Modeling of Water Treatment Plant using Hybrid Petri Nets

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***Abstract*—Hybrid Petri nets are modeling framework for a system that contains discrete and continuous states. In this paper, we construct a model of water treatment plant using Hybrid Petri nets. We choose hybrid Petri nets because the water treatment plant contains discrete and continuous parts. The discrete part consists of the state of pumps and the continuous part consists of water volume in each processing unit. The Petri Nets obtained were further simplified to facilitate simulation. Through simulation we can determine the amount of water production with certain specifications.**

***Index Terms*—Hybrid Petri nets, modeling, water treatment plant.**

I. Introduction

HYBRID Petri Nets (HPN) are a class of hybrid systems which uses the Petri net representation. Essentially, in HPN there are two kinds of states, namely continuous and discrete states. These models have been widely used in many areas. We will mention some of them here. The authors of [1] design a control of Hybrid Dynamic Systems (HDS) modeled as Hybrid Petri Net (HPN) systems such that the HPN reaches a desired marking. Then, authors of [2] focus on Generalized Hybrid Petri nets (GHPN), as a tool for model-based exploration of environmental systems. Finally, the authors of [3] present a modular approach for modeling and analysis of the complex systems (in communication or transport systems area).

Water treatment plant (WTP) is a building to process raw water into clean water. There are many research on WTP in the literature. First of all, the authors of [4] have combined first principle equation of Darcy's law on cognitive implications and definitive neural network (ANN) predictive models to represent the dead-end ultrasound (UF) process. Then, the authors of [5] have applied a dynamic programming algorithm to design a conventional treatment system with minimal cost. Furthermore, the authors of [6] demonstrate the application of system analysis to the design of a water treatment system based on the concept and practice of optimization theory.

In our previous study, we have constructed a model of a water treatment plant using continuous Petri net [7]. In [7], we have modeled the quantity of waters in several units of WTP. In reality, the WTP has several pumps. The pumps cannot be modeled by using continuous Petri nets because the state of the pump is discrete, namely on and off. As an extension, in this work we also construct the model of the pump. Therefore, in this paper, we use HPN to model a water treatment plant.

The structure of the paper is as follows. Section 2 discusses the preliminaries, which consists of three parts. The first part is the explanation of WTP, the second part is the discussion on Hybrid Petri Nets, and the third part is the definition of Timed Hybrid Petri Nets. Section 3 discusses the HPN model of WTP. Finally, Section 4 concludes the work.

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II. Models and Preliminaries

This section consists of three subsections. In first section, we discuss the components and mechanisms of water treatment plants. Then we discuss the definition and some explanations of Hybrid Petri Nets and Timed Hybrid Petri Nets in next section.

A. *Water Treatment Plant*

Water treatment is any process that makes water more acceptable for a specific end-use. Water treatment removes contaminants or reduces their concentration so that the water becomes fit for its desired end-use. Water treatment is done in a building called water treatment plant (WTP). The main processing units of WTP include coagulation and flocculation, sedimentation, filtration, and disinfection [5]. A graphical representation of a WTP is depicted in Figure 1. In WTP shown in Figure 1, there are 3 main constructions, namely: intake, the main building, and reservoir.

Intake is the first building to the influx of water from the water source, such as river. Usually at Intake, there is a bar screen that serves to filter out objects that participate in the stagnant water. There is a pump and a tube in this building. Volume of the tube is around 64 m^3. The pump is used to transfer the water in the tube to the coagulation unit. Notice that the pump can be used if there is some water in the tube. We assume that the pump can be used if there is at least 30 m^3 water in the tube. Otherwise, if the volume of water in the tube is less than 30 m^3, the pump cannot be used. Main Building usually consist of four parts: coagulation unit, flocculation unit, sedimentation unit, filtration unit, and disinfectant unit. We will discuss each unit in more detail.

- The coagulation process is carried out via the destabilization process of colloidal particles by rapid mixing. More precisely, in the coagulation unit, raw water is mixed with chemicals of predetermined concentration before the water treatment process takes place. The addition of these chemicals uses a small pipe located at the top of the coagulation unit. The volume of coagulation unit is 24

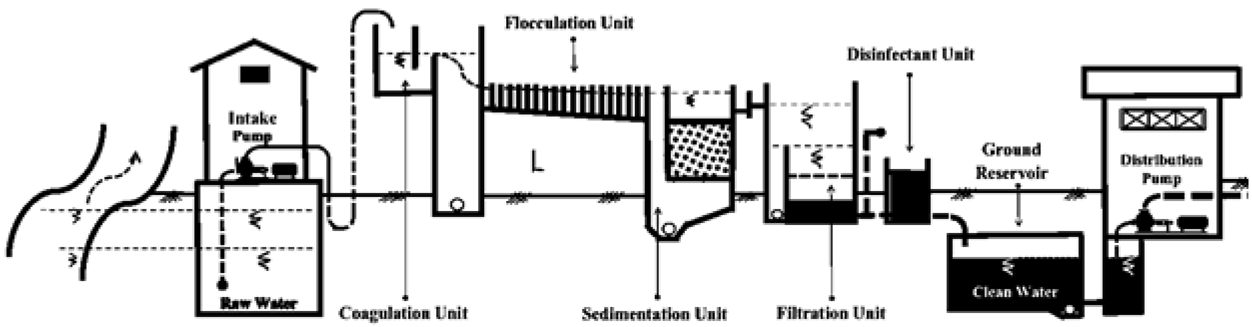

Fig. 1. Graphical representation of the water treatment plant [8]

m^3. By using the height difference, the water is headed to the flocculation unit.

- Flocculation unit is intended to form and enlarge floc. Technically, this is done by stirring slowly (slow mixing). Within this unit, there are a number of barriers of height 10 cm so that the water will flow in a zig-zag motion and in a slow speed to form a floc. The volume of water in the flocculation unit is approximately 44.5 m^3. Then, the water will enter the sedimentation unit. Sedimentation unit serves to precipitate colloidal particles having triplets as results obtained from the previous unit. This unit uses the principle of specific gravity.
- The sedimentation unit is divided into three tubes of the same size, which is 33 m^3. In this unit, there are tube settlers which are used to separate the dirt floc and the clean water. The volume of water that can be processed in this tube is about 25 m^3. Then the water will flow to the filtration unit by using a height difference.
- Filtration unit is used to filter using granular media. An additional process is usually carried out by adding chlorine disinfection, ozonation, UV, and others before entering the reservoir. There are four tubes of filtration of the same size that is equal to 37.5 m^3. The filtration process is done by using coral, sand silica and activated carbon. Furthermore, the water from the filtration unit will flow into the disinfectant unit.
- In disinfectant unit, adds a disinfectant to remove the bacteria in water that has a volume of 18 m3, which is relatively small compared to other processing units. In the disinfectant unit, the water is added by some chemicals to ensure the water quality. Clean water is then stored in the ground reservoir.

Reservoir serves as a temporary shelter of clean water before it is distributed. Ground reservoir is divided into two sections with a total volume of 675 m^3. At the ground reservoir distribution pumps are used to distribute clean water to consumers.

B. Hybrid Petri Nets

A marked autonomous hybrid Petri net is a sextuple $R = (P, T, Pre, Post, m_0, H)$ fulfilling the following conditions [9]:

- $P = \{P_1, P_2, \cdots, P_n\}$ is a finite, not empty, set of places;
- $T = \{T_1, T_2, \cdots, T_n\}$ is a finite, not empty, set of transitions;
- $P \cap T = \emptyset$ empty set, i.e. the sets P and T are disjointed;
- $h : P \cup T \to \{D, C\}$, called "hybrid function", indicates for every node whether it is a discrete node or a continuous one;
- $Pre : P \times T \to Q_+$ or N is the input incidence application;
- $Post : P \times T \to Q_+$ or N is the input incidence application;
- $m_0 \to R_+$ or N is the initial marking.

Hybrid Petri nets (HPN) are a combination of discrete Petri nets and continuous Petri nets. There are two kinds of places, namely discrete places and continuous places. The discrete places are graphically denoted by single circle, whereas the continuous places are graphically denoted by double circle. The number of tokens in a discrete place is a nonnegative integer, whereas the number of tokens in a continuous place is a nonnegative real number. The amount of tokens in a place represents certain conditions have been satisfied.

An arc connects a place to a transition or a transition to a place. The place can be either discrete or continuous. The transition can be either discrete or continuous. Each arc has a weight. The weight of an arc can be either a positive integer or a positive rational number.

A transition is associated with an event. Firing a transition represents the occurrence of the corresponding event. A transition can be fired if the transition is enabled. A transition is enabled if the input places satisfy certain criteria. The input place of a transition is defined as the set of places such that there is an arc from the place to the transition. After a transition is fired, some conditions may change. Such conditions are represented by output places. The output place of a transition is defined as the set of places such that there is an arc from the transition to the place.

In HPN, there are two kinds of transitions, namely discrete transitions and continuous transitions. The amount of firing of discrete transitions is a nonnegative integer, whereas the amount of firing of a continuous transition is a nonnegative real number. A discrete transition is enabled if for each input place, the number of tokens is greater than or equal to the weight of the arc from the place to the transition. A continuous transition is enabled if the number of tokens in each input place is greater than zero. If a transition is fired once (either discrete or continuous), the tokens in each input place is reduced by the weight of the corresponding arc from the place to the transition and the tokens in each output place is added by the weight of the corresponding arc from the transition to the place. If a discrete transition is fired several times, the number of reduced and added tokens is multiplied by the number of firing. If a continuous transition is fired in some quantity, the number of reduced and added tokens is multiplied by the firing quantity.

A macro-marking for a hybrid PN is a set of markings $m^* = (m^D, m^{C*})$, such that:

- The partial marking m^D is either a marking of the discrete part, or a macro-marking of the discrete part if this discrete part is not bounded.
- The partial marking m^{C*} is a macro-marking of the continuous part.

In a hybrid PN, a change of macro-marking, hence a change of set of enabled transitions, or of enabling degrees of D-transitions, can occur only if an event belonging to one of the following types occurs.

- C1-event: the marking of a marked C-place becomes zero
- C2-event: an unmarked C-place becomes marked
- D1-event: firing of a D-transition
- D2-event: enabling degree of a D-transition changes because of the marking of a C-place

C. Timed Hybrid Petri Nets

Timed hybrid PN are an extension of hybrid PN by adding a timing mechanism. More precisely, according to [9], a timed hybrid PN is a pair $(R, tempo)$ such that:

- R is a marked autonomous hybrid PN
- Tempo is a function from the set T of transitions to the set of positive or zero rational numbers:
  - If $T_j = T^D$, d_j = tempo (T_j) = timing associated with T_j
  - If $T_j = T^C$, $U_j = \frac{1}{tempo(T_j)}$ = flow rate associated with T_j

In timed hybrid PN, $tempo$ is related with the timing mechanism. If D-enabling degree is 1, the flow rate U_i associated with transition T_i corresponds to its maximal speed. If D-enabling degree is more than one, the maximal firing speed of transition T_i is the product of its flow rate and its D-enabling degree, i.e. $V_i = U_i \cdot D(T_i, m)$.

III. RESULTS AND DISCUSSION

In this section, we discuss the Hybrid Petri Net model for the Water Treatment Plants discussed in previous section. First, we will give the complete HPN model. Then, we will describe each component of the HPN model in detail.

In Figure 1 HPN model for the WTP consists of 12 places (6 discrete places and 6 continuous places) and 9 transitions (5 discrete transitions and 4 continuous transitions). Places P_1 and P_2 represent the availability of source water. If there is enough water in the source, then place P_2 has a token and place P_1 does not have any token. On the other hand, if there is almost no water in the source, then place P_1 has a token and place P_2 does not have any token. Initially, we assume that the water source is not available. Thus P_1 has a single token. Changes in the availability of source water can be done by firing either transitions T_1 or T_2.

Places P_7 and P_8 represent the available capacity and water volume in the tube at the intake building. Since the volume of tube is 64 m3, the total marking of P_7 and P_8 is exactly 64. Initially, there is no water in the tube. Thus, the marking of P_7 is 64 and the marking of P_8 is 0. When the water enters the tube, the marking of P_8 increases and the marking of P_7 decreases. Transition T_6 is associated with the following event: the water from source enters the tube. This can happen if there is enough water in the source (P_2 has a token). Thus, there is an arc from T_6 to P_2 and from P_2 to T_6. Places P_3 and P_4 represent the status of the pump in the intake building. If there is a token in P_4, the pump is turned on. If there is a token in P_3, the pump is turned off. Observe that the total number of tokens in P_3 and P_4 is 1. When we turn on the pump, transition T_3 is fired and when we turn off the pump, transition T_4 is fired. In order to be able to turn on the pump, there has to be at least 30 m^3 of water in the tube. Thus, there is an arc from T_7 to P_4 and P_4 to T_7 of weight 30.

Places P_9 and P_10 represent the available capacity and water volume in the main building, respectively. According to the discussion in the previous section, the total volume of water that can be processed in the main building is 311.5 m^3. Initially, there is no water in the main building. Thus, the token in P_9 is 311.5 and place P_10 does not have any token. If there is a flow of water entering the main building, then the amount of tokens in P_9 decreases and the amount of tokens in P_10 increases. Places P_5 and P_6 represent whether the main building is full of water or not. If there is a token in P_5, the main building is not full of water. Once the main building is full of water, transition T_5 is enabled. When transition T_5 is fired, the token moves from P_5 to P_6. Thus, if the main building is full of water, there is a token in P_6. Transition T_8 is enabled when the water is able to enter the reservoir. Since the flow of water from the main building to the reservoir is using the height difference, the water can flow to the reservoir whenever the main building is full of water. Thus, there is an arc from T_8 to P_6 and P_6 to T_8.

Places P_{11} and P_{12} represent the available capacity and the amount of water in the ground reservoir, respectively. Initially, the ground reservoir is empty. Thus, the amount of tokens in P_{11} is 675 and the amount of tokens in P_{12} is 0. Notice that the token at place P_{11} can decrease and at the same time the token at place P_{12} can increase by firing transition T_8. This means that the transition T_8 is an event where clean water enters the reservoir. Observe that there is an arc that connects place P_8 with the transition T_8, this arc means that if all the tanks are fully filled, the volume of water that enters the intake is the same with the volume of clean waters that enters the reservoir. Finally, transition T_{11} represents the distribution of water.

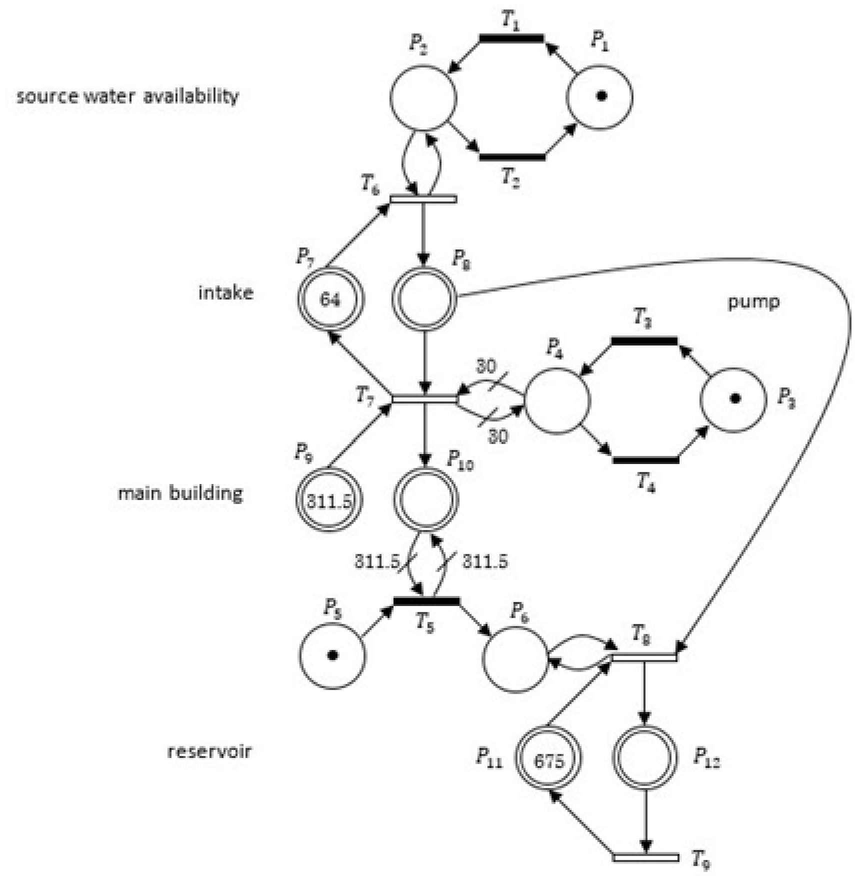

Fig. 2. Hybrid Petri net model of the water treatment plant

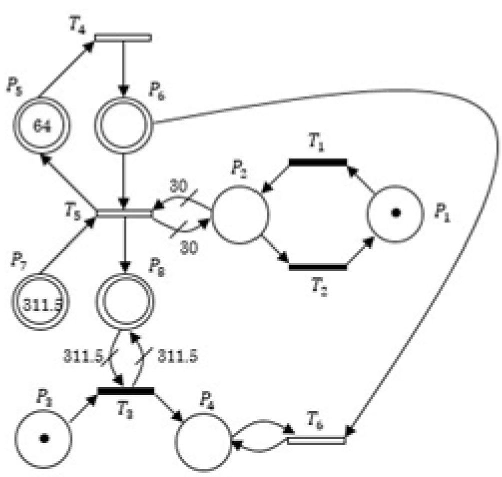

Fig. 3. Simplified Hybrid Petri Net model

In the next part, we extend the current HPN to Timed Hybrid Petri Net (THPN). In general, complexity of the procedure to generate THPN from HPN depends on the number of places. Notice that the HPN model has many places. Thus, we will reduce the number of places by raising several assumptions. First of all, we assume that the source water is always available, we do not model the reservoir, and we do not model the distribution of water after leaving the main building. The simplified HPN model is shown in Figure 1.

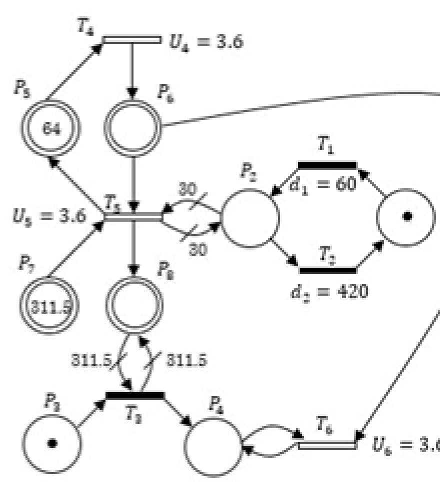

Fig. 4. Timed Hybrid Petri Net model

Description:

$$
\begin{aligned}
P_1 &= \text{The pump is off} \\
P_2 &= \text{The pump is on} \\
P_3 &= \text{Water volume in the main building is not full}
\end{aligned}
$$

P_4 = Water volume in the main building is full
P_5 = Water capacity in intake building
P_6 = Water volume in intake building
P_7 = Water capacity in main building
P_8 = Water volume in main building
T_1 = Switch on the pump
T_2 = Switch off the pump
T_3 = The main building is full of water
T_4 = Source water from river to the intake building
T_5 = Water from intake building to main building
T_6 = Water from main building to ground reservoir

We run a simulation using timed hybrid Petri nets with the following specifications:

- Debit of source water $U_4 = 3.6$ m^3/minute
- Debit of water from intake to main building $U_5 = 3.6$ m^3/minute
- Debit of water from main building to reservoir (clean water) $U_6 = 3.6$ m^3/minute

The water pump is working for the first 7 hours and break for 1 hour. In this case, $d_1 = 60$ minutes and $d_2 = 420$ minutes. For more details, see Figure 1.

In the initial conditions, the engine pump has not worked, so that for the first 60 minutes the event is just filling the source water in the intake with discharge $U_4 = 3.6$ m^3/minute. Thus, for 60 minutes the intake has abundant. After the 60th minute the engine pump is on and brings water from the intake to the main building with discharge $U_4 = 3.6$ m^3/minute. As $U_4 = U_5$, so the intake is always full, i.e. $P_4 = 64$ and $P_3 = 0$.

For the next 60 minutes (until 120 minutes), the main building will fill as much as 216 m^3, so that $P_8 = 216$ and $P_7 = 311.5 - 216 = 95.5$. Then in (60+86)-th minute, the capacity of P_8 is 309.6. The maximum capacity of P_8 is 311.5, which happens at (60+86)-th minute and 31.67 seconds. After that, the debit of the produced clean water is the same with the debit of water flow that goes into the main building, that is 3.6 m^3. In 147th, 148th, 149th, 150th and 180th minute, the amount of produced water is 1.7 m^3, 5.3 m^3, 8.9 m^3, 12.5 m^3 and 120.5 m^3 respectively.

Every 60 minutes, the produced clean water is 3.6 (60) = 216 m^3. So, until 480 minutes, the clean water produced is 120.5 + 3.6(60)(5) = 1200.5 m^3. Finally, the water pump at the intake is turned off, so that for 60 minutes no clean water is produced (1 cycle is complete).

IV. Conclusions

We have modeled a water treatment plant including the pump as a Hybrid Petri net. Then, we have extended the model to timed hybrid Petri nets. Along this direction, we are interested in analyzing the obtained Hybrid Petri Net model by using techniques available in the literature, such as in [8]. Besides that, we are also interested in modeling the water treatment system using other modeling frameworks

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