L-Fuzzy Filters of a Poset

Berhanu Assaye Alaba, Mihret Alamneh and Derso Abeje

***Abstract*—Many generalizations of ideals and filters of a lattice to an arbitrary poset have been studied by different scholars. The authors of this paper introduced several generalizations of L-fuzzy ideal of a lattice to an arbitrary poset in [1]. In this paper, we introduce several L-fuzzy filters of a poset which generalize the L-fuzzy filter of a lattice and give several characterizations of them.**

***Index Terms*—Poset, Filter, L-fuzzy closed filter, L-fuzzy Frink filter, L-fuzzy V-Filter, L-fuzzy semi-filter, L-fuzzy filter, l-L-fuzzy filter.**

I. INTRODUCTION

WE have found several generalizations of ideals and filters of a lattice to arbitrary poset (partially ordered set) in a literature. Birkhoff in [2, p. 59] introduced a closed or normal ideals who gives accredit to the work of Stone in [3]. Next, in 1954 the second type of ideal and filter of a poset called Frink ideal and Frink filter have been introduced by O. Frink [4]. Following this P. V. Venkatanarasimhan developed the theory of semi ideals and semi filter in [5] and ideals and filters for a poset in [6], in 1970. These ideals (respectively, filters) are called ideals (respectively, filter) in the sense of Venkatanarasimhan or V-ideals (V-filters) for short. Later Halaś [7], in 1994, introduced a new ideal and filter of a poset which seems to be a suitable generalization of the usual concept of ideal and filter in a lattice. We will simply call it ideal (respectively, filter) in the sense of Halaš.

Moreover, the concept of fuzzy ideals and filters of a lattice has been studied by different authors in series of papers [8], [9], [10], [11] and [12]. The aim of this paper is to notify several generalizations of L-fuzzy filters of a lattice to an arbitrary poset whose truth values are in a complete lattice satisfying the infinite meet distributive law and give several characterizations of them. We also prove that the set of all L-fuzzy filters of a poset forms a complete lattice with respect to point-wise ordering "$\subseteq$". Throughout this work, L means a non-trivial complete lattice satisfying the infinite meet distributive law: $x \wedge \sup S = \sup\{x \wedge s : s \in S\}$ for all $x \in L$ and for any subset S of L.

II. PRELIMINARIES

We briefly recall certain necessary concepts, terminologies and notations from [2], [13] and [14]. A binary relation "$\leq$" on a non-empty set Q is called a partial order if it is reflexive, anti-symmetric and transitive. A pair $(Q, \leq)$ is called a partially ordered set or simply a poset if Q is a non-empty set and "$\leq$" is a partial order on Q. When confusion is unlikely, we use simply the symbol Q to denote a Poset $(Q, \leq)$. Let Q be a poset and $S \subseteq Q$. An element x in Q is called a lower bound (respectively, an upper bound) of S if $x \leq a$ (respectively, $x \geq a$) for all $a \in S$. We denote the set of all lower bounds and upper bounds of S by S^l and S^u, respectively. That is $S^l = \{x \in Q : x \leq a \ \forall \ a \in S\}$ and $S^u = \{x \in Q : x \geq a \ \forall \ a \in S\}$. S^{ul} shall mean $\{S^u\}^l$ and S^{lu} shall mean $\{S^l\}^u$. Let $a, b \in Q$. Then $\{a\}^u$ is simply denoted by a^u and $\{a, b\}^u$ is denoted by $(a, b)^u$. Similar notations are used for the set of lower bounds. We note that $S \subseteq S^{ul}$ and $S \subseteq S^{lu}$ and if $S \subseteq T$ in Q then $S^l \supseteq T^l$ and $S^u \supseteq T^u$. Moreover, $S^{lul} = S^l$, $S^{ulu} = S^u$, $\{a^u\}^l = a^l$ and $\{a^l\}^u = a^u$. An element x_0 in Q is called the least upper bound of S or supremum of S, denoted by $supS$ (respectively, the greatest lower bound of S or infimum of S, denoted by $infS$) if $x_0 \in S^u$ and $x_0 \leq x$ $\forall x \in S^u$ (respectively, if $x_0 \in S^l$ and $x \leq x_0 \ \forall x \in S^l$). An element x_0 in Q is called the largest (respectively, the smallest) element if $x \leq x_0$ (respectively, $x_0 \leq x$) for all $x \in Q$. The largest (respectively, the smallest) element if it exists in Q is denoted by 1 (respectively, by 0). A poset $(Q \leq)$ is called bounded if it has 0 and 1. Note that if $S = \emptyset$ we have $S^{lu} = (\emptyset^l)^u = Q^u$ which is equal to the empty set or the singleton set $\{1\}$ if Q has the largest element 1

Now we recall definitions of filters of a poset that are introduced by different scholars.

Definition 2.1 (Dual of [2]): A subset F of a poset $(Q, \leq)$ is said to be a closed or a normal filter in Q if $F^{lu} \subseteq F$.

Definition 2.2 ([4]): A subset F of a poset $(Q, \leq)$ is said to be a Frink filter in Q if $S^{lu} \subseteq F$ whenever S is a finite subset of F.

Definition 2.3 ([5]): A non-empty subset F of a poset $(Q, \leq)$ is called a semi-filter or an order filter of Q if $a \leq b$ and $a \in F$ *implies* $b \in F$.

Definition 2.4 ([6]): A subset F of a poset $(Q, \leq)$ is said to be a V-filter or a filter in the sense of Venkatannarasimhan if F is a semi-filter and for any nonempty finite subset S of F , if $\inf S$ exists, then $\inf S \in F$.

Definition 2.5 ([7]): A subset F of a poset $(Q, \leq)$ is called a filter in Q in the sense of Halaš if $(a, b)^{lu}$ contained in F whenever $a, b \in F$.

Note that every filter of a poset Q defined above contains Q^u.

Remark 2.6: The following remarks are due to R. Halaš and J. Rachůnek [15].

1) If $(Q \leq)$ is a lattice then a non-empty subset F of Q is a filter as a poset if and only if it is a filter as a lattice $(Q \leq)$.
2) If a poset does not have the largest element then the empty subset $\emptyset$ is a filter in $(Q \leq)$ (since $\emptyset^{lu} = (\emptyset^l)^u = Q^u = \emptyset$).

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B.A. Alaba and M. Alamneh are with the College of Science, Bahir Dar University, Bahir Dar, Ethiopia. E-mails: berhanu_assaye@yahoo.com, deab02@yahoo.com

D. Abeje is with the Department of Mathematics, College of Science, University of Gondar, Gondar, Ethiopia. E-mail: mihretmahlet@yahoo.com

Definition 2.7: Let A be any subset of a poset Q. Then the smallest filter containing A is called a filter generated by A and is denoted by $[A)$. The filter generated by a singleton set $\{a\}$, is called a principal filter and is denoted by $[a)$
Note that for any subset S of Q if $\inf S$ exists then $S^{lu} = [\inf S)$.

The followings are some characterizations of filters generated by a subset S of a poset Q. We write $T \subset\subset S$ to mean T is a finite subset of S.

1) The closed or normal filter generated by S, denoted by $[S)_C$, is $[S)_C = \bigcup\{T^{lu} : T \subseteq S\}$ where the union is taken over all subsets T of S.
2) The Frink filter generated by S, denoted by $[S)_F$, is $[S)_F = \bigcup\{T^{lu} : T \subset\subset S\}$, where the union is taken over all finite subsets T of S
3) Define $B_1 = \bigcup\{(a,b)^{lu} : a,b \in S\}$ and $B_n = \bigcup\{(a,b)^{lu} : a,b \in B_{n-1}\}$ for each positive integer $n \geq 2$, inductively. Then the filter generated by S in the sense of Halaš, denoted by $[S)_H$, is $[S)_H = \bigcup\{B_n : n \in \mathscr{N}\}$ where $\mathscr{N}$ denotes the set of positive integers.
4) If $a \in Q$ then $[a) = \{x \in Q : x \leq a\} = a^l$ is the principal ideal generated by a.

Definition 2.8 ([7]): A filter F of a poset Q is called an l-filter if $(x,y)^l \cap F \neq \emptyset$ for all $x,y \in F$.
Note that an easy induction shows that F is an l-filter if $B^l \cap F \neq \emptyset$ for every non-empty finite subset B of F.

Theorem 2.9 ([7]): Let $\mathscr{F}(Q)$ be the set of filters of a poset Q and A and B be l-filters of Q. Then the supremum $A \vee B$ of A and B in $\mathscr{F}(Q)$ is $A \vee B = \bigcup\{(a,b)^{lu} : a \in A, b \in B\}$.

Definition 2.10 ([16]): An L-fuzzy subset η of a poset Q is a function from Q into L.
Note that if L is a unit interval of real numbers [0,1], then the L-fuzzy subset η is the fuzzy subsets of Q which is introduced by L. Zadeh [17]. The set of all L-fuzzy subsets of Q is denoted by L^Q.

Definition 2.11 ([11]): Let $\eta \in L^Q$. Then for each $\alpha \in L$ the set $\eta_\alpha = \{x : \eta(x) \geq \alpha\}$ is called the level subset or level cut of η at α.

Lemma 2.12 ([9]): Let $\eta \in L^Q$. Then $\eta(x) = \sup\{\alpha \in L : x \in \eta_\alpha\}$ for all $x \in Q$.

Definition 2.13 ([16]): Let $\nu, \sigma \in L^Q$. Define a binary relation "$\subseteq$" on L^Q by $\nu \subseteq \sigma$ if and only $\nu(x) \leq \sigma(x)$ for all $x \in Q$.
It is simple to verify that the binary relation "$\subseteq$" on L^Q is a partial order and it is called the *point wise ordering*.

Definition 2.14 ([18]): Let θ and η be in L^Q. Then the union of fuzzy subsets θ and η of X, denoted by $\theta \cup \eta$, is a fuzzy subset of Q defined by $(\theta \cup \eta)(x) = \theta(x) \vee \eta(x)$ for all $x \in Q$ and the intersection of fuzzy subsets θ and η of Q, denoted by $\theta \cap \eta$, is a fuzzy subset of X defined by $(\theta \cap \eta)(x) = \theta(x) \wedge \eta(x)$ for all $x \in Q$.
More generally, the union and intersection of any family $\{\eta_i\}_{i\in\Delta}$ of L-fuzzy subsets of Q, denoted by $\bigcup_{i\in\Delta}\eta_i$ and $\bigcap_{i\in\Delta}\eta_i$ respectively, are defined by:
$(\bigcup_{i\in\Delta}\eta_i)(x) = \sup_{i\in\Delta}\eta_i(x)$ and $\bigcap_{i\in\Delta}\eta_i = \inf_{i\in\Delta}\eta_i(x)$ for all $x \in Q$, respectively.

Definition 2.15 ([10]): An L-fuzzy subset η of a lattice Q with 1 is said to be an L -fuzzy filter of Q; if $\eta(1) = 1$ and $\eta(a \wedge b) = \eta(a) \wedge \eta(b)$ for all $a,b \in Q$.

Definition 2.16: Let η be L- fuzzy subset of a poset Q. The smallest fuzzy filter of Q containing η is called a fuzzy filter generated by η and is denoted by $[\eta)$.

III. L-Fuzzy Filters of a Poset

In this section, we notify the concept of L-fuzzy filters of a poset and give several characterizations of them. Throughout this paper, Q stands for a poset $(Q, \leq)$ with 1 unless otherwise stated. We begin with the following

Definition 3.1: An L-fuzzy subset η of Q is called an L-fuzzy closed filter if it fulfills the following conditions:

1) $\eta(1) = 1$ and
2) for any subset S of Q, $\eta(x) \geq \inf\{\eta(a) : a \in S\}$ $\forall x \in S^{lu}$.

Lemma 3.2: A subset F of Q is a closed filter of Q if and only if its characteristic map χ_F is an L-fuzzy closed filter of Q.

Proof: Suppose F is a closed filter of Q. Since 1 is in $F^{lu} \subseteq F$, we have $\chi_F(1) = 1$. Again let S be any subset of Q and $x \in S^{lu}$. Then if $S \subseteq F$, we have $S^{lu} \subseteq F^{lu} \subseteq F$ and $\chi_F(a) = 1$ for all $a \in S$. Therefore $\chi_F(x) = 1 = \inf\{\chi_F(a) : a \in S\}$. Again if $S \nsubseteq F$, then there is $c \in S$ such that $c \notin F$ and hence $\chi_F(c) = 0$ and hence $\chi_F(x) \geq 0 = \inf\{\chi_F(a) : a \in S\}$. Thus in either cases, $\chi_F(x) \geq \inf\{\chi_F(a) : a \in S\}$ for all $x \in S^{lu}$ and $S \subseteq Q$. Therefore, χ_F is an L-fuzzy closed filter of Q. Conversely, suppose χ_F is an L-fuzzy closed filter. Since $\chi_F(1) = 1$, we have $1 \in F$, that is $\{1\} = Q^u \subseteq F$. Let $x \in F^{lu}$. Then by hypotheses, $\chi_F(x) \geq \inf\{\chi_F(a) : a \in F\} = 1$. This implies $\chi_F(x) = 1$ and hence $x \in F$. Therefore, $F^{lu} \subseteq F$ and hence F is a closed filter. This proves the result. ■

The following result characterizes the L-fuzzy closed filter of Q in terms of its level subsets.

Lemma 3.3: Let η be in L^Q. Then η is an L- fuzzy closed filter of Q if and only if η_α is a closed filter of Q for all $\alpha \in L$.

Proof: Let η be an L- fuzzy closed filter of Q and $\alpha \in L$. Then $\eta(1) = 1 \geq \alpha$ and hence $1 \in \eta_\alpha$, i.e., $\{1\} = Q^u \subseteq \eta_\alpha$. Again let $x \in (\eta_\alpha)^{lu}$. Then $\eta(x) \geq \inf\{\eta(a) : a \in \eta_\alpha\} \geq \alpha$ and hence $x \in \eta_\alpha$. Therefore $(\eta_\alpha)^{lu} \subseteq \eta_\alpha$ and hence η_α is a closed filter.

Conversely, let η_α is a closed filter of Q for all $\alpha \in L$. In particular η_1 is a closed filter. Since $1 \in (\eta_1)^{lu} \subseteq \eta_1$, we have $\eta(1) = 1$.

Again let S be any subset of Q. Put $\alpha = \inf\{\eta(a) : a \in S\}$. Then $\eta(a) \geq \alpha$ $\forall a \in S$ and hence $S \subseteq \mu_\alpha$. This implies $S^{lu} \subseteq \mu_\alpha^{lu} \subseteq \mu_\alpha$. Now $x \in S^{lu} \Rightarrow x \in \eta_\alpha \Rightarrow \eta(x) \geq \alpha = \inf\{\eta(a) : a \in S\}$. Therefore η is an L-fuzzy closed filter of Q. This proves the result. ■

Lemma 3.4: Let η be fuzzy closed filter of a poset Q. Then η is iso-tone, in the sense that $\eta(x) \leq \eta(y)$ whenever $x \leq y$.

Proof: Let $x,y \in Q$ such that $x \leq y$. Put $\eta(x) = \alpha$. Since η is a fuzzy closed filter, η_α is a closed filter of Q and hence $(\eta_\alpha)^{lu} \subseteq \eta_\alpha$. Now $\eta(x) = \alpha \Rightarrow x \in \eta_\alpha \Rightarrow x^u = \{x\}^{lu} \subseteq (\eta_\alpha)^{lu} \subseteq \eta_\alpha$. Thus $x \leq y \Rightarrow$ $y \in x^u \Rightarrow y \in \eta_\alpha$ and hence $\eta(x) = \alpha \leq \eta(y)$. This proves the result. ■

Theorem 3.5: Let $(Q, \leq)$ be a lattice. Then an L-fuzzy subset η of Q is an L- fuzzy closed filter in the poset Q if and only if an L-fuzzy filter in the lattice Q.

Proof: Let η be an L-fuzzy filter in the poset Q and $a,b \in Q$. Then $\eta(1) = 1$ and since $S = \{a,b\} \subseteq Q$ and $a \wedge b \in S^{lu}$, we have $\eta(a\wedge b)\geq \inf\{\eta(x):x\in S\}=\eta(a)\wedge\eta(b)$. Again since η is iso-tone, we have $\eta(a\wedge b)\leq\eta(a)$ and $\eta(a\wedge b)\leq \eta(b)$ and hence we have $\eta(a)\wedge\eta(b)\leq\eta(a\wedge b)$. Therefore $\eta(a\wedge b)=\eta(a)\wedge\eta(b)$ and hence η is an L-fuzzy filter in the lattice Q. Conversely suppose μ be an L-fuzzy filter in the lattice Q. Then $\eta(1)=1$ and $\eta(a\wedge b)=\eta(a)\wedge\mu(b)\ \forall a,b\in Q$. Let $S\subseteq Q$ and $x\in(S)^{lu}$. Then x is an upper bound of $(S)^l$. Since $\inf S\in(A)^l$, we have $x\geq\inf S$ and hence we have $\eta(x)\geq\eta(\inf S)=\inf\{\eta(a):a\in S\}$. Therefore η is an L-fuzzy closed filter in the poset Q. This proves the result. ■

Lemma 3.6: The intersection of any family of L-fuzzy closed filters is an L-fuzzy closed filter.

Theorem 3.7: Let $[S)_C$ be a closed filter generated by a subset S of Q and χ_S be its characteristic functions. Then the $[\chi_S)=\chi_{[S)_C}$.

Proof: Since $[S)_C$ is a closed filter of Q containing S, by Lemma 3.2, we have $\chi_{[S)_C}$ is a fuzzy closed filter. Again since $S\subseteq[S)_C$, clearly we have $\chi_S\subseteq\chi_{[S)_C}$. Now, we show that it is the smallest L- fuzzy closed filter containing χ_S. Let η be an L-fuzzy closed filter such that $\chi_S\subseteq\eta$. Then $\eta(a)=1$ for all $a\in S$. Now we claim $\chi_{[S)_C}\subseteq\eta$. Let $x\in Q$. If $x\notin[S)_C$, then $\chi_{[S)}(x)=0\leq\eta(x)$. If $x\in[S)_C$, then $x\in T^{lu}$ for some subset T of S and hence $\eta(x)\geq\inf\{\eta(b):b\in T\}=1=\chi_{[S)_C}(x)$. Hence in either cases, $\chi_{[S)_C}(x)\leq\eta(x)$ for all $x\in Q$ and hence $\chi_{[S)_C}\subseteq\eta$. This proves the theorem. ■

In the following theorem we characterize a fuzzy closed filter generated by a fuzzy subset of Q in terms of its level closed filters.

Theorem 3.8: Let $\eta\in L^Q$. Then the L-fuzzy subset $\hat{\eta}$ of Q defined by $\hat{\eta}(x)=\sup\{\alpha\in L:x\in[\eta_\alpha)_C\}$ for all $x\in Q$ is a fuzzy closed filter of Q generated by η, where $[\mu_\alpha)_C$ is a closed filter generated by η_α.

Proof: Now we show $\hat{\eta}$ is the smallest fuzzy closed filter containing η. Let $x\in Q$ and put $\eta(x)=\beta$. Then $x\in\eta_\beta\subseteq[\eta_\beta)_C\Rightarrow\beta\in\{\alpha\in L:x\in[\eta_\alpha)_C\}$. Thus $\eta(x)=\beta\leq\sup\{\alpha\in L:x\in[\eta_\alpha)_C\}=\hat{\eta}(x)$ and hence $\eta\subseteq\hat{\mu}$. Again since $\{1\}=Q^u\subseteq[\eta_\alpha)_C$ for all $\alpha\in L$, clearly we have $\hat{\eta}(1)=1$. Let S be any subset of Q and $x\in S^{lu}$. Now $\inf\{\hat{\eta}(a):a\in S\}=\inf\{\sup\{\alpha_a:a\in[\eta_{\alpha_a})_C\}:a\in S\}=\sup\{\inf\{\alpha_a:a\in S\}:a\in[\eta_{\alpha_a})_C\}$. Put $\lambda=\inf\{\alpha_a:a\in S\}$. Then $\lambda\leq\alpha_a$ for all $a\in S$ and hence $[\eta_{\alpha_a})_C\subseteq[\eta_\lambda)_C\ \forall a\in S$. Therefore $S\subseteq[\eta_\lambda)_C$ and hence $x\in S^{lu}\subseteq[\eta_\lambda)^{lu}\subseteq[\eta_\lambda)$. So

$$\begin{aligned}\inf\{\hat{\eta}(a):a\in S\} &= \sup\{\inf\{\alpha_a:a\in S\}:a\in[\eta_{\alpha_a})\}\\ &\leq \sup\{\lambda\in L:x\in[\eta_\lambda)\}\\ &= \hat{\eta}(x)\end{aligned}$$

Therefore $\hat{\eta}$ is an L-fuzzy closed filter. Again let θ be any L-fuzzy closed filter of Q such that $\eta\subseteq\theta$. Then $\eta_\alpha\subseteq\theta_\alpha$ and θ_α is a closed filter for all $\alpha\in L$ and hence $[\eta_\alpha)\subseteq[\theta_\alpha)=\theta_\alpha$. Thus for any $x\in Q$, $\hat{\eta}(x)=\sup\{\alpha\in L:x\in[\eta_\alpha)\}\leq\sup\{\alpha\in L:x\in\theta_\alpha\}=\theta(x)$ and hence $\hat{\eta}\subseteq\theta$. This proves that $\hat{\eta}=[\eta)$. ■

In the following, we give an algebraic characterization of L-fuzzy Closed filter generated by fuzzy subset of Q.

Theorem 3.9: Let $\eta\in L^Q$. Then the fuzzy subset $\overline{\eta}$ defined by

$$\overline{\eta}(x)=\begin{cases}1 & if x=1\\ \sup\{\inf_{a\in S}\eta(a):x\in S^{lu},S\subseteq Q\} & if x\neq 1\end{cases}$$

is a fuzzy closed filter of Q generated by η.

Proof: It is enough to show that $\bar{\eta}=\hat{\eta}$ where $\hat{\eta}$ is an L-fuzzy subset given in the above theorem. Let $x\in Q$. If $x=1$, then $\bar{\eta}(x)=1=\hat{\eta}(x)$. Let $x\neq 0$. Put $A_x=\{\inf_{a\in S}\eta(a):S\subseteq Q$ and $x\in S^{lu}\}$ and $B_x=\{\alpha:x\in[\eta_\alpha)_C\}$. Now we show $\sup A_x=\sup B_x$. Let $\alpha\in A_x$. Then $\alpha=\inf_{a\in A}\eta(a)$ for some subset S of Q such that $x\in S^{lu}$. This implies that $\alpha\leq\eta(a)$ for all $a\in S$ and hence $S\subseteq\eta_\alpha\subseteq[\eta_\alpha)$. Thus $S^{lu}\subseteq(\eta_\alpha]^{lu}\subseteq[\eta_\alpha)$ and hence $x\in[\eta_\alpha)$. Therefore $\alpha\in B_x$. Thus $A_x\subseteq B_x$ and hence $\sup A_x\leq\sup B_x$. Again let $\alpha\in B_x$.Then $x\in[\eta_\alpha)$. Since $[\mu_\alpha)_C=\bigcup\{S^{lu}:S\subseteq\ \eta_\alpha\}$, we have $x\in S^{lu}$ for some subset S of η_α. This implies $\eta(a)\geq\alpha$ for all $a\in S$ and hence $\inf\{\eta(a):a\in S\}\geq\alpha$. Thus $\beta=\inf\{\eta(a):a\in S\}\in A_x$. Thus for each $\alpha\in B_x$ we get $\beta\in A_x$ such that $\alpha\leq\beta$ and hence $\sup A_x\geq\sup B_x$. Therefore $\sup A_x=\sup B_x$ and hence $\overline{\eta}=\hat{\eta}$. ■

The above result yields the following.

Theorem 3.10: Let $\mathscr{FCF}(Q)$ be the set of all L-fuzzy closed filters of Q. Then $(\mathscr{FCF}(Q),\subseteq)$ forms a complete lattice with respect to the point wise ordering " $\subseteq$ ", in which the supremum $sup_{i\in\Delta}\mu_i$ and the inifimum $\inf_{i\in\Delta}\eta_i$ of any family $\{\eta_i:i\in\Delta\}$ in $\mathscr{FCF}(Q)$ are given by:
$sup_{i\in\Delta}\eta_i=\overline{\bigcup_{i\in\Delta}\{\eta_i\}}$ and $\inf_{i\in\Delta}\eta_i=\bigcap_{i\in\Delta}\eta_i$.

Corollary 3.11: For any L-fuzzy closed filters η and ν of Q, the supremum $\eta\vee\nu$ and the infimum $\eta\wedge\nu$ of η and ν in $\mathscr{FCF}(Q)$ respectively are:
$\eta\vee\nu=\overline{\eta\cup\nu}$ and $\eta\wedge\nu=\eta\cap\nu$.
Now we introduce the fuzzy version of a filter (dual ideal) of a poset introduced by O. Frink [4].

Definition 3.12: An L-fuzzy subset η of Q is an L-fuzzy Frink filter if it satisfies the following conditions:

1) $\eta(1)=1$ and
2) for any finite subset F of Q, $\eta(x)\geq\inf\{\eta(a):a\in F\}$ $\forall x\in F^{lu}$

Lemma 3.13: Let $\eta\in L^Q$. Then η is an L-fuzzy Frink filter of Q if and only if η_α is a Frink filter of Q for all $\alpha\in L$.

Lemma 3.14: Let η be fuzzy Frink filter of a poset Q. Then η is iso-tone, in the sense that $\eta(x)\leq\eta(y)$ whenever $x\leq y$.

Corollary 3.15: A subset S of Q is a Frink filter of Q if and only if its characteristic map χ_S is an L-fuzzy Frink filter of Q.

Theorem 3.16: Let $(Q,\leq)$ be a lattice and $\eta\in L^Q$. Then η is an L- fuzzy Frink filter in the poset Q if and only it an L-fuzzy filter in the lattice Q.

Lemma 3.17: The intersection of any family of L-fuzzy Frink-filters is an L-fuzzy Frink filter.

Theorem 3.18: Let $[S)_F$ be a Frink-filter generated by subset S of Q and χ_S be its characteristic functions. Then $[\chi_S)=\chi_{[S)_F}$.
In the following theorems, we give characterizations of L-Fuzzy Frink filters generated by fuzzy subset of Q.

Theorem 3.19: Let $\eta\in L^Q$. Define a fuzzy subset $\hat{\eta}$ of Q by $\hat{\eta}(x)=\sup\{\alpha\in L:x\in[\eta_\alpha)_F\}$ for all $x\in Q$ where $[\eta_\alpha)_F$ a

Frink filter generated by η_α, where $[\eta_\alpha)_F$ is a Frink filter generated by η_α. Then $\hat{\eta}$ is an L-fuzzy Frink filter of Q generated by η.

In the following, we give an algebraic characterization of L-fuzzy Frink filters generated by fuzzy subset of Q.

Theorem 3.20: Let η be a fuzzy subset of Q. Then the fuzzy subset $\overrightarrow{\eta}$ defined by

$$\overrightarrow{\eta}(x) = \begin{cases} 1 & if x = 1 \\ \sup\{\inf_{a\in F} \eta(a) : F \subset\subset Q,\, x \in F^{lu}\} & if x \neq 1 \end{cases}$$

is a Frink fuzzy filter of Q generated by η.

Theorem 3.21: Let $\mathscr{F}\mathscr{F}\mathscr{F}(Q)$ be the of all L-fuzzy Frink filter of Q. Then $(\mathscr{F}\mathscr{F}\mathscr{F}(Q), \subseteq)$ forms a complete lattice with respect to point wise ordering "$\subseteq$", in which the supremum and the infimum of any family $\{\eta_i : i \in \Delta\}$ in $\mathscr{F}\mathscr{F}\mathscr{F}(Q)$ respectively are: $sup_{i\in\Delta}\eta_i = \overrightarrow{\bigcup_{i\in\Delta}\{\eta_i\}}$ and $\inf_{i\in\Delta}\eta_i = \bigcap_{i\in\Delta}\eta_i$.

Corollary 3.22: For any L-fuzzy Frink ideals η and ν of Q in the supremum $\eta \vee \nu$ and the infimum $\eta \wedge \nu$ of η and ν in $\mathscr{F}\mathscr{F}\mathscr{F}(Q)$ respectively are: $\eta \vee \nu = \overrightarrow{\eta \cup \nu}$ and $\eta \wedge \nu = \eta \cap \nu$.

Now we introduce the fuzzy version of semi-filters and V-filters of a poset introduced by P.V. Venkatanarasimhan [5] and [6].

Definition 3.23: η in L^Q is said to be an L-fuzzy semi-filter or L-fuzzy order filter if $\eta(x) \leq \eta(y)$ whenever $x \leq y$ in Q.

Definition 3.24: η in L^Q is said to be an L- fuzzy V-filter if it satisfies the following conditions:

1) for any $x, y \in Q$ $\eta(x) \leq \eta(y)$ whenever $x \leq y$ and
2) for any non-empty finite subset B of Q, if $\inf B$ exists then $\eta(\inf B) \geq \inf\{\eta(b) : b \in B\}$.

Theorem 3.25: Every L-fuzzy Frink filter is an L-fuzzy V-filter.

Proof: Let η be an L-fuzzy Frink filter and let $x, y \in Q$ such that $x \leq y$. Put $\eta(x) = \alpha$. Since η is an L-fuzzy Frink filter, η_α is a Frink filter of Q. Now $\eta(x) = \alpha \Rightarrow x \in \eta_\alpha \Rightarrow \{x\}^{lu} \subseteq \eta_\alpha$. Now $x \leq y \Rightarrow y \in x^u = x^{lu} \subseteq \eta_\alpha \Rightarrow \eta(x) = \alpha \leq \eta(y)$. Again let B be any nonempty subset of Q such that $\inf B$ exists in Q. Then $\inf B \in B^{lu}$ and hence $\eta(\inf B) \geq \inf\{\eta(a) : a \in B\}$. Therefore η is an L-fuzzy V-filter. ■

Now we introduce the fuzzy version filters of a poset introduced by Halaš [7] which seems to be a suitable generalization of the usual concept of L-fuzzy filter of a lattice.

Definition 3.26: $\eta \in L^Q$ is called an L- fuzzy filter in the sense of Halaš if it fulfills the followings:

1) $\eta(1) = 1$ and
2) for any $a, b \in Q$, $\eta(x) \geq \eta(a) \wedge \eta(b)$ for all $x \in (a,b)^{lu}$

In the rest of this paper, an L-fuzzy filter of a poset will mean an L-fuzzy filter in the sense of Halaš.

Lemma 3.27: $\eta \in L^Q$ is an L-fuzzy filter of Q if and only if η_α is a filter of Q in the sense of Halaš for all $\alpha \in L$.

Corollary 3.28: A subset S of Q is a filter of Q in the sense of Halaš if and only if its characteristic map χ_S is an L-fuzzy filter of Q.

Lemma 3.29: If η is an L-fuzzy filter of Q, then the following assertions hold:

1) for any $x, y \in Q$ $\eta(x) \leq \eta(y)$ whenever $x \leq y$.
2) for any $x, y \in Q$, $\eta(x \wedge y) \geq \mu(x) \wedge \eta(y)$ whenever $x \wedge y$ exists.

Theorem 3.30: Let $(Q, \leq)$ be a lattice. Then an L-fuzzy subset η of Q is an L-fuzzy filter in the poset Q if and only if an L-fuzzy filter is in the lattice Q.

Theorem 3.31: Let $[S)_H$ be a filter generated by subset S of Q in the sense of Halaš and χ_S be its characteristic functions. Then $[\chi_S) = \chi_{[S)_H}$.

Lemma 3.32: The intersection of any family of L-fuzzy filters is an L- fuzzy filter.

Now we give characterization of an L- fuzzy filter generated by a fuzzy subset of a poset Q.

Definition 3.33: Let η be a fuzzy subset of Q and $\mathscr{N}$ be a set of positive integers. Define fuzzy subsets of Q inductively as follows: $B_1^\eta(x) = \sup\{\eta(a) \wedge \eta(b) : x \in (a,b)^{lu}\}$ and $B_n^\eta(x) = \sup\{B_{n-1}^\eta(a) \wedge B_{n-1}^\eta(b) : x \in (a,b)^{lu}\}$ for each $n \geq 2$ and $a, b \in Q$.

Theorem 3.34: The set $\{B_n^\eta : n \in \mathscr{N}\}$ forms a chain and the fuzzy subset $\hat{\eta}$ defined by $\hat{\eta}(x) = \sup\{B_n^\eta(x) : n \in \mathscr{N}\}$ is a fuzzy filter generated by η.

Proof: Let $x \in Q$ and $n \in \mathscr{N}$. Then

$$\begin{aligned} B_{n+1}^\eta(x) &= \sup\{B_n^\eta(a) \wedge B_n^\eta(b) : x \in (a,b)^{lu}\} \\ &\geq B_n^\eta(x) \wedge B_n^\eta(x) \ (since\ x \in x^u = (x,x)^{lu}) \\ &= B_n^\eta(x)\ \forall\, x \in Q. \end{aligned}$$

Therefore $B_n^\eta \subseteq B_{n+1}^\eta$ for each $n \in \mathscr{N}$ and hence $\{B_n^\eta : n \in \mathscr{N}\}$ is a chain. Now we show $\hat{\eta}$ is the smallest fuzzy filter containing η.

$$\begin{aligned} \text{Since } \hat{\eta}(x) &= \sup\{B_n^\eta(x) : n \in \mathscr{N}\} \\ &\geq B_1^\eta(x) \\ &= \sup\{\eta(a) \wedge \eta(b) : x \in (a,b)^{lu}\} \\ &\geq \eta(x) \wedge \eta(x) \ \ (\text{since } x \in (x,x)^{lu}) \\ &= \eta(x)\ \ \forall\, x \in Q. \end{aligned}$$

Therefore $\eta \subseteq \hat{\eta}$. Let $a, b \in L$ and $x \in (a,b)^{lu}$.

$$\begin{aligned} \text{Now } \hat{\eta}(x) &= \sup\{B_n^\eta(x) : n \in \mathscr{N}\} \\ &\geq B_n^\eta(x)\ \text{ for all } n \in \mathscr{N} \\ &= \sup\{B_{n-1}^\eta(y) \wedge B_{n-1}^\eta(z) : x \in (y,z)^{lu}\} \\ &\quad \text{for all } n \geq 2. \\ &\geq B_{n-1}^\eta(a) \wedge B_{n-1}^\eta(b)\ \ \forall n \geq 2 \\ &\quad (\text{since } x \in (a,b)^{ul}) \\ &= B_m^\eta(a) \wedge B_m^\eta(b)\ \ \forall\, m \in \mathscr{N} \end{aligned}$$

$$\begin{aligned} \text{Thus } \hat{\eta}(x) &\geq \sup\{B_m^\eta(a) \wedge B_m^\eta(b) : m \in \mathscr{N}\} \\ &= \sup\{B_m^\eta(a) : m \in \mathscr{N}\} \wedge \\ &\quad \sup\{B_m^\eta(b) : m \in \mathscr{N}\} \\ &= \hat{\eta}(a) \wedge \hat{\eta}(b). \end{aligned}$$

Therefore $\hat{\eta}$ is a fuzzy filter. Again let θ be any L-fuzzy filter of Q such that $\eta \subseteq \theta$. Now let $a, b \in Q$ and $x \in (a,b)^{lu}$. Then $\theta(x) \geq \theta(a) \wedge \theta(b) \geq \eta(a) \wedge \eta(b)$. This implies $\theta(x) \geq \sup\{\eta(a) \wedge \eta(b) : x \in (a,b)^{ul}\} = B_1^\eta(x)$. Therefore $\theta(x) \geq B_1^\eta(x)$ for all $x \in (a,b)^{lu}$. Again for any $x \in (a,b)^{lu}$ we have $\theta(x) \geq \theta(a) \wedge \theta(b) \geq B_1^\eta(a) \wedge B_1^\eta(b)$. This implies $\theta(x) \geq \sup\{B_1^\eta(a) \wedge B_1^\eta(b) : x \in (a,b)^{lu}\} = B_2^\eta(x)$. Thus by induction we have $\theta(x) \geq B_n^{\eta}(x)\ \forall n \in \mathcal{N}$ and $\forall\, x \in (a,b)^{lu}$. Thus for any $x \in Q$, we have

$$\begin{aligned}
\hat{\eta}(x) &= sup\{B_n^{\eta}(x) : n \in \mathcal{N}\} \\
&= \sup\{B_{n-1}^{\eta}(a) \wedge B_{n-1}^{\eta}(b) : n \in \mathcal{N}, x \in (a,b)^{lu}\} \\
&\leq \sup\{\theta(a) \wedge \theta(b) : x \in (a,b)^{lu}\} \\
&\quad (\, since,\ a,b \in (a,b)^{lu}.) \\
&\leq \theta(x)
\end{aligned}$$

Therefore $\theta \supseteq \hat{\eta}$. This proves the theorem. ■

The above result yields the following.

Theorem 3.35: Let $\mathscr{F}\mathscr{F}(Q)$ be the set of all L-fuzzy filter of Q. Then $(\mathscr{F}\mathscr{F}(Q), \subseteq)$ forms a complete lattice with respect to the point wise ordering "$\subseteq$", in which the supremum and the infimum of any family $\{\eta_i : i \in \Delta\}$ in $\mathscr{F}\mathscr{F}(Q)$ respectively are: $(sup_{i\in\Delta}\eta_i)(x) = \sup\{B_n^{\bigcup_{i\in\Delta}\eta_i}(x) : n \in \mathcal{N}\}$ and $(\inf_{i\in\Delta}\eta_i)(x) = (\bigcap_{i\in\Delta}\eta_i)(x)$ for any $x \in Q$.

Corollary 3.36: For any L-fuzzy filter η and ν of Q, the supremum $\eta \vee \nu$ and the infimum $\eta \wedge \nu$ of η and ν in $\mathscr{F}\mathscr{F}(Q)$ respectively are: $(\eta \vee \nu)(x) = \sup\{B_n^{\eta\cup\nu}(x) : n \in \mathcal{N}\}$ and $(\eta \wedge \nu)(x) = (\eta \cap \nu)(x)$ for any $x \in Q$.

Theorem 3.37: The following implications hold, where all of them are not equivalent:

1) L-fuzzy closed filter $\Longrightarrow$ L-fuzzy Frink filter $\Longrightarrow$ L-fuzzy V-filter $\Longrightarrow$ L-fuzzy semi-filter.
2) L- fuzzy closed filter $\Longrightarrow$ L-fuzzy Frink filter $\Longrightarrow$ L-fuzzy filter $\Longrightarrow$ L- fuzzy semi-filter.

The following examples show that the converse of the above implications do not hold in general.

Example 3.38: Consider the Poset $([0,1], \leq)$ with the usual ordering. Define a fuzzy subset $\eta : [0,1] \longrightarrow [0,1]$ by

$$\eta(x) = \begin{cases} 1 & if x \in (\frac{1}{2}, 1] \\ 0 & if x \in [0, \frac{1}{2}] \end{cases}$$

Then η is an L- fuzzy Frink filter but not an L- fuzzy closed filter.

Example 3.39: Consider the poset $(Q, \leq)$ depicted in the figure below. Define a fuzzy subset $\nu : Q \longrightarrow [0,1]$ by $\nu(1) = \nu(a') = 1$, $\nu(a) = \nu(b) = \nu(c) = \nu(d) = \nu(0) = 0.2$, $\nu(b') = 0.6$, $\nu(c') = 0.5$ and $\nu(d') = 0.7$. Then ν is an L-fuzzy filter

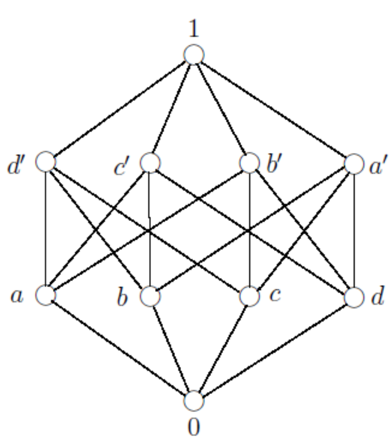

Fig. 1. A Poset.

but not an L- fuzzy Frink-filter.

Example 3.40: Consider the poset $(Q, \leq)$ depicted in the figure below. Define a fuzzy subset $\theta : Q \longrightarrow [0,1]$ by $\theta(U) = 1$, $\theta(L) = \theta(M) = 0.8$ and $\theta(N) = 0.6$. Then θ is an L-fuzzy

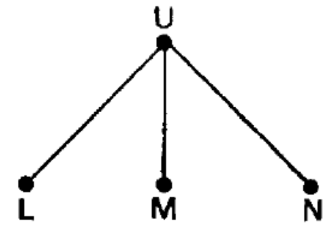

Fig. 2. A Poset.

V-filter but not an L- fuzzy Frink-filter.

Example 3.41: Consider the poset $(Q, \leq)$ depicted in the figure below. Define a fuzzy subset $\sigma : Q \longrightarrow [0,1]$ by $\sigma(1) = 1$, $\sigma(a) = 0.8$, $\sigma(b) = 0.9$ and $\sigma(0) = 0.2$.

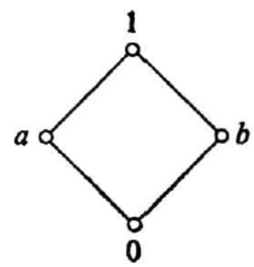

Fig. 3. A Poset.

Then σ is an L-fuzzy semi-filter but not an L-fuzzy filter.

Theorem 3.42: Let $x \in Q$ and $\alpha \in L$. Define an L- fuzzy subset α^x of Q by

$$\alpha^x(y) = \begin{cases} 1 & if\ y \in [x) \\ \alpha & if\ y \notin [x) \end{cases}$$

for all $y \in Q$. Then α^x is an L-fuzzy filter of Q.

Proof: By the definition of α^x, we clearly have $\alpha_x(1) = 1$. Let $a,b \in Q$ and $y \in (a,b)^{lu}$. Now if $a,b \in [x)$, then we have $(a,b)^{lu} \subseteq [x)$ and $\alpha^x(a) = \alpha^x(b) = 1$. Thus $\alpha^x(y) = 1 = 1 \wedge 1 = \alpha^x(a) \wedge \alpha^x(b)$. Again if $a \notin [x)$ or $b \notin [x)$, we have $\alpha^x(a) \wedge \alpha^x(b) = \alpha$ and hence $\alpha^x(y) \geq \alpha = \alpha^x(a) \wedge \alpha^x(b)$. Therefore in either cases we have $\alpha^x(y) \geq \alpha^x(a) \wedge \alpha^x(b)$ for all $y \in (a,b)^{lu}$ and hence α^x is an L-fuzzy filter. ■

Definition 3.43: The L-fuzzy filter α^x defined above is called the α-level principal fuzzy filter corresponding to x.

Definition 3.44: An L-fuzzy filter μ of a poset Q is called an l-L-fuzzy filter if for any $a,b \in Q$, there exists $x \in (a,b)^l$ such that $\mu(x) = \mu(a) \wedge \mu(b)$.

Lemma 3.45: An L-fuzzy filter μ of Q is an l-L-fuzzy filter of Q if and only if μ_α is an l-filter of Q for all $\alpha \in L$.

Proof: Suppose μ is an l-L-fuzzy filter and $\alpha \in L$. Since μ is an L- fuzzy filter, μ_α is a filter of Q. Let $a,b \in \mu_\alpha$. Then $\mu(a) \geq \alpha$ and $\mu(b) \geq \alpha$ and hence $\mu(a) \wedge \mu(b) \geq \alpha$. Also since μ is an l- L- fuzzy filter there exists $x \in (a,b)^l$ such that $\mu(x) = \mu(a) \wedge \mu(b)$ and hence $\mu(x) \geq \alpha$. Therefore $x \in \mu_\alpha \cap (a,b)^l$

and hence $\mu_\alpha \cap (a,b)^l \neq \emptyset$. Therefore μ_α is an l-filter of a poset Q. Conversely suppose μ_α is an l-filter of a poset Q for all $\alpha \in L$. Then μ is an L-fuzzy filter. Let $a,b \in Q$ and put $\alpha = \mu(a) \wedge \mu(b)$. Then $\mu_\alpha \cap (a,b)^l \neq \emptyset$. Let $x \in \mu_\alpha \cap (a,b)^l$. Then $x \in \mu_\alpha$ and $x \in (a,b)^l$. This implies $\mu(x) \geq \alpha = \mu(a) \wedge \mu(b)$ and $x \leq a$, $x \leq b$. Since μ is iso-tone we have $\mu(x) \leq \mu(a)$ and $\mu(x) \leq \mu(b)$ and hence $\mu(x) \leq \mu(a) \wedge \mu(b)$. Therefore there exists $x \in (a,b)^l$ such that $\mu(x) = \mu(a) \wedge \mu(b)$ and hence μ is an l-L-fuzzy filter. ■

Corollary 3.46: Let $(Q, \leq)$ be a poset with 0 and let $x \in Q$ and $\alpha \in L$. Then the α-level principal fuzzy filter corresponding to x is an l-L-fuzzy filter.

Remark 3.47: Every L-fuzzy filter is not an l-L-fuzzy filter. For example consider the poset $(Q, \leq)$ depicted in the figure below and define a fuzzy subset $\mu : Q \longrightarrow [0,1]$ by $\mu(1) = 1$, $\mu(c) = \mu(d) = 0.9$, $\mu(a) = \mu(b) = \mu(0) = 0.7$. Then μ is an L-fuzzy filter but not an l- L-fuzzy filter.

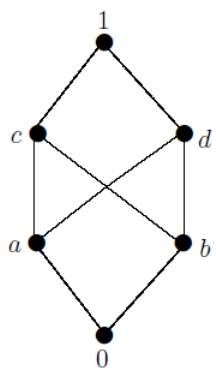

Fig. 4. A Poset.

Theorem 3.48: Every l-L-fuzzy filter is an L- fuzzy Frink filter.

Proof: Suppose η is an l-L-fuzzy filter. Let F be a finite subset of Q. Then there exists $y \in F^l$ such that $\eta(y) = inf\{\eta(a) : a \in F\}$.

$$\begin{aligned} \text{Again } x \in F^{lu} &\Rightarrow s \leq x \, \forall s \in F^l \\ &\Rightarrow y \leq x \ (\text{since } y \in F^l) \\ &\Rightarrow \eta(x) \geq \eta(y) = \inf\{\eta(a) : a \in F\} \\ &\Rightarrow \eta(x) \geq \inf\{\eta(a) : a \in F\} \end{aligned}$$

Therefore η is an L-fuzzy Frink filter. ■

Theorem 3.49: Let η and θ be l- L-fuzzy filters of Q. Then the supremum $\eta \vee \theta$ of η and θ in $\mathscr{F}\mathscr{F}(Q)$ is given by: $(\eta \vee \theta)(x) = \sup\{\eta(a) \wedge \theta(b) : x \in (a,b)^{lu}\}$ for all $x \in Q$.

Proof: Let σ be an L-fuzzy subset of Q defined by $\sigma(x) = \sup\{\eta(a) \wedge \theta(b) : x \in (a,b)^{lu}\} \, \forall x \in Q$. Now we claim σ is the smallest L-fuzzy filter of Q containing $\eta \cup \theta$. Let $x \in Q$.

$$\begin{aligned} \text{Now } \sigma(x) &= \sup\{\eta(a) \wedge \theta(b) : x \in (a,b)^{lu}\} \\ &\geq \eta(x) \wedge \theta(1), \ (\text{since } x \in (x,1)^{lu}) \\ &= \eta(x) \wedge 1 = \eta(x) \end{aligned}$$

and hence $\sigma \supseteq \eta$. Similarly we can show $\sigma \supseteq \theta$ and hence $\sigma \supseteq \eta \cup \theta$.

Let $a,b \in Q$ and $x \in (a,b)^{lu}$. Now

$$\begin{aligned} \sigma(a) \wedge \sigma(b) &= \sup\{\eta(c) \wedge \theta(d) : a \in (c,d)^{lu}\} \wedge \\ &\quad \sup\{\eta(e) \wedge \theta(f) : b \in (e,f)^{lu}\} \\ &= \sup\{\eta(c) \wedge \theta(d) \wedge \eta(e) \wedge \theta(f) : \\ &\quad a \in (c,d)^{lu}, b \in (e,f)^{lu}\} \\ &\leq \sup\{\eta(c) \wedge \eta(e) \wedge \theta(d) \wedge \theta(f) : \\ &\quad a,b \in (c,d,e,f)^{lu}\} \end{aligned}$$

Again since η and θ are l-L-fuzzy filters, for each c,e and d,f there are $r \in (c,e)^l$ and $s \in (d,f)^l$ such that $\eta(r) = \eta(c) \wedge \eta(e)$ and $\theta(s) = \theta(d) \wedge \theta(f)$. Now

$$\begin{aligned} r \in (c,e)^l and\ s \in (d,f)^l &\Rightarrow \{c,d,e,f\}^{lu} \subseteq \{s,r\}^{lu} \\ &\Rightarrow a,b \in \{s,r\}^{lu} \\ &\Rightarrow (a,b)^{lu} \subseteq \{s,r\}^{lu} \\ &\Rightarrow x \in \{s,r\}^{lu} \end{aligned}$$

Thus $\sigma(a) \wedge \sigma(b) \leq \sup\{\eta(c) \wedge \eta(e) \wedge \theta(d) \wedge \theta(f) : a,b \in (c,d,e,f)^{lu}\} \leq \sup\{\eta(r) \wedge \theta(s) : x \in (r,s)^{lu}\} \leq \sigma(x)$ for all $x \in (a,b)^{lu}$ and hence σ is an L-fuzzy filter.

Let ϕ be any L-fuzzy filter of Q such that $\eta \cup \theta \subseteq \phi$. Now for any $x \in Q$, we have

$$\begin{aligned} \sigma(x) &= \sup\{\eta(a) \wedge \theta(b) : x \in (a,b)^{lu}\} \\ &\leq \sup\{\phi(a) \wedge \phi(b) : x \in (a,b)^{lu}\} \\ &\leq \phi(x) \end{aligned}$$

and hence $\sigma \subseteq \phi$. Therefore $\sigma = (\eta \cup \theta] = \eta \vee \theta$, that is σ is the supremum of η and θ in $\mathscr{F}\mathscr{F}(Q)$. ■

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