Dual Hesitant N-Soft Sets

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***Abstract*—In this article, we introduce a new hybrid model called Dual Hesitant N-Soft Sets (DHNSS). This new model is clarified and reformulated as a combination of dual and hesitancy with N-soft sets. We investigate the basic operations of DHNSS. Our novel model is illustrated with real life examples. Moreover, we propose the decision making algorithm to see the application of this model.**

***Index Terms*—N-Soft sets, Hesitant N-Soft sets, dual Hesitant N-Soft sets**

I. INTRODUCTION

MULTI-CRITERIA decision-making consists of criteria, alternatives, evaluation values based on experts, and decision methods. Nevertheless, criteria evaluation sometimes found uncertainty or incomplete information. Thus, it can make decision-makers doubt making the right choice. Theories that can handle uncertainties are applied in various fields such as marketing, government, medication, and social choices. The fuzzy set theory [1] is one of the theories that can overcome uncertainty problems. It has been developed in tremendous variations [2], [3]. When the membership degree of the alternatives is general because of hesitation, then we can use the hesitant fuzzy set [4], [5]. The hybrid of hesitancy with other models can be seen for examples in [6], [7], [8], [9], [10].

On the other hand, soft set theory [11] handles uncertainty, involves criteria and evaluation of objects, and considers parameterization. The popular topics in soft set theory used evaluations in binary data or real numbers between 0 and 1. Ali *et al.* [12] propounded elementary algebraic operations on soft sets. Maji *et al*. [13] proposed decision-making problems using soft sets for the first time. Afterwards, Fatimah *et al.* [14], [15] referred to other extended soft set models related to statistics, i.e., probabilistic soft sets and dual probabilistic soft sets.

However, we can get non-binary information like rating systems in medicine, movie selections, hotel preferences etc. Herawan and Deris [16] generated n binary-valued information design in soft sets where each parameter has its own hierarchies. Instead of rankings as appraisal, Ali *et al.* [17] worked with the ranking of elements of soft sets parameters.

Fatimah *et al.* [18] were motivated by practical concerns and proposed an N-soft set definition. For example, it does not deserve any star rating (•), 'one star' (*), 'two stars' (**), 'three stars' (***), 'four stars' (****), and 'five stars' (*****). N-soft set [18] are the novel formula for the parameterized descriptions of objects that have a finite number of ordered grades. Several researchers welcomed the N-soft set and developed this theory further with various models. We can also apply N-soft sets for numerous parameters by using parameter reductions [19]. Akram *et al.* [20] firstly combined N-soft sets with fuzzy sets. Recently, the extension of fuzzy N-soft sets can be seen in [21], [22], and multi-fuzzy N-soft set [23].

Let us now describe the roadmap that led to the arrival of Dual Hesitant N-Soft Sets. Hesitancy can naturally set in a situation of approximate descriptions using the configuration of hesitant N-soft sets [24]. Previous successful attempts are limited to [25] and [24]. Hesitant fuzzy N-soft sets [26] is a model that incorporates hesitant fuzzy sets and N-soft sets. These elements model what peculiar rates are accustomed to objects when parameterizations criteria are ranked, which can be placed as partial degrees of membership, and they allow for hesitancy when we characterize such membership values.

Thus, we formalize a novel concept that we call dual hesitant N-soft sets in this paper. We propose a model that permits to collect all the hesitant information both on reference and non-reference grades. We achieve our purpose by an appropriate hybridization of N-soft sets and dual hesitant.

This paper is organized as follows. Section 2 gives the background about N-soft sets and their extensions. Section 3 defines our model, inclusive of its basic properties and score. In Section 4 we propose an algorithm and validate the model with a real example. We give conclusions and lines for further research in Section 5.

II. N-SOFT SETS AND EXTENSIONS

This section recalls some of N-soft set definitions introduced by Fatimah et al. [18], and its extensions i.e., fuzzy N-soft sets [20], and hesitant N-soft sets [26].

Let O express the objects under deliberation and P the set of parameters, $T \subseteq P$. Let $G = \{0, 1, 2, \cdots, N-1\}$ be the set of sequence ratings where $N \in \{2, 3, \cdots\}$.

A. N-Soft Sets

Definition 1: [18] A triple (F, T, N) is named an *N-Soft Set* (NSS) on O if F is mapping from T to $2^{O\times G}$, with the characteristic that for each $t \in T$ and $o \in O$ there stands a distinctive $(o, g_t) \in O \times G$ such that $(o, g_t) \in F(t)$, $g_t \in G$.

The exegesis of the pair $(o, g_t) \in F(t)$, is that the component o belongs to the set of t-estimations of the set O with the grade g_t. Its tabular illustration can be seen in the Table below I.

We demonstrate these ideas with a concise example:

Example 1: The selection of a lecturer in a university is based on star ratings provided by an election board, which involves the vice-chancellor, dean, head of program, and psychologist. Let $O = \{o_1, o_2, o_3, o_4\}$ the set of applicants that attend in university interview and P be the set of characteristics

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TABLE I
N-SOFT SET (F,T,N)

(F,T,N)	t_1	t_2	...	t_q
o_1	g_{11}	g_{12}	...	g_{1q}
o_2	g_{21}	g_{22}	...	g_{2q}
...	...	...	...	...
o_p	g_{p1}	g_{p2}	...	g_{pq}

"judgment of applicants by election board". The subset $T \subseteq P$ such that $T = \{t_1, t_2, t_3\}$ is used.

The evaluation values stand for 'bad' (•), 'adequate' (*), 'good' (**), and 'excellent' (***). The results can be shown in Table II.

TABLE II
ASSESSMENT RESULTS FROM ELECTION BOARD

$(F,T,4)$	t_1	t_2	t_3
o_1	**	***	**
o_2	•	**	*
o_3	*	*	**
o_4	***	**	*

We adjust the rating in Table II by replacing • with 0, (*) with 1, and so forth. A 4-soft set can be acquired from Table III.

TABLE III
$(F,T,4)$ BASED ON EXAMPLE 1

$(F,T,4)$	t_1	t_2	t_3
o_1	2	3	2
o_2	0	2	1
o_3	1	1	2
o_4	3	2	1

B. Fuzzy N-Soft Sets

If Definition 1 applies partial membership degrees, then the set becomes a fuzzy N-soft sets [20]:

Definition 2: [20] a *Fuzzy N-Soft Set* (FNSS), symbolized by (F,N)-soft set, is a pair (μ, K) when $K = (F,T,N)$ is an N-soft set on O with $N \in \{2,3,\cdots\}$, and $\mu : T \to \bigcup_{t\in T} \mathscr{F}(F(t))$ is a utilization with the characteristic that $\mu(t) \in \mathscr{F}(F(t))$ for each $t \in T$.

$\mathscr{F}(F(t))$ is the set of all fuzzy sets on $F(t)$. Definition 2 asserts that the function μ relates with every feature a fuzzy set on the description of this feature by F. Therefore, for each $t \in T$ and $o \in O$ there is an exclusive pair $(o, g_t) \in O \times G$ such that $g_t \in G$ and $\langle (o,g_t), \mu_t(o) \rangle \in \mu(t)$. Consequently, it means that $\mu_t(o) = \mu(t)(o,g_t)$.

There are developments of the two definitions above that can facilitate opinions that contain doubts. The model that properly allows for hesitancy in the evaluation by N-soft sets is the core of [24], while a model that emerges from the merger of fuzzy N-soft sets and hesitancy was stated in [26]. Torra [27], [28] claimed hesitant fuzzy set as follows:

Definition 3: [27], [28] Suppose O is any set. A *hesitant fuzzy set*, abbreviated as HFS, on O is defined in a function h that for every component o of O reverts a subset $h(o)$ of $[0,1]$. Thus, $h(o) \in \mathscr{P}([0,1])$ for every $o \in O$.

$\mathscr{P}([0,1])$ declares the set of all subsets of $[0,1]$. Also, we use HFS(O) to indicate the set of all HFSs on O.

C. Hesitant N-Soft Sets

Definition 4: [24] A *Hesitant N-Soft Set* (HNSS) on O, (H,T,N), if H is a mapping as follows $H : T \to 2^{O\times G}$ such that $\forall\, t \in T$ and $o \in O$ there exists at least one pair $(o, g_t) \in O \times G$ such that $(o,g_t) \in H(t),\ g_t \in G$.

According to Definition 4, with every feature, the mapping H establishes a non-empty aggregation of pairs denoted by objects and probable ratings. Thus, the construct of HNSS expands the consequent three extraordinary models:

(i) If $N = 2$ then an HNSS becomes as an incomplete soft set, which is a soft set with missing data.

(ii) If for every $t \in T$ and $o \in O$ then H relates *exactly* a pair $(o,g_t) \in O \times G$ such that $(o,g_t) \in H(t),\ g_t \in G$, is an N-soft set.

(iii) In consequence, if in adjunct to the condition (ii) above we add $N = 2$ then we acquire a soft set.

Tabular illustration of the HNSS delineated in Table IV.

The mapping $\eta : O \times T \to \mathscr{P}^*([0,1])$ such that for all $t \in T$ and $o \in O$, then $\eta(o,t) = \{g \in G : (o,g) \in H(t)\}$ where $\mathscr{P}^*([0,1])$ expresses the set of non-empty subsets of $[0,1]$.

TABLE IV
HESITANT N-SOFT SET TABULAR

(H,T,N)	t_1	...	t_q
o_1	$\{\eta^1_{11}, \eta^2_{11}, ..., \eta^{l(11)}_{11}\}$	...	$\{\eta^1_{1q}, \eta^2_{1q}, ..., \eta^{l(1q)}_{1q}\}$
...	...	...	...
o_p	$\{\eta^1_{p1}, \eta^2_{p1}, ..., \eta^{l(p1)}_{p1}\}$	...	$\{\eta^1_{pq}, \eta^2_{pq}, ..., \eta^{l(pq)}_{1q}\}$

III. DUAL HESITANT N-SOFT SETS

Several real examples in Fatimah et al. [18] prove that N-soft set is applicable in decision making problem which is not only can solve binary but also multinary evaluations. In this Section, we introduce our new model. The following definitions introduce a novel model that emerges from the hybridization of N-soft sets ([18],[24]) and dual hesitancy ([29], [30], [31]). A real case shows a model that we propose is natural in standard decision-making situations (see Example 2 in Section 4).

A. Proposed Model: Dual Hesitant N-Soft Sets

Definition 5: A *Dual Hesitant N-Soft Element* (DHNSE) is a pair $d = (h,g)$ with $h, g \subseteq \{0,1,2,\cdots,N-1\}$ such that $\gamma^+ + \eta^+ \leq N-1$, $\gamma^+ = \sup\{\gamma : \gamma \in h\}$, $\eta^+ = \sup\{\eta : \eta \in g\}$.

In Definition 5, h and g denote all possible preference grades and non preference grades of the element $x \in X$ respectively based on agent's preference.

Definition 6: Let $\mathscr{D}_N = \{d_t = (h,g)\}$ is the set of dual hesitant N-soft elements. A *Dual Hesitant N-Soft Set* (DHNSS) on O, $(\mathscr{H},T,N)$, if $\mathscr{H}$ is a mapping as follows $\mathscr{H} : T \to 2^{O\times \mathscr{D}_N}$ such that $\forall t \in T$ and $o \in O$ there exists exactly one pair $(o,d_t) \in O \times \mathscr{D}_N$ such that $(o,d_t) \in \mathscr{H}(t)$, and $d_t \in \mathscr{D}_N$.

Based on Definition 6, d_t is the assessment about the extent to which option $o \in O$ satisfies attribute $t \in T$. In other words, when $O = \{o_1, o_2, ..., o_p\}$ and $T = \{t_1, t_2, ..., t_q\}$ are finite, and $(o_i, d_{tj}) \in O \times \mathscr{D}_N$, we denote $d_{tj} = d_{ij} = (h_{ij}, g_{ij})$ for every i, j.

B. Basic Operations for Dual Hesitant N-Soft Sets

Definition 7: [30] Let $d_1 = (h_1, g_1)$ and $d_2 = (h_2, g_2)$ be two DHFEs. The union of DHFEs is defined as $d_1 \cup d_2 = \bigcup_{h \in (h_1 \cup h_2), g \in (g_1 \cap g_2)}$ where $h \geq max(h_1^-, h_2^-)$ and $g \leq min(g_1^+, g_2^+)$.
Let $h_i^- = min\{\gamma \mid \gamma \in h_i\}$, $g_i^- = min\{\eta \mid \eta \in g_i\}$, $h_i^+ = max\{\gamma \mid \gamma \in h_i\}$, $g_i^+ = max\{\eta \mid \eta \in g_i\}$ where $(i = 1, 2)$.

Definition 8: Let $d_1 = (h_1, g_1)$ and $d_2 = (h_2, g_2)$ be two DHNSEs. The *union* of DHNSEs is defined as $d_1 \cup d_2 = (h, g)$ where $h = \{\gamma \in h_1 \cup h_2$ such that $\gamma \geq max(h_1^-, h_2^-)\}$ and $g = \{\eta \in g_1 \cup g_2$ such that $\eta \leq min(g_1^+, g_2^+)\}$.

$d_1 \cup d_2 = \bigcup_{h \in (h_1 \cup h_2), g \in (g_1 \cap g_2)}$ where $h \geq max(h_1^-, h_2^-)$ and $g \leq min(g_1^+, g_2^+)$.

Definition 9: Let $d_1 = (h_1, g_1)$ and $d_2 = (h_2, g_2)$ be two DHNSEs. The *intersection* of DHNSEs is defined as $d_1 \cap d_2 = (h, g)$ where $h = \{\gamma \in h_1 \cup h_2$ such that $\gamma \leq min(h_1^+, h_2^+)\}$ and $g = \{\eta \in g_1 \cup g_2$ such that $\eta \geq max(g_1^-, g_2^-)\}$.

$d_1 \cap d_2 = \bigcap_{h \in (h_1 \cap h_2), g \in (g_1 \cup g_2)}$ where $h \leq min(h_1^+, h_2^+)$ and $g \geq max(g_1^-, g_2^-)$.

Inspired by ideas from [18], we can define the intersection and union of DHNSSs. Let $(\mathscr{H}_1, T_1, N_1)$ and $(\mathscr{H}_2, T_2, N_2)$ are DHNSSs on objects O, and τ_1 and τ_2 are mappings of their functional representations.

Definition 10: The *restricted intersection* of DHNSSs is denoted by $(\mathscr{H}_1, T_1, N_1) \cap_{\mathscr{R}} (\mathscr{H}_2, T_2, N_2)$. It is defined as $(J, T_1 \cap T_2, min(N_1, N_2))$ whose functional representation is $\tau_{RI} : O \times (T_1 \cap T_2) \rightarrow \mathscr{P}^*(\mathscr{D}_N)$ such that $\forall\ t_j \in T_1 \cap T_2$ and $o_i \in O$, $\tau_{ij}^k \in \tau_{RI}(o_i, t_j) \Leftrightarrow \tau_{ij}^k \in h_1 \cup h_2$ and $\tau_{ij}^k \leq min\{h_1^+, h_2^+\}$, where $h_1 = \tau_1(o_i, t_j)$ and $h_2 = \tau_2(o_i, t_j)$.

Definition 11: The *restricted union* of DHNSSs is denoted by $(\mathscr{H}_1, T_1, N_1) \cup_{\mathscr{R}} (\mathscr{H}_2, T_2, N_2)$. It is defined as $(M, T_1 \cap T_2, max(N_1, N_2))$ whose functional representation is $\tau_{RU} : O \times (T_1 \cap T_2) \rightarrow \mathscr{P}^*(\mathscr{D}_N)$ such that $\forall\ t_j \in T_1 \cap T_2$ and $o_i \in O$, $\tau_{ij}^k \in \tau_{RU}(o_i, t_j) \Leftrightarrow \tau_{ij}^k \in h_1 \cup h_2$ and $\tau_{ij}^k \geq max\{h_1^-, h_2^-\}$, where $h_1 = \tau_1(o_i, t_j)$ and $h_2 = \tau_2(o_i, t_j)$.

IV. DECISION MAKING AND APPLICATIONS

Below we proceed to show that our model permits to give a decision making mechanism. We define an algorithm for reaching a decision in problems that are characterized by DHNSSs. In order to prove the importance and feasibility of the algorithm, we also apply it into the real group decision-making (GDM) problem in [24]. The score that is used for this example is the arithmetic score.

Definition 12: The arithmetic scores of DHNSE, $d(h, g)$ is as follows:
$s_a(d) = \frac{\sum\{\gamma_i : \gamma_i \in h\}}{|h|} - \frac{\sum\{\eta_i : \eta_i \in g\}}{|g|}$.

Definition 13: Let (H, T, N) be a DHNSS. For every $o_i \in O$, its arithmetic score is $s_a(o_i) = \frac{\sum_{j=1}^{q} s_a(d_{ij})}{|T|}$.

Algorithm 1: The Algorithm of DHNSS Based on Arithmetic Score

1) Select a score s for DHNSSs (arithmetic score).
2) Input $O = \{o_1, o_2, \cdots, o_p\}$ as a universe of objects, and $T = \{t_1, t_2, \cdots, t_q\}$ as a set of attributes.
3) Input $G = \{0, 1, 2, \cdots, N-1\}$, $N \in \{2, 3, \cdots\}$, and $\mathscr{D}_N = \{d_t = (h, g)\}$ is the set of dual hesitant N-soft elements for every $t \in T$ and $o \in O$ there arises at least a pair $(o, d_t) \in O \times \mathscr{D}_N$ such that $(o, d_t) \in \mathscr{H}(t)$, $d_t \in \mathscr{D}_N$.
4) Compute the DHNSS $(\mathscr{H}, T, N)$.
5) Compute arithmetic scores $s_a(o_i)$ of DHNSSs, $\forall o_i \in O$.
6) Any of the alternatives for which $s_a(o_l) = max_{i=1,2,\cdots,p} s_a(o_i)$ can be selected.

The steps of the Algorithm 1 are summarized in Figure 1.

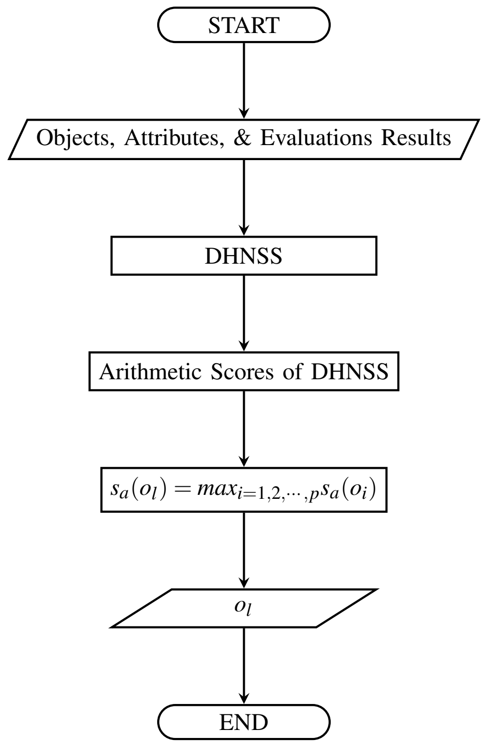

Figure 1. Flowchart of Algorithm 1

Example 2: [24] Let $V = \{o_1, o_2, o_3\}$ be victims of hepatic encephalopathy, P be the "evaluation of victims by various experts" related to symptoms of hepatic encephalopathy. Let $T = \{t_1, t_2, t_3, t_4\}, T \subseteq P$ i.e., t_1: level of consciousness, t_2: personality and intellect neurologic signs, t_3: electroencephalogram and t_4: abnormalities. The severity of hepatic encephalopathy can be classified by the following decreasing scale:

- 4: Coma with or without response to painful stimuli.
- 3: Somnolent but can be aroused.
- 2: Lethargy or apathy, minimal disorientation, inappropriate behavior, obvious personality changes.
- 1: Trivial lack of awareness, euphoria or anxiety, shortened attention span.
- 0: No abnormality detected.

Table V captures the information provided by four experts who give their evaluations of patients o_1, o_2, o_3 based on the attributes t_1, t_2, t_3, t_4. For o_1, and t_1, Expert 1 provides the evaluation "3", Expert 2 submits the evaluation "2", Expert

TABLE V
EXPERTS EVALUATION IN EXAMPLE 2

	O/T	t_1	t_2	t_3	t_4
D_1	o_1	3	0	3	2
	o_2	2	3	1	1
	o_3	3	4	4	3
D_2	o_1	2	1	2	1
	o_2	2	3	1	0
	o_3	2	3	4	3
D_3	o_1	1	0	3	2
	o_2	3	3	2	2
	o_3	3	2	4	3
D_4	o_1	0	1	2	3
	o_2	1	3	0	3
	o_3	4	1	4	3

3 submits the evaluation "1", and Expert 4 submits the evaluation "0". We can gather this information into H5SSs for each expert (cf. Definition 4) as $d_{11} = \langle o_1, t_1, \{0,1,2,3\}\rangle$ and so forth. Table VI shows all opinions in Example 2.

TABLE VI
THE H5SS IN EXAMPLE 2

$(H,T,5)$	t_1	t_2	t_3	t_4
o_1	$\{0,1,2,3\}$	$\{0,1\}$	$\{2,3\}$	$\{1,2,3\}$
o_2	$\{1,2,3\}$	$\{3\}$	$\{0,1,2\}$	$\{0,1,2,3\}$
o_3	$\{2,3,4\}$	$\{1,2,3,4\}$	$\{4\}$	$\{3\}$

Based on Definition 5, we should notice that $\gamma^+ + \eta^+ \leq N-1$, $\gamma^+ = \sup\{\gamma : \gamma \in h\}$, $\eta^+ = \sup\{\eta : \eta \in g\}$. Therefore DH5SS in Example 2 could be not unique as can be seen in Table VII and Table VIII.

TABLE VII
TABULAR REPRESENTATION OF THE DH5SS IN EXAMPLE 2 VERSION 1

$(\mathscr{H},T,5)$	t_1	t_2	t_3	t_4
o_1	$\{\{0,1,2,3\},\{0,1\}\}$	$\{\{0,1\},\{1,2,3\}\}$	$\{\{2,3\},\{0,1\}\}$	$\{\{1,2,3\},\{1\}\}$
o_2	$\{\{1,2,3\},\{0,1\}\}$	$\{\{3\},\{0,1\}\}$	$\{\{0,1,2\},\{0,2\}\}$	$\{\{0,1,2,3\},\{1\}\}$
o_3	$\{\{2,3,4\},\{0\}\}$	$\{\{1,2,3,4\},\{0\}\}$	$\{\{4\},\{0\}\}$	$\{\{3\},\{1\}\}$

TABLE VIII
TABULAR REPRESENTATION OF THE DH5SS IN EXAMPLE 2 VERSION 2

$(\mathscr{H},T,5)$	t_1	t_2	t_3	t_4
v_1	$\{\{0,1,2,3\},\{1\}\}$	$\{\{0,1\},\{0,1,2\}\}$	$\{\{2,3\},\{0\}\}$	$\{\{1,2,3\},\{0,1\}\}$
v_2	$\{\{1,2,3\},\{0,1\}\}$	$\{\{3\},\{0,1\}\}$	$\{\{0,1,2\},\{0,1,2\}\}$	$\{\{0,1,2,3\},\{0\}\}$
v_3	$\{\{2,3,4\},\{0\}\}$	$\{\{1,2,3,4\},\{0\}\}$	$\{\{4\},\{0\}\}$	$\{\{3\},\{0,1\}\}$

Based on Algorithm 1, we can obtain the choice values of Table VII using the arithmetic scores as can be seen in Table IX.

- $s_a(d_{11}) = \frac{0+1+2+3}{4} - \frac{0+1}{2} = 1$
- $s_a(d_{12}) = \frac{0+1}{2} - \frac{1+2+3}{3} = -1.5$
- $s_a(d_{13}) = \frac{2+3}{2} - \frac{0+1}{2} = 2$
- $s_a(d_{14}) = \frac{1+2+3}{3} - 1 = 1$

Thus, $s_a(d_1) = \frac{1-1.5+2+1}{4} = 0.625$

Therefore, the severity of hepatic encephalopathy in its victims is as $o_3 > o_2 > o_1$ (cf. Table X).

V. CONCLUSION

N-soft sets [18] can deal with both binary and non-binary evaluations. The model by N-soft sets is unable to make decisions when data collection produces hesitancy. Thus, Akram et. all [24] proposed an extended model of N-soft sets i.e., hesitant N-soft sets.

TABLE IX
GENERAL TABULAR OF ARITHMETIC SCORES FOR EXAMPLE 2

	t_1	t_2	t_3	t_4	$s_a(d_i)$
o_1	$s_a(d_{11})$	$s_a(d_{12})$	$s_a(d_{13})$	$s_a(d_{14})$	$s_a(d_1)$
o_2	$s_a(d_{21})$	$s_a(d_{22})$	$s_a(d_{23})$	$s_a(d_{24})$	$s_a(d_2)$
o_3	$s_a(d_{31})$	$s_a(d_{32})$	$s_a(d_{33})$	$s_a(d_{34})$	$s_a(d_3)$

TABLE X
ARITHMETIC SCORES EXAMPLE 2

	t_1	t_2	t_3	t_4	$s_a(d_i)$
o_1	1	-1.5	2	1	0.625
o_2	1	2.5	0	0.5	1
o_3	3	$\frac{10}{4}$	4	2	2.875

This research article is a novel hybrid model called dual hesitant N-soft sets, which is a blend of dual hesitant sets with N-soft sets. This model as an answer for non reference grades, which is the natural problem such as in medical decision that a patient has a right to know the positive (preference grade) or negative (non preference grade) decision with complex medical information. It guarantees a reliable model to approach decision-making. We have illustrated it with a real example and we have investigated its basic operations. In the future, we expect to extend our research work on its incomplete information and parameter reduction.

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