developed and some findings there are still limitations in our model. We only consider one item, one purchase price and constant deterioration rate. Some insights for further research are still available to be researched such as considering discount scheme from the supplier either all-unit discount or incremental discount, developing model for multi-item and considering non constant deterioration rate, e.g. Weibull deterioration rate.

## ACKNOWLEDGMENTS

The authors would like to express the gratitude to The Directorate of Research, Technology and Community Service, Directorate General of Higher Education Re-public of Indonesia for funding this research under the grant scheme numbers 180/E5/PG.02.00.PL/2023, 040/SP2H/RT-MONO/LL4/2023, and III/LPPM/2023-07/122-PE. Counterpart funds from the Institute of Research and Community Service Parahyangan Catholic University under the grant scheme number III/LPPM/2023-06/112-P are greatly appreciated.

## REFERENCES

[1] L. Chen and X. Chen and M.F. Keblis and G. Li, "Optimal pricing and replenishment policy for deteriorating inventory under stock-level-dependent, time-varying and price-dependent demand," *Computers & Industrial Engineering*, vol. 135, pp. 1294–1299, 2019.

[2] U. Chaudhari and A. Bhadoriya and M.Y. Jani and B. Sarkar, "A generalized payment policy for deteriorating items when demand depends on price, stock and advertisement under carbon tax regulations," *Mathematics and Computer in Simulations*, vol. 207, pp. 556–574, 2023.

[3] M. A.-A. Khan and A. A. Shaikh and I. Konstantaras and A.K. Bhunia and L.E. Cárdenas-Barrón, "Inventory models for perishable items with advanced payment, linearly time-dependent holding cost and demand dependent on advertisement and selling price," *International Journal of Production Economics*, vol. 230, p. 107804, December 2020.

[4] N. Loedy and D. Lesmono and T. Limansyah, "An Inventory-Dependent Demand Model with Deterioration, All-Units Discount, and Return," in *IOP Conf Series: Journal of Physics*, vol. 1108, 2018, p. 012010.

[5] M. Önal and A. Yenipazarli and O.E. Kundakcioglu, "A mathematical model for perishable products with price- and displayed-stock-dependent demand," *Computers & Industrial Engineering*, vol. 102, pp. 246–258, December 2016.

[6] M. Sebatjane and O. Adetunji, "Optimal lot sizing and shipment decisions in a three-echelon supply chain for growing items with inventory level- and expiration date-dependent demand," *Applied Mathematical Modelling*, vol. 90, pp. 1204–1225, 2021.

[7] S. Ouret, "Production control problem with semi-Markov jump under stochastic demands and deteriorating inventories," *Applied Mathematical Modelling*, vol. 107, pp. 85–102, 2022.

[8] H.-L. Yang, "An optimal replenishment cycle and order inventory model for deteriorating items with fluctuating demand," *Supply Chain Analytics*, vol. 3, p. 100021, 2023.

[9] S. Priyan and P. Mala, "Optimal inventory system for pharmaceutical products incorporating quality degradation with expiration date: A game theory approach," *Operations Research for Health Care*, vol. 24, p. 100245, 2020.

[10] Z.T. Balkhi and L. Benkherouf, "On an inventory model for deteriorating items with stock dependent and time-varying demand rates," *Computers & Operations Research*, vol. 31, pp. 223–240, 2004.

[11] V.K. Mishra and L.S. Singh and R. Kumar, "An inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging," *Journal of Industrial Engineering International*, vol. 9, 2013.

[12] L. Feng and Y.-C. Chan and L. E. Cárdenas-Barrón, "Pricing and lot-sizing policies for perishable goods when the demand depends on selling price, displayed stocks, and expiration date," *International Journal of Production Economics*, vol. 185, pp. 11–20, 2017.

[13] T. Limansyah and D. Lesmono, "A Mathematical Model for Inventory and Price-Dependent Demand with All-Units Discount," in *IOP Conf. Series: Journal of Physics*, vol. 1490, 2020, p. 012051.

[14] A. Macias-López and L. E. Cárdenas-Barrón and R.E. Peimbert-Garcia and B. Mandal, "An inventory model for perishable items with price-, stock-, and time-dependent demand rate considering shelf-life and nonlinear holding costs," *Mathematical Problems in Engineering*, vol. 2021, p. 6630938, 2021.

[15] M. Palanivel and M. Suganya, "Partial backlogging inventory model with price and stock level dependent demand, time varying holding cost and quantity discounts," *Journal of Management Analytics*, vol. 9, no. 1, pp. 32–59, 2022.

[16] V. Pando and L.A. San-José and J. Sicilia and D. Alcaide-López-de-Pablo, "Maximization of the return on inventory management expense in a system with price- and stock-dependent demand rate," *Computers & Operations Research*, vol. 127, p. 105134, March 2021.

[17] N.H. Shah and M.K. Naik, "Inventory Policies for Price-Sensitive Stock-Dependent Demand," *International Journal of Mathematical, Engineering and Management Sciences*, vol. 3, no. 3, pp. 245–257, 2018.

[18] N.H. Shah and K. Rabari and E. Patel, "A Deteriorating Inventory Model under Overtime Production and Credit Policy for Stock- and Price Sensitive Demand Function," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 5, no. 2, pp. 85–98, August 2022.

[19] S. Saha and N. Sen, "An inventory model for deteriorating items with time and price dependent demand and shortages under the effect of inflation," *International Journal of Mathematics in Operational Research*, vol. 14, no. 3, pp. 377–388, 2019.

[20] M. Bakker and J. Riezebos and R.H. Teunter, "Review of inventory systems with deterioration since 2011," *European Journal of Operational Research*, vol. 221, pp. 275–284, 2012.

[21] T. Limansyah and D. Lesmono and I. Sandy, "Economic order quantity model with deterioration factor and all-units discount," in *IOP Conf: Series: Journal of Physics*, vol. 1490, 2020, p. 012052.

[22] S.W. Setiawan and D. Lesmono and T. Limansyah, "A Perishable Inventory Model with Return," in *IOP Conf. Series: Materials Science and Engineering*, vol. 335, 2018, p. 012049.

[23] S.K. Goyal and B.C. Giri, "Recent trends in modeling of deterioration inventory," *European Journal of Operational Research*, vol. 134, pp. 1–16, 2001.