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Equivalent Single Layer Approach for Buckling Analysis of Stiffened Panel under Bi-Axial Compression and Lateral Pressure

Teguh Putranto, Ardi Nugroho Yulianto, Dedi Budi Purwanto, Dony Setyawan

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Abstract— **Ship structures consisting of stiffened plates are subjected to several loading conditions during service, which can lead to buckling. As a result of panel buckling, the overall strength of the ship hull girder is reduced, which is what determines the ultimate strength of the hull girder. The ultimate strength analysis can be accomplished with finite element (FE) simulation, but detailed modeling can be time-consuming. Due to these reasons, it is more advantageous and cost-effective to replace the three-dimensional (3D) stiffened panel model with a two-dimensional (2D) equivalent single layer (ESL) plate. This shift from 3D to 2D is premised on the ESL accuracy in describing the various buckling modes of stiffened panels, which are determined by panel topology and boundary conditions. Therefore, the performance of a stiffened panel represented by an equivalent single layer plate (ESL) is evaluated in different modes of buckling. Considering that ESL is asymmetric in nature, the buckling modes of stiffened panels are significantly affected by any modification to their geometry. In this paper, we are concerned with two modes of buckling: (i) plate and web stiffeners buckle locally, and (ii) buckling of the stiffeners due to lateral-torsional forces. According to the results, ESL is capable of accurately predicting the effect of local buckling in combination of biaxial compression and lateral pressure.**

Keywords— Biaxial compression; Buckling analysis; Equivalent single layer; Lateral pressure; Stiffened panel.

I. INTRODUCTION

The use of stiffened panels, a structural element consisting of plates and stiffeners attached longitudinally, is widespread across a wide range of applications, including marine, civil, and aerospace. As a result of stiffeners, bending stiffness can be significantly improved by using fewer materials. A stiffened panel is commonly used in marine structures to achieve both adequate strength and minimal weight. As part of the ship design process, this work is completed to ensure that the panels are capable of withstanding longitudinal bending because of sagging or hogging. Although detailed finite element (FE) modelling can produce accurate strength assessments, the process is costly and time-consuming if the entire structure is considered.

The behavior of the buckling is a significant indicator of the degree to which structure rigidity has decreased. Buckling mode is a phenomenon where an out-of-plane deformation of structures happens under compressive loads. When a slender structural element, such as a column, beam, or plate, is subjected to compressive loads, it can lose its stability and undergo large out-of-plane deformations. This can result in a loss of load-carrying capacity, reduced stiffness, and even structural collapse. Geometry and boundary conditions have an impact on the buckling behavior of structures [1]. Different structural elements and configurations can exhibit various buckling modes. These modes can range from simple axial deformation to more complex bending, torsional, or combination modes, depending on the geometry and loading conditions. buckling involves out-of-plane deformation, which means that the structure bends or twists in a direction perpendicular to its original plane. This behavior contrasts with in-plane deformation, where the structure experiences deformation within its original plane due to loads like tension or compression. Over the past decade, numerous studies have been conducted on the buckling of structures to evaluate the effects of local and global buckling on load responses [2]–[4].

Simple and clamped boundary conditions were applied to the stiffened panel to produce different buckling behavior on stiffened [5] and sandwich panels [6]. Stiffened panels consist of a main structural element with additional stiffeners attached to enhance its strength and rigidity. In general, sandwich panels are composed of two outer layers (skins) and an inner core material. Both types of panels are commonly used in various engineering applications due to their structural efficiency. Applying these boundary conditions to your stiffened and sandwich panels and analyzing their buckling behavior is essential for understanding their stability characteristics. Based on the boundary conditions and the panel geometry, different buckling modes can emerge. These modes could involve bending, torsion, or a combination of deformations.

According to DNVGL, buckling effects are more prevalent in slender structures as a result of the geometry of structures [7]. Slender structures are characterized by having one or more dimensions that are significantly

Teguh Putranto is with Departement of Naval Architecture, Institut Teknologi Sepuluh Nopember, Surabaya, 60111, Indonesia. E-mail: theories@na.its.ac.id (Corresponding Author)

Ardi Nugroho Yulianto is with Departement of Naval Architecture, Institut Teknologi Sepuluh Nopember, Surabaya, 60111, Indonesia.

Dedi Budi Purwanto is with Departement of Naval Architecture, Institut Teknologi Sepuluh Nopember, Surabaya, 60111, Indonesia.

Dony Setyawan is with Departement of Naval Architecture, Institut Teknologi Sepuluh Nopember, Surabaya, 60111, Indonesia.

larger than the others. In engineering, a structure is considered slender when its length-to-width or length-to-height ratios are relatively high. For example, a long column or a tall beam can be considered slender. Because slender structures have relatively low resistance to buckling compared to their resistance to other deformation modes like bending or tension, buckling effects can dominate their failure mechanisms. This is especially true when compressive loads are applied. Buckling occurs when the critical load is reached.

The buckling behavior of an Equivalent Single Layer (ESL) can be considered both locally and globally. This method is based on homogenization and is characterized by stiffness matrices that are equal to its three-dimensional structures when using the ESL method, two-dimensional single layer. It is particularly common for layered composite structures like sandwich panels or laminates. ESL attempts to capture the essential mechanical response while reducing the complexity of the analysis [8]. An effective mechanical property of a composite material or structure can be represented mathematically as if it were a solid, homogeneous substance. This technique is particularly useful when dealing with composite materials made up of different layers or phases with distinct material properties [9]. Global buckling phenomena were predicted accurately by ESL model representing sandwich panel with defining correctly bending and transverse shear stiffnesses [10]. An accurate representation of the panel's behavior requires an understanding of the bending and transverse shear stiffnesses of the skins and core [11]. Bending stiffness relates to the panel's resistance to bending deformations, while transverse shear stiffness accounts for shear deformations within the core. For local buckling analysis in ESL, UGENS subroutine in Abaqus enables to consider the non-linearity of stiffness matrices of ESL element [12]. This method was implemented in ESL to capture local buckling due to non-linear stiffness and global buckling caused by geometrical nonlinearity. The successful implementation of such a method requires careful calibration, validation against experimental data, and potentially the use of advanced analysis techniques. This approach can provide valuable insights into the buckling behavior of complex structures, aiding in their design and ensuring their structural integrity and safety.

In this paper, The ESL model represents a 3D stiffened panel and measures local and buckling behaviors for combined biaxial compression and lateral pressure loading. ESL stiffness matrices are calculated by simulating unit cells in six boundary conditions. ESL with non-linear stiffness matrix is performed using UGENS subroutine to allow the change of stiffness in each strain. We examined the effects of simply supported and clamped boundary conditions using ESL and 3D FEM. In comparison with 3D FEM, ESL has a good agreement.

II. METHOD

The analyses are performed by modifying the panel configurations, boundary conditions consisted of simply supported (SS) and clamped (CS), and displacement ratios of biaxial compression (β=v/u=0.0, 0.25 and 0.5). The lateral pressure is applied as 5 kPa on the plate. Detailed FE modeling of stiffened panel is called 3D FEM, while a 2D layer that has same stiffnesses as such stiffened panel is called ESL. Each 3D FEM and ESL is simulated based on the variation and the name of simulation is written, for example: UC1-0.25-SS meaning that stiffened panel with UC1 is subjected to biaxial compression (β=0.25) and using a simply supported boundary condition.

A. ESL Methodology

Procedures of ESL application, see **Figure 1**, can be described as follows:

1. Step 1: Unit cell (UC) definition. Unit cell size is a square shape which the length and width are equal to stiffeners spacing. Local imperfection in UC is determined by eigenvalue buckling analysis.
2. Step 2: Generate stiffness matrices. Unit cell is subjected to six boundary condition consisted of uniaxial compression in two different direction, shear, bending in two different direction, and torsion.
3. Step 3: ESL simulation. ESL stiffness matrices are defined from the result of Step 2. ESL size is equal to the 3D stiffened panel.

B. Panel Configurations

Strength of stiffened panel is dependent of buckling characteristics that cause a reduction of load response. Under axial compression, thin plates will experience out-of-plane buckling deformation at small loads [13]. In terms of web height of stiffener, slender structures are very sensitive leading to over critical buckling limit. In this paper, three different size configurations of stiffened panel which include stocky and slender structures are modeled as shown in **TABLE 1**.

Net plate thickness, as shown in **Equation** $t_w \geq \frac{h_w}{C_w}\sqrt{\frac{R_{eH}}{235}}$ (1, is calculated based on the DNVGL considering slenderness requirement (DNVGL, 2015), where: Cw is the coefficient of stiffener types.

$$t_w \geq \frac{h_w}{C_w}\sqrt{\frac{R_{eH}}{235}} \tag{1}$$

C. FE Modeling

In the case of 3D FEM and ESL models, ABAQUS is used to perform finite element simulations. The modified Riks statics method is chosen to solve the numerical simulation. This method is effective in resolving an unstable static response.

For 3D FEM, the geometry and boundary condition are shown in **Figure 2**. The element type used is S4R to decrease the computational effort. Simply supported boundary condition is applied to models where no vertical displacement is present at the edges of the plate. Nodes located within the neutral axis are subject to displacement. The local element thickness is increased by five times the web thickness to prevent local buckling on the edge of the web due to centered displacement. In addition, equation constraint, using periodic v, is applied in the web edges to allow the corresponding nodes moving together in the x2 direction. A 3D FEM is also applied to the clamped support to investigate the effect of boundary conditions. The clamped support is like the simply supported; however, another constraint is added in the web edges where the nodes, using periodic u, are applied to move together in the *x1* direction.

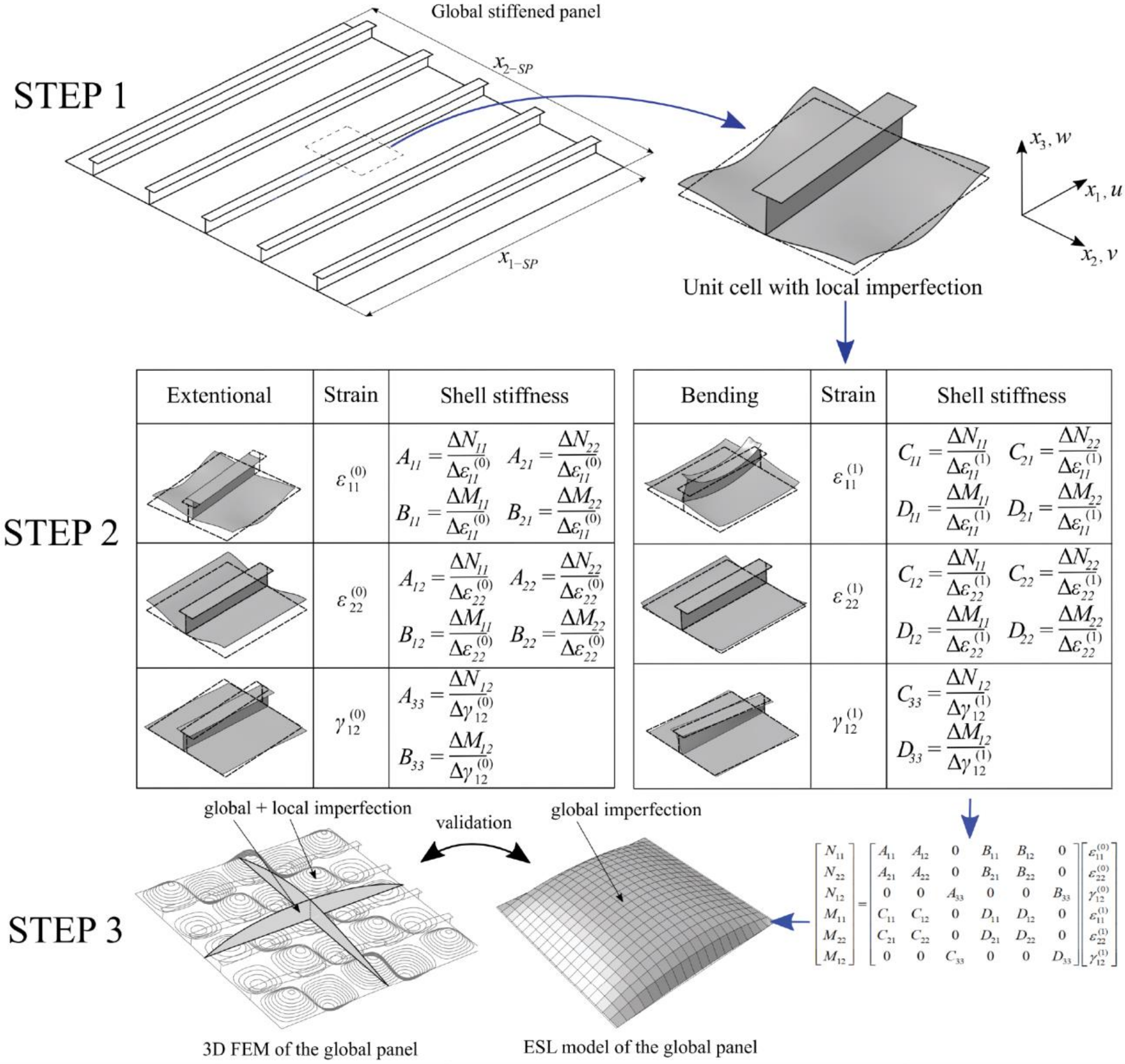

Figure 1. Flowchart of ESL methodology.

TABLE 1.
DIMENSION OF UNIT CELLS. ALL UNITS IN MM.

Stiffened panel names	Plate Length lp × breadth bp × thickness tp	Stiffener web Height hw × thickness tw
Unit cell 1 (UC1)	600 × 600 × 10	125 × 7.8
Unit cell 2 (UC2)	600 × 600 × 6	125 × 7.8
Unit cell 3 (UC3)	600 × 600 × 10	475 × 7.8

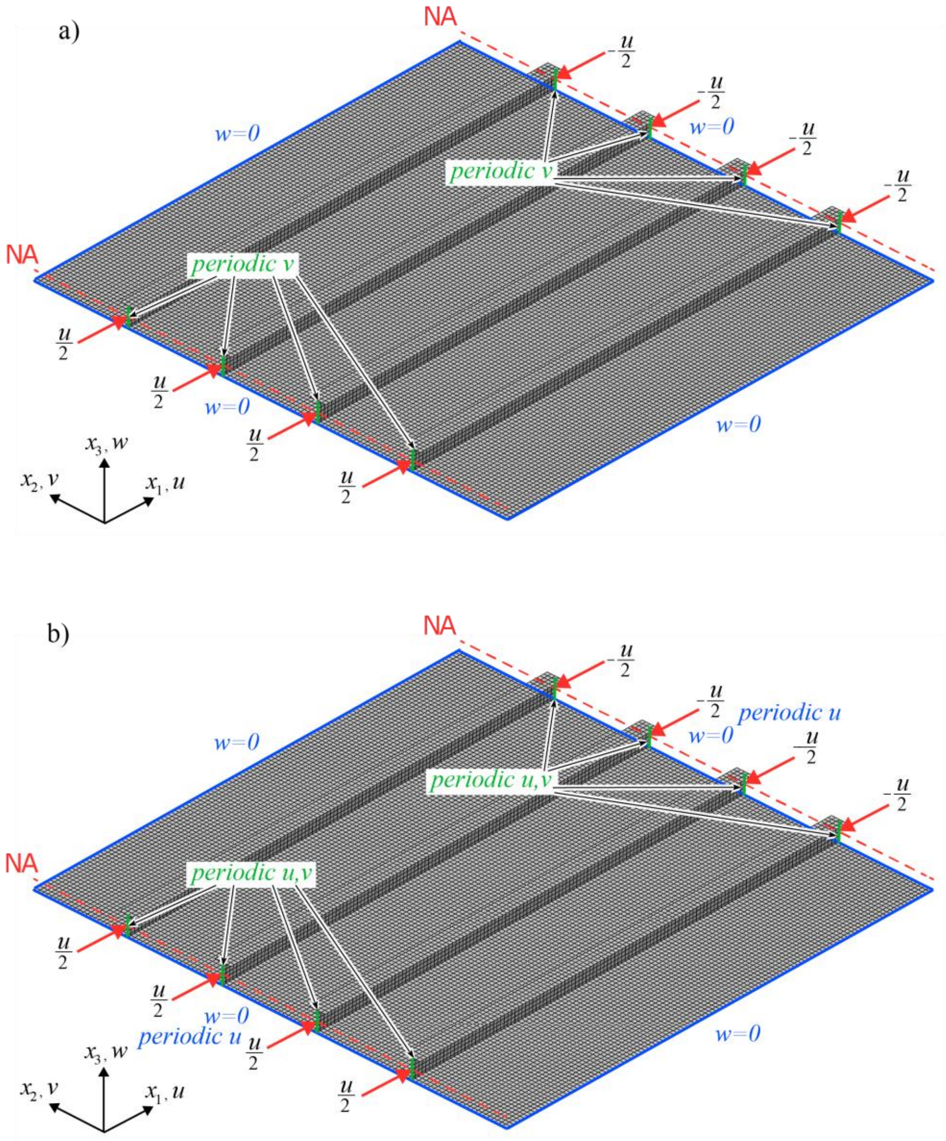

Figure 2. a) Simply supported and b) clamped boundary conditions applied in 3D stiffened panel for uniaxial compression. For biaxial compression, the displacement is added in the nodes of plate edges that perpendicular with the *x2* direction.

For ESL model, the element type used is shell general section that requires the ABD matrices. The boundary condition (BC) applied are simply supported and clamped BCs. For simply supported, the nodes in all edges are zero for translation in the *x3* direction. For clamped support, the constraints are same as the simply supported; however, the nodes in the applied displacement u are also zero for rotation in the *x2* direction. The displacement is applied in the nodes where is parallel to the stiffener direction. Additionally, in order to keep the rigid body of ESL, the one node in the middle of ESL is zero for translation in the *x2* direction.

Rigid body behavior refers to the motion of an object as a whole, without any deformation or distortion. In this case, you're making sure that the structure (ESL) maintains its rigid body behavior despite the applied displacements. This might involve constraining specific degrees of freedom at certain nodes to prevent unintended translations or rotations.

There is a specific node located at the middle of the ESL (Elastic Stiffened Laminate) that has constraints placed on it. These constraints ensure that this node cannot undergo translation in the *x2* direction (presumably a direction perpendicular to the stiffener direction).

D. Material Properties and Stiffnesses

For ESL and 3D FEM, the elastic material properties are used. For 3D FEM, the type of properties selected is an isotropic elasticity where the Young's modulus, E = 206 GPa and Poisson's ratio, υ = 0.3 are defined. While for ESL model, the stiffnesses are defined as shown in **Figure 3**. The UGENS subroutine is invoked to consider the non-linearity of stiffness matrices. To consider local buckling in the unit cell level, the imperfection scaling factor is applied of 0.01 meaning slight imperfection.

Local buckling refers to a phenomenon where certain parts of a structural element, such as a beam or a column, experience instability due to compressive forces. A localized buckling of the material can reduce the load-carrying capacity of the structure.

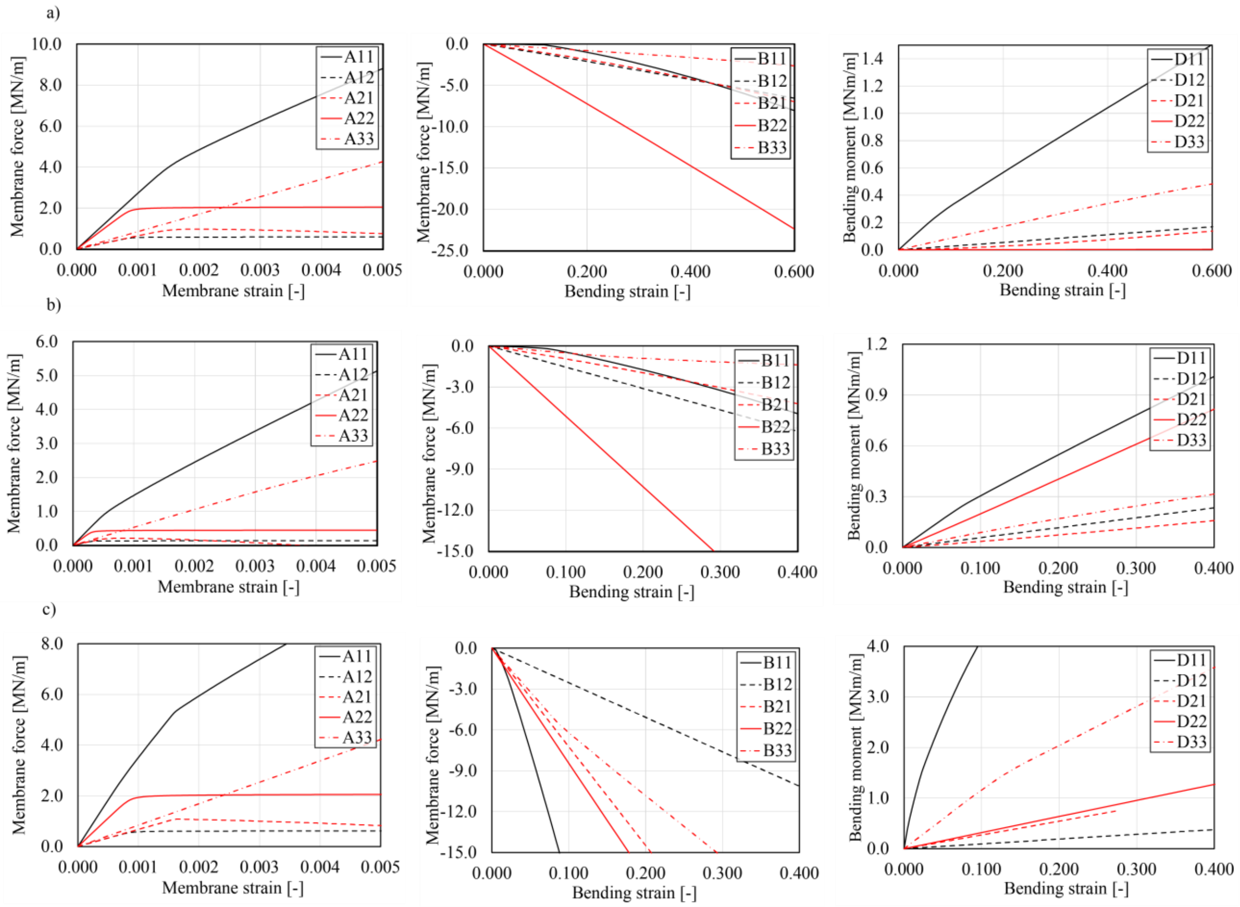

Figure 3. Non-linear stiffness of ESL for a) UC1, b) UC2, and c) UC3.

III. Results and Discussion

To define the average stress-strain responses of stiffened panel under compression, load-end shortening curves are used. A load-end shortening curve is a function of the axial load and displacement under compression. The axial load is a summation of forces on the nodes where the displacement u is applied. This simulation is run with the displacement of 5 mm. To investigate the buckling behaviors and load responses of stiffened panel, the analyses are conducted based on the effect of:

A. Panel Configurations

The reduction of plate thickness (UC2) causes the bifurcation point or initial buckling behavior to start earlier rather than the plate thickness without reduction (UC1). This phenomenon can be seen in Figs. 4-5 explaining the effect of panel dimensions on the responses using simply supported BC. UC2 likely refers to a specific unit cell or configuration of your structure where the plate thickness has been reduced. Altering the thickness of a structural element can have a significant impact on its buckling behavior. Thinner plates generally have a higher susceptibility to buckling due to their reduced stiffness and resistance to compressive loads. It appears that reducing the plate thickness in UC2 causes the initial buckling behavior or the bifurcation point to occur earlier than in the case of the original plate thickness (UC1). This means that the thinner plate is more prone to buckling under the same load or conditions compared to the thicker plate. This is consistent with the behavior of slender structures, where reducing stiffness, such as by reducing thickness, can lead to earlier onset of buckling.

Additionally, for slender stiffener (UC3), the response is stiffer from the beginning compared to UC1; however, the initial buckling of UC3 is caused by the local web buckling behavior. UC3 refers to a specific configuration of your structural component, likely a stiffener. A stiffener is a structural element used to provide additional rigidity and strength to a main structural member. In UC3, the stiffener's response is stiffer right from the beginning of loading compared to the reference case UC1. This could be due to factors like a more favorable geometry, different material properties, or other design considerations that enhance the initial stiffness of the stiffener. For UC1 with different biaxial compression, the difference in the responses is clearly visible in the post-buckling regime, while the initial stiffness did not differ significantly. UC1 refers to the baseline configuration of your structural element. Biaxial compression implies that the element is subjected to compressive forces in two orthogonal directions. The differences in biaxial compression levels can lead to varying behavior in the element. Even though the initial stiffness of UC1 did not show significant differences with varying biaxial compression levels, the behavior in the post-buckling regime is visibly distinct. This could mean that the structural element responds differently in terms of deflections, modes of deformation, load-carrying capacity, or stability as it undergoes further deformation after the initial buckling.

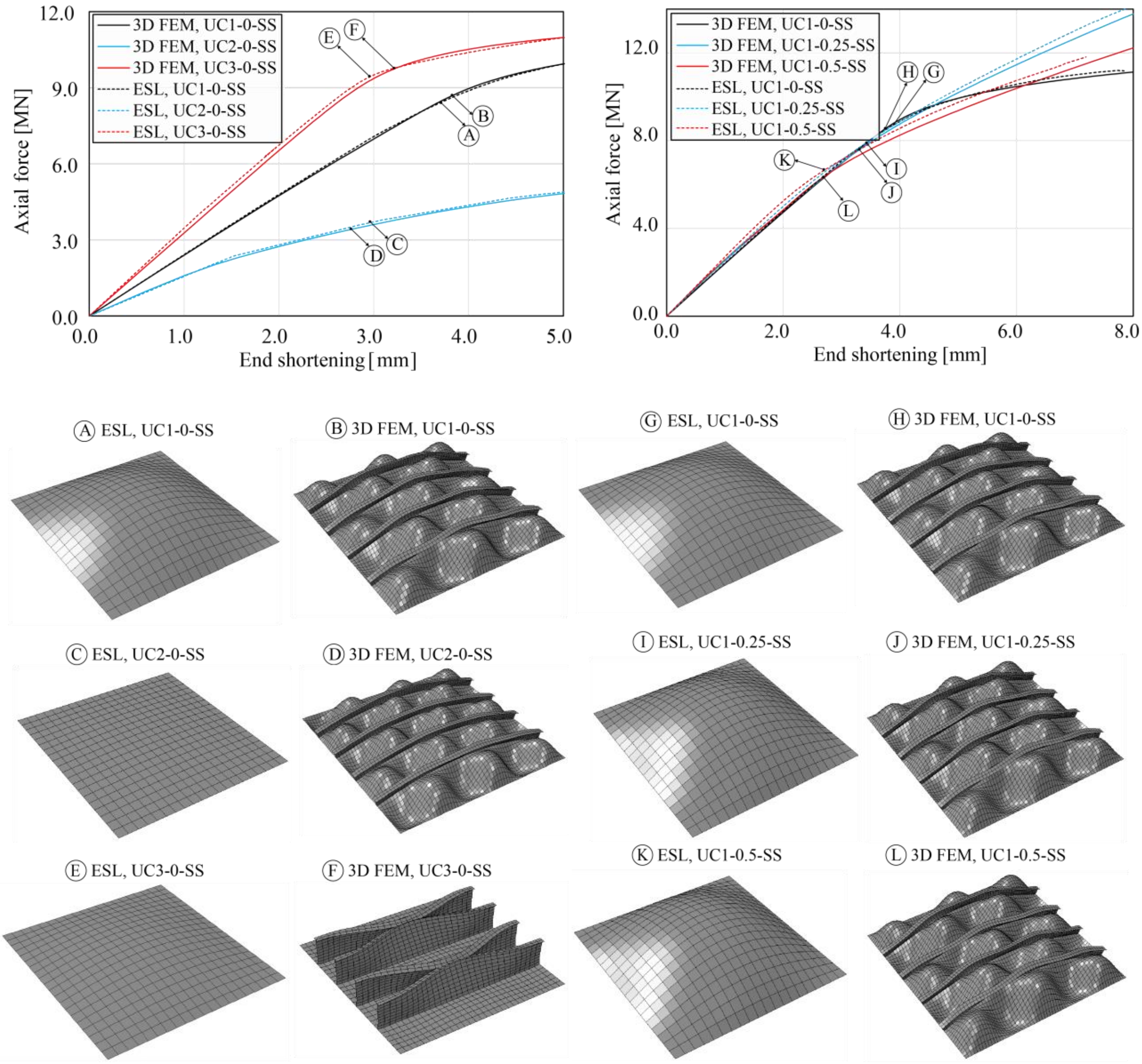

Figure 4. Load-end shortening curves and deformed shapes for UC1, UC2, and UC3 using simply supported boundary condition.

Non-linear ESL can capture the effect of local and global buckling behaviors on the plate and stiffener which can be seen in **Figure 4**. In the post-buckling stage, the UC1 and UC2 experience local and global buckling on plate. The UC3 behaves in tripping mode. All behaviors can be captured well by ESL. The comparison of response loads between non-linear ESL and 3D FEM shows a good agreement that point buckling of non-linear ESL can be predicted accurately for axial and biaxial compressions.

B. Boundary Conditions

The effect of boundary condition (BC) influences the stiffness reduction of stiffened panel, see in Fig. 6, especially for UC2 and UC3 where they are stocky and slender structures. In UC2, the simply supported BC affects global and local deformation, while the clamped BC causes local plate and web buckling. In UC3, local web buckling behavior causes lateral-torsional buckling and tripping mode shapes. In the post-buckling regime, the clamped BC is stronger than simply supported BC. The curve pattern for clamped application is same as simply supported BC.

The response of non-linear ESL can consider the local and global buckling behaviors whether the boundary condition used is simply supported or clamped BCs. Initial stiffness is the same for both conditions. However, in the post-buckling condition, clamped support applied to ESL is stiffer than simply supported. This agreement is obtained the same by comparing the load-end shortening curves of ESL and 3D FEM.

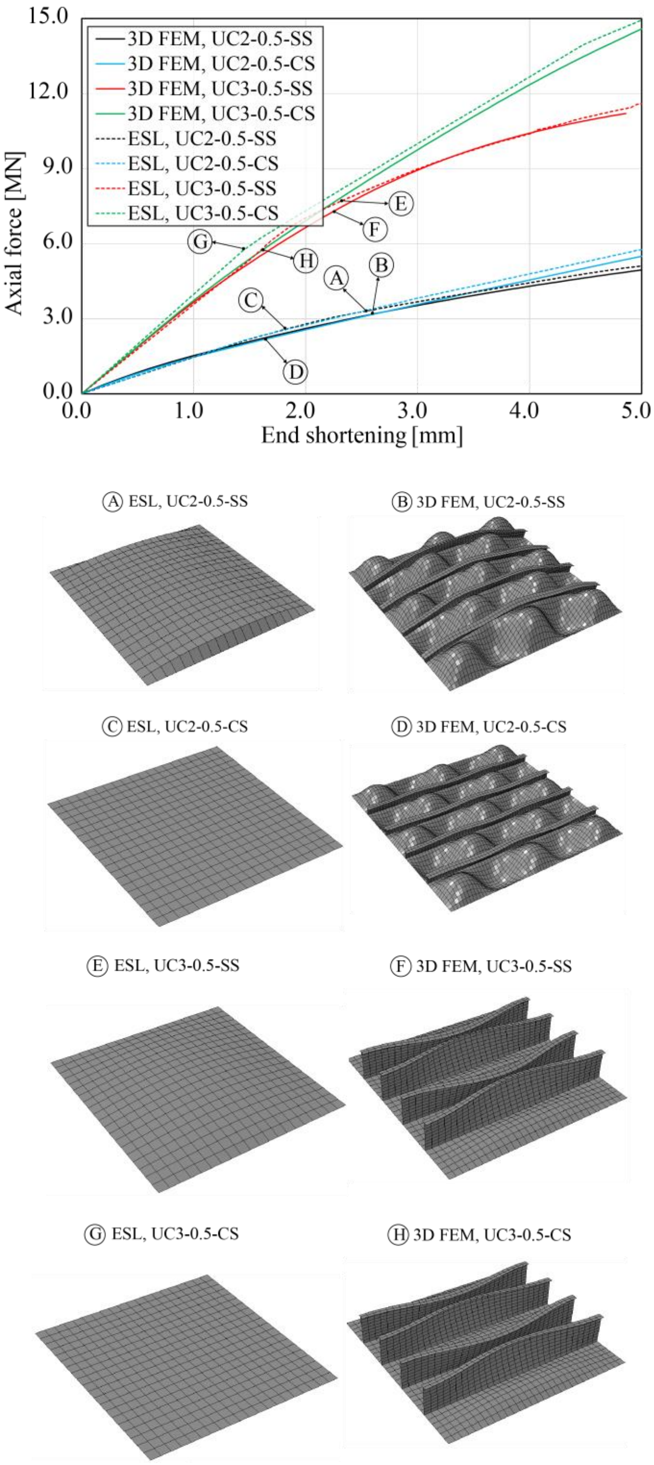

Figure 5. Load-end shortening curves and deformed shapes for biaxial compression using simply and clamped supports BCs.

IV. CONCLUSION

This research proposes a homogenization technique which transforms a 3D stiffened panel become an equivalent single layer (ESL) for assessing strength of some buckling behaviors. Application of non-linear ESL allows the change of stiffness matrices during simulation. The strength analysis is conducted based on the variation of panel configurations, boundary conditions, and biaxial compression in the elastic condition.

ESL is accurate in predicting global buckling but might not visually capture local buckling behavior. Local buckling involves smaller-scale deformations or instabilities in specific regions of a structure. While ESL might not directly show these deformations visually, it can still account for their effects by considering non-linear stiffness matrices.

ESL considers the load response resulting from local buckling using non-linear stiffness matrices. These matrices describe how the stiffness of the structure changes as it deforms, which is essential for capturing the structural behavior that experiences both global and local buckling.

Under elastic conditions, this study examines the post-buckling behavior of stiffened panels. This means that the analysis considers deformations that occur after buckling but without entering the plastic deformation range. Elastic behavior is characterized by reversible deformations, while plastic behavior involves permanent deformations.

The study recognizes its limitations and suggests that further work is needed to analyze post-buckling strength. This would involve considering elasto-plastic properties, where the material undergoes both elastic and plastic deformations, and exploring different loading conditions that go beyond small displacements.

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