

The Application of Max-Plus Algebra to Determine The Optimal Time of Ikat Kupang Woven Production

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Abstract—The problem of scheduling is a problem that becomes part of project management that aims to plan the implementation of activities in a project in a structured manner with a clear time limit. Kupang ikat woven is an Indonesian handicraft that the process of making it through the stages of structured activities with a long time limit. Thus, in this study we intend to estimate the optimal time required to manufacture the Kupang ikat woven by applying the Max-Plus algebra method. In this case, the linkages between activities within a project can be transformed into a matrix form which can be analyzed using Max-Plus algebra method. This matrix will be applied in the calculation to get the solution needed in project scheduling, in this case is the optimum time required of making Kupang ikat woven.

Index Terms—Max-Plus Algebra, Scheduling Projects, Optimum time, Kupang Ikat Woven.

I. INTRODUCTION

IKAT Kupang woven is one kind of Indonesian handicraft which made form cloth that comes from East Nusa Tenggara (NTT) province. Generally, Ikat Kupang woven handicraft has great potential to be developed in order to increase the potential of the regional economy and the community of weaving craftsmen in particular. However, the problems that arise in the making of traditional Ikat Kupang woven is no time calculations and fixed scheduling. So far, the calculation time of making Ikat Kupang woven only based on estimates without any indicator of the right time [1]. The process of making Ikat Kupang woven itself has several stages, namely: arrangement of yarn on the tool, binding motif and decoration, coloring, and weaving [2]. So because there is no optimal time in the process of making, as a result the process of making Ikat Kupang woven from one stage to another stage has a delay time which less of productive.

In 2016, Lusiana P. conducted a study on the calculation time of making ikat kupang woven using the critical path method (CPM) method. From the result of the research, it was found that to produce one handicraft ikat kupang woven took 20 days of making process. Another method that can be used to get optimal time is Algebra Max-Plus.

The Max-Plus algebra is an algebraic structure consisting of real numbers sets joined with $-\infty$, where the standard

operations of addition, \oplus , defined as operation of taking a maximum and the standard operations of multiplication, \otimes , defined as operation of standard addition on real number sets. Related to this problem, Max-Plus algebra can be used to optimize the manufacturing time on Kupang bundling making process, so that the making time can be used efficiently and effectively. So with reference to the above background then in this study will be discussed on how to determine the optimal time for making ikat kupang woven using Algebra Max-Plus.

II. LITERATURE STUDY

A. Ikat Kupang Woven

One of the traditional weaving craft is relatively well known by the people of East Nusa Tenggara, namely ikat kupang woven. Named "weaving" because before being colored, the threads to be woven are tied with raffia ropes on certain parts, then dyed into the dyeing fluid. The section tied to the raffia, once opened, remains white, while the unattached parts of the raffia become colored in color to the liquid. The color composition of the threads is colored and there are parts that remain white. At the time of woven will form patterns of decoration with certain colors. The yarn used for weaving is made of cotton or silk, which is specially used for ikat weaving [2]. The following will describe the process of making fabrics ikat kupang woven that has several stages:

- a. Provision of raw materials
Raw material is the initial capital of the crafters because without any raw materials, the production process of woven fabric can not work.
- b. Structuring the yarn on the tool
Structuring the yarn is done by inserting the feed yarn (yarn in transverse position) repeatedly and alternating on the thread lungsi (yarn in longitudinal position).
- c. Binding motifs and decorations
The part of the thread to be left is white tied with a raffia rope, while the unbound part will be colored.
- d. Coloring
The yarn is dipped into the color liquid obtained from the concoction of leaves and vegetation. The coloring process takes a long time to allow the dye to completely seep into the yarn.
- e. Drying
After dyed, the colored yarn is drained and dried by air-aerated.

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f. Unlock the motive bond

After drying, the raffia straps that bind the yarn are opened and the result of the threads has a combination of white and dyed colors.

g. Weaving

After the staining process is complete and the thread has completely dried, then the thread is mounted on a loom or ATBM (non-machine loom).

In this case, we assumed that both of cost and quantities of order is neglected. So that, based on above explanation, both of activities and time durations on the Kupang Ikat Woven production is described on Table I and network illustration of that is described on Figure 1.

TABLE I
LIST ACTIVITIES AND TIME DURATIONS OF IKAT KUPANG WOVEN PRODUCTION

Activities	Declaration	Following Activities	Time (day)
A	Provision of raw materials	B	4
B	Structuring the yarn on the tool	C,D	1
C	Binding motifs and decorations	F	4
D	Coloring	E	1
E	Drying	F	2
F	Unlock the motive bond	G	1
G	Weaving	I	10

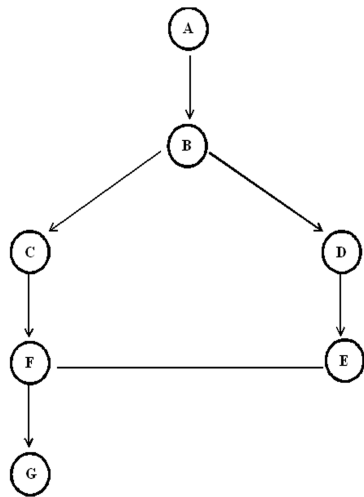


Fig. 1. Network of production.

B. Max-Plus Algebra

The Max-Plus algebra is an algebraic structure consisting of real numbers where the standard operations of addition and multiplication are replaced by the operation of taking a maximum and the operation of standard addition, respectively. More precisely, let \mathbb{R}_{max} denote the set $\mathbb{R} \cup \{-\infty\}$, let \oplus be a binary operator on \mathbb{R}_{max} with $x \oplus y = \max(x, y)$, and let \otimes be a binary operator on \mathbb{R}_{max} with $x \otimes y = x + y$. Then the Max-Plus Algebra is the algebraic structure consisting of \mathbb{R}_{max} and the binary operations \oplus and \otimes , denoted by $(\mathbb{R}_{max}, \oplus, \otimes)$ [3].

The Max-Plus Algebra can be extended to matrices. Operations on matrix elements are required to perform calculations on scheduling project. Let matrix set $n \times n$ in Max-Plus algebra denoted by $\mathbb{R}_{max}^{n \times m}$. For $n \in \mathbb{N}$, where $n \neq 0$, defined that $\underline{n} \stackrel{def}{=} \dots$. Let $A \in \mathbb{R}_{max}^{n \times m}$ where the i^{th} row and the j^{th} column denoted by $a_{i,j}$ for $i \in n$ and $j \in m$. In this case matrix A stated as:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,m} \end{bmatrix}$$

Occasionally, element $a_{i,j}$ also denoted as:

$$[A]_{i,j}, i \in n, j \in m$$

An addition matrix $A, B \in \mathbb{R}_{max}^{n \times m}$ denoted as $A \oplus B$ defined by:

$$\begin{aligned} [A \oplus B] &= a_{i,j} \oplus b_{i,j} \\ &= \max\{a_{i,j}, b_{i,j}\} \end{aligned}$$

for $i \in n$ and $j \in m$. While, for $A \in \mathbb{R}_{max}^{n \times p}$ and $B \in \mathbb{R}_{max}^{p \times m}$, a multiplication of matrix $A \otimes B$, defined as : with $i \in n$ and

$$\begin{aligned} [A \otimes B] &= \bigotimes_{k=1}^p a_{i,k} \otimes b_{k,j} \\ &= \max_{k \in p} \{a_{i,k} + b_{k,j}\} \end{aligned}$$

$j \in m$. The multiplication of this matrix is similar with the multiplication of general matrix algebra, where $+$ and \times operation replaced by \max and $+$, respectively. Then, for $A \in \mathbb{R}_{max}^{n \times n}$, the k^{th} rank of A denoted by $A^{\otimes k}$ defined as follow:

$$A^{\otimes k} = \underbrace{A \otimes A \otimes \cdots \otimes A}_k$$

for $k \in \mathbb{N}$ and $k \neq 0$. The following are the steps to find project scheduling solution using Max-Plus algebra :

- 1) Formulate the existing data into diagram. Give additional activities (α, ω) on diagram. Each activities i that has no predecessor activity, give direction from α to i with weight 0. Each activities j that has no activity after it, give direction from j to ω with the weight of the completion time of activity j . Give direction on activity i toward activity j that are related to the same weight with the length of activity process of i .
- 2) Create a Max-Plus Matrix X , where element $x_{i,j}$ is weight from activity i to weight j . If i and j are not related, then element $x_{i,j}$ has value $-\infty$.
- 3) Calculate $X^* = X \oplus X^2 \oplus \cdots \oplus X^{n+1}$, where n is frequency of activities before added by α, ω .
- 4) The optimum completing time is $x_{\alpha\omega}^*$.
- 5) Find a value of the furthest route vector (V) and vector slack (S). Both of that vector has size $n \times 1$ without contain α, ω .

$$\begin{aligned} v_i &= x_{\alpha}^* \otimes x_{i\omega}^* \\ s_i &= x_{\alpha\omega}^* - v_i \end{aligned}$$

The critical path is a element on vector slack that has value ($s_i = 0$).

To get the optimum time, we should to find the matrix X^* where is:

$$X^* = X \oplus X^2 \oplus \dots \oplus X^8$$

$$= \begin{pmatrix} -\infty & 0 & 4 & 5 & 5 & 6 & 9 & 10 & 20 \\ -\infty & -\infty & 4 & 5 & 5 & 6 & 9 & 10 & 20 \\ -\infty & -\infty & -\infty & 1 & 1 & 2 & 5 & 6 & 16 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 4 & 5 & 15 \\ -\infty & -\infty & -\infty & -\infty & -\infty & 1 & 3 & 4 & 14 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 2 & 3 & 13 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 1 & 11 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 10 \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & -\infty \end{pmatrix}$$

The optimum time fot Ikat Kupang Woven production is $X_{\alpha\omega}^* = 0.20$. So, the optimum time to produce Ikat Kupang Woven is 20 days.

The next steps is find critical path. Searching process of critical path has advantageous to know whichever activity that can't delayed its implementation. Critical path obtained by getting vector V where the i th (v_i) element of it obtained from :

$$v_i = x_{\alpha i}^* \otimes x_{i\omega}^*$$

and the i th element of vector S stated as:

$$s_i = x_{\alpha\omega}^* - v_i \tag{1}$$

where $i = A, B, C, D, E, F, G$. sehingga

$$V = \begin{pmatrix} v_A \\ v_B \\ v_C \\ v_D \\ v_E \\ v_F \\ v_G \end{pmatrix} = \begin{pmatrix} x_{\alpha A}^* \otimes x_{A\omega}^* \\ x_{\alpha B}^* \otimes x_{B\omega}^* \\ x_{\alpha C}^* \otimes x_{C\omega}^* \\ x_{\alpha D}^* \otimes x_{D\omega}^* \\ x_{\alpha E}^* \otimes x_{E\omega}^* \\ x_{\alpha F}^* \otimes x_{F\omega}^* \\ x_{\alpha G}^* \otimes x_{G\omega}^* \end{pmatrix} = \begin{pmatrix} 0 \otimes 20 \\ 4 \otimes 16 \\ 5 \otimes 15 \\ 5 \otimes 14 \\ 6 \otimes 13 \\ 9 \otimes 11 \\ 10 \otimes 10 \end{pmatrix} = \begin{pmatrix} 20 \\ 20 \\ 20 \\ 19 \\ 19 \\ 20 \\ 20 \end{pmatrix} \tag{2}$$

and

$$S = \begin{pmatrix} s_A \\ s_B \\ s_C \\ s_D \\ s_E \\ s_F \\ s_G \end{pmatrix} = \begin{pmatrix} x_{\alpha\omega}^* - v_A \\ x_{\alpha\omega}^* - v_B \\ x_{\alpha\omega}^* - v_C \\ x_{\alpha\omega}^* - v_D \\ x_{\alpha\omega}^* - v_E \\ x_{\alpha\omega}^* - v_F \\ x_{\alpha\omega}^* - v_G \end{pmatrix} = \begin{pmatrix} 20 - 20 \\ 20 - 20 \\ 20 - 20 \\ 20 - 19 \\ 20 - 19 \\ 20 - 20 \\ 20 - 20 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

Critical path is an element on vector S that has value 0. While $s_i \neq 0$ show time duration for activity that can't be delayed. So, activity that can't be delayed are activity A, B, C, F,dan G. While, activity E and G can be delayed only for one day. This result is suitable with Prastiwi (2016).

IV. CONCLUSIONS

To find the optimal time on Ikat Kupang Woven production, we can use Max-Plus Algebra methode. From the result and discussion, we can conclude that we need 20 days to produce a Ikat Kupang Woven.

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