Mathematical Model for Ship Berthing Allocation in A Multipurpose Terminal: Study Case from Tanjung Perak Port Indonesia

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Abstract—In the operation of a multipurpose terminal in a port (terminal that can serve and be berthed to more than one type of a ship), the position of planning an optimal berth allocation for ships is essential to maintain. Berth allocation planning (BAP) is considered tactical planning that can impact the performance of a terminal and the cost of a ship in a port. That is because the longer the waiting time of a ship that waits to be berthed, the smaller number of ships will be served by the terminal, and the higher the cost will be borne by the ships. This paper, based on a case study in Terminal Jamrud of Tanjung Perak Port of Indonesia, will discuss a Mixed Integer Linear Programming (MILP) mathematical model of a BAP to minimize the ships’ waiting time to berth with consideration to the tailored conditions of the multipurpose terminal. The results show that the optimization model yielded reductions in the datasets 1 to 3 tested on the North/West and 4 to 6 on the South pier. There are savings in the waiting time reduced in hours, which are 403 (12.82%), 189 (26.18%), 418 (30.74%) for the North/West pier and 34 (16.43%), 1663 (88.13%), 475 (46.75%) for the South pier.

Keywords—Berth Allocation Planning, Optimization, MILP, Multipurpose Terminal.

I. INTRODUCTION

A port is a stopping place (terminal) for ships after sailing and an important node in traffic that connects land and sea. Apart from that, it also acts as a turnaround for various types of trade cargo flows[1]. It is estimated that around 80% of the volume of international trade in goods is carried by sea. Those numbers are even higher in most developing countries [2]. Hence, it is crucial to maintain a well-operated port or terminal since the busier the port, the more complicated the planning of the incoming and outgoing vessels and cargo, which affects the whole port or terminal operation.

Jamrud Terminal is one of the busiest terminals at Tanjung Perak Port, Indonesia, and it serves various cargoes (multipurpose terminal). Jamrud terminal is divided into three parts, namely North Jamrud, West Jamrud for international shipping, and South Jamrud for domestic shipping. According to a case study conducted by [3] at the Jamrud Terminal, some ships must sometimes wait several days to dock at the pier because there are still other ships being serviced or berthed, which causes a queue phenomenon in loading and unloading services.

The Berth Allocation Problem (BAP) is one of the main problems in terminal operational planning [4]. Berth allocation is part of port operational planning, which is included in the category of tactical planning to minimize berthing delays [5][6]. In implementing berth allocation, creating a mooring window containing a plot of the ship's mooring schedule is necessary. The berth allocation problem occurs when ships arrive every time, and the terminal operator must place the ships onto the berths to be serviced (loaded or unloaded or both) as soon as possible. Several studies discussed BAP in the port terminal and its effect on the terminal’s operational efficiency.

[7] stated that regarding the diversity of the technical equipment and terminal configurations, there are numerous optimization models for planning seaside operations in container terminals. As for the bulk or multipurpose terminal, the model should follow its characteristics. A study by [8] also stated that there are complexities and uncertainties inherent in bulk port operations that can disrupt normal functioning, requiring

Figure. 1. A picture of ship queue waiting to berth at Terminal Jamrud

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swift real-time actions at the planning level after researching a BAP case of the SAQR port in Ras Al Khaimah, UAE, which studied the key issues, mainly focusing on waiting times at the berth. There are also a few other considerations for the BAP in a bulk or multipurpose port terminal.

[9] studied the maritime industrial port complex in São Luís, Brazil. These ports handle various goods crucial to the Brazilian economy, such as coffee, soy, iron ore, and petrol derivatives. The efficient management of vessel berthing impacts the operational movement and storage of bulk cargo. Such natural restriction of bulk or multipurpose terminals is also discussed by [10], which presents real conditions, and the model included allocation of berth and quay cranes for multiple berths, multiple wharves, and maintain customer satisfaction. Efficient operations are essential to avoid fines, prevent size, production, and transport capacity is crucial.

The importance of BAP towards overall terminal efficiency is discussed in [11], which extended the research by investigating the dynamic and continuous berth allocation problem (BAP) concerning tidal constraints and seeking to minimize the total service time of berthed vessels. Another form of restriction comes from rules made by the terminal. A study by [12] offered a mathematical model for the Continuous Berth Allocation Problem (CBAP) that considers cargo operating limitations along the pier at a multipurpose terminal. The model included the specific berthing restriction or criteria for supporting offshore oil platforms where specific cargos can only operate in certain berthing positions. This study has the closest similarity with Jamrud Terminal of Tanjung Perak Port regarding its rules on ship berthing activities. We will discuss this later in Chapter II.

Eventually, the BAP will also be useful to other important plans in the port’s terminal to ensure operational efficiency [13]. Such an example of the importance of BAP towards overall terminal efficiency is discussed in [14], which focuses on the integrated planning of dry bulk port terminals where balancing stock size, production, and transport capacity is crucial. Efficient operations are essential to avoid fines, prevent accidents, and maintain customer satisfaction.

Several studies have discussed the BAP, which took place in Indonesia, and most of them are container terminals. [15] proposed a template design for BAP with multiple wharves. [16] talked about the dynamic allocation of berth and quay cranes for multiple berths, while [17] proposed discrete-event systems modeling and the algorithm to solve the integration of berth and quay crane allocation. The uncertainty condition is discussed in [18], which analyzes the ships’ arrival and departure time uncertainty, while [19] considers environmental uncertainty in relation to the allocation problem of berth and quay cranes. Another perspective on solving the BAP is proposed in [20], which talks about the risk management approach. All those Indonesian port studies are currently solving the BAP of a container terminal.

In this paper, the problem of BAP in a multipurpose terminal will be discussed through a proposed formulation of a mathematical model that represents the process and conditions of ship berthing activity in the Jamrud multipurpose terminal. No literature has yet been found about the BAP of a bulk or multipurpose terminal in Indonesia. In Chapter II of this paper, we will discuss the method used for the BAP in the Jamrud multipurpose terminal. The results and discussion will be analyzed in Chapter III, and Chapter IV will be the conclusion of this paper.

II. METHOD

This paper discusses how the allocation of ships that will berth at a multi-purpose pier is carried out optimally by considering the limitations of the terminal planner. A research method was developed, as shown in Figure 2, to achieve this goal. In this case, it is necessary to include research objects, terminal and pier characteristics, historical arrivals and ship services, formulation of mathematical models that represent real conditions, and computational results from optimization models.

A. Research Location

The location of this research is in Jamrud Terminal in Tanjung Perak Port, Surabaya. The terminal is divided into some areas, which are Jamrud Utara (North), Jamrud Barat (West), and Jamrud Selatan (South). In this case, based on the current terminal’s practice, the West Jamrud is also included in the North Jamrud. Later, the North/West Jamrud will include those two piers as one.
The North/West pier is prioritized for international cargo, while the South pier is prioritized for domestic cargo. This is because it is in a different draught at the Jamrud terminal, where the North/West pier has a deeper draught than the South pier, so it is considered capable of serving the majority of international vessels by the berthing planners.

Figure 3 above illustrates the distribution of berths for certain cargoes at Jamrud Terminal. The North Pier in the 0m-400m cade is used exclusively for passenger ships and may not be used by cargo ships. Thus, it will not be included in this paper. Meanwhile, 400m-800m is used for ships carrying general cargo, which prioritizes using a ship crane for loading and unloading, but it does not rule out the possibility that during this period, dry bulk ships can dock, which requires an HMC (Harbor Mobile Crane) for loading and unloading. The 800m-1200m cade can be prioritized for berthing ships that require HMC for loading and unloading.

Jamrud Terminal serves domestic container cargo at the South Pier. Apart from container cargo, the South pier specifically serves domestic ship cargo along the pier (0m-800m).

**B. Data and Sources**
The data used in this study are:
1) An interview was conducted with PELINDO to identify the business process of berth allocation planning.
2) Ships and Terminal data on the Turn Round Time (TRT), which includes the ship’s arrival and departure at port and its berthing time.
3) Assumptions used in this research are the approaching time when the ship is coming and leaving the pier and the not operating time when the ship is waiting to be lashed and unlished in the pier and waiting to be permitted to depart by the authority.

**C. Research Tools and Instruments**
The software used in this study is:
1) Microsoft Excel to arrange, model, and analyze the data related to the ships and the terminal performance.
2) LINGO 18 to formulate and compute the optimization model of the berth allocation problem.

**TABLE 1. THE DIVISION OF BERTHS ALONG THE PIER IN JAMRUD TERMINAL**

<table>
<thead>
<tr>
<th>No</th>
<th>Area</th>
<th>Length (m)</th>
<th>Depth (m)</th>
<th>Type of Ship</th>
<th>Prioritization</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>North Jamrud</td>
<td>400-800</td>
<td>11</td>
<td>General Cargo</td>
<td>International</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800-1200</td>
<td>11</td>
<td>Dry Bulk, General Cargo</td>
<td>International</td>
</tr>
<tr>
<td>2</td>
<td>West Jamrud</td>
<td>0-210</td>
<td>7</td>
<td>Dry &amp; Liquid Bulk, General Cargo</td>
<td>International</td>
</tr>
<tr>
<td>3</td>
<td>South Jamrud</td>
<td>0-210</td>
<td>9</td>
<td>Dry Bulk, Container</td>
<td>Domestic &amp; International</td>
</tr>
<tr>
<td></td>
<td></td>
<td>210-800</td>
<td>8.5</td>
<td>General Cargo</td>
<td>Domestic</td>
</tr>
</tbody>
</table>
D. The Classification of Berth Allocation Problem

The berth allocation problem refers to the problem of placing ships on moorings in ports in planning. Therefore, the BAP decision will determine the location and place where the ship is berthed on a pier [21]. There are two types of constraints in the problem: spatial constraints and temporal constraints. Spatial constraints relate to the dimensions of the pier, the size of the ship, the depth of the terminal’s pier, and the partitions of the pier. Meanwhile, the main temporal constraints relate to the ship’s arrival schedule and the ship's berthing schedule.

Based on Figure 3 and Table 1, it is known that in each of the Terminal Jamrud (North/West, South), there are no partitions of specific berths. It means that the ships can be berthed along the pier (e.g., cade 400-510; 511-672; and else) as long as the berthed ships don’t violate the cargo-restrictive pier or the pier’s capacity. Hence, it can be said that the Jamrud Terminal consists of multiple continuous berths, as classified in Figure 4. Note that later, each of Jamrud Terminal’s pier (North/West and South) will be divided into one or more parts and follows the pier’s division as illustrated in Figure 5.

As for the temporal constraints, static arrival will be used in the model for two reasons. First, the current practice of berth allocation planning in Terminal Jamrud conducted by the authority requires the ships to have arrived at the port area before requesting to be planned to berth. Second, since the goal of this paper is to evaluate the manual planning to the optimized one, the requirement of ship arrival conditions will remain the same.

In conducting the berth allocation plan for either the North/West Jamrud or South Jamrud pier, several things are taken into consideration by the berth planners, including:

1) Priority of ship arrival based on its anchoring time.
2) The ship's document readiness to process to dock.
3) Placement of ship berthing locations (cades) at the pier based on the type of cargo.
4) Limited dock zone regarding ship length.
5) The presence of an already berthed ship at the Jamrud Terminal pier when planning the berth allocation.

Points 1) and 2) are used by the ship planners to determine when a ship will be berthed; in this case, point 2) is a non-technical factor, so it is not considered within the scope of the research. This research aims to minimize the waiting time for ships from anchoring to berth so that ships can shorten the length of time they spend in port.

Points 3), 4), and 5) are used to determine the position of the ship berthing at the North/West Jamrud and South Jamrud piers, where the North/West Jamrud pier is divided into cade 400-800 meters for general cargo, bag, unitized, and cade 800-1210 meters for bulk cargo. Dry, liquid bulk. In its implementation, the planners did not fully adhere to these provisions, so there were still ships that berthed outside the designated berthing zones.

E. The Proposed Mathematical Model

In this research, an integer linear programming optimization method is developed to describe the state of the research problem. The model used in this research is adapting the proposed model in [12] that considered the Continuous Berth Allocation Problem (CBAP) with multiple cargo zones to minimize the ships’ waiting time to berth at the port.

Later in this paper’s model, a variable of a vessel i approach time (ai) is added to the model. The approach time is the time or number of hours used during the pilotage service, from the time the ship moves from anchorage to the tie-down of the rope at the mooring and vice versa when the ship leaves the port, which will affect the waiting time to berth of a vessel.

The Terminal Jamrud situation is also categorized as CBAP, where the pier isn’t segregated into several berths so that ships can berth in any position (cade) along the pier, unlike the discrete or hybrid as in Figure 4. In this situation, a cargo zone restriction is also added to the CBAP at Terminal Jamrud, as in Figure 5, while still maintaining the continuous berth conditions.
Figure 6 is an illustration of the main parameters and decision variables in the model. The X-axis represents units of time, and the Y-axis represents the length of the pier on the diagram. A vessel $i$ is depicted in a square where on the Y-axis side there is a length $l_t$, and on the X-axis side there is an operating time (activities at the berth) $t_i$. There is vessel $i$ arrival time (anchoring) $at_i$, and berthing time $bt_i$. So the waiting time for a ship to berth is $wt(b_t_i) = bt_i + a_t_i - at_i$ and after docking $ub_t_i = bt_i + t_i$. Meanwhile, the position/point of the berthed ship is $p_i$ and will occupy the pier from $bp_i$ to $bp_i + l_t$.

As for the segregation of the cargo zones, the starting of cargo zone type 1 is $a_1$ and the end zone of cargo type 1 is $b_1$, and so on.

Sets
- $N$: Number of vessels;
- $C$: Number of cargo zones;

Parameter
- $D$: Length of the pier;
- $TH$: Time horizon of the planning;
- $a_c$: Starting position of cargo zone $c \in C$;
- $b_c$: End position of cargo zone $c \in C$;
- $l_i$: Time spent at berth by vessel $i \in N$;
- $l_i$: Length of vessel $i \in N$ (Length Overall +10% to include safety distance);
- $at_i$: Arrival time of vessel $i \in N$ at port waiting to berth;
- $c_i$: Type of cargo $c \in N$ carried by the vessel $i \in N$;
- $a_t$: Approaching time vessel $i \in N$ towards the pier;

Decision Variables
- $bt_i$: The start position of a berthing vessel $i \in N$ in X-axis (time);
- $ub_i$: The end position of a berthing vessel $i \in N$ in X-axis (time);
- $bp_i$: The start position of a berthing vessel $i \in N$ in Y-axis (pier);
- $X_{ij}$: binary variables concerning the vessel’s time of berth. Equal to 1 if ship $j \in N$ is berthed completely to the right above (Y-axis) the vessel $i \in N$ (does not overlap) on the planning diagram. Otherwise, it equals 0 if it overlaps in position at the pier.

Objective Functions
Minimize
\[ \sum_{i \in N} (ub_i - at_i) \]  
Equation (1), the objective function, is the total time the ship is in port (anchoring, approaching, berthing, unberthing) which must be minimized. Calculated by subtracting the unberthing time and the arrival time of vessel $i$.

Constraints
\[ bt_i - bt_i - t_i - (X_{ij} - 1)TH \geq 0 \quad \forall i,j \in N, i \neq j \]  
Constraint (2) is to ensure that no vessels overlap in time (X-axis) in the planning diagram.

\[ bp_j - bp_j - l_t - (Y_{ij} - 1)D \geq 0 \quad \forall i,j \in N, i \neq j \]  
Constraint (3) ensures that no ships overlap in the pier (Y-axis) in the planning diagram.

\[ X_{ij} + X_{ji} + Y_{ij} + Y_{ji} \geq 1 \quad \forall i,j \in N, i \neq j \]  
Constraint (4) requires at least a value of 1 in the sum of these variables, to ensure that there is no overlap between ships on the X and Y axes on the planning diagram. An illustration of this situation is pictured in Figure 7.

For example, Figure 7(a) shows a situation where on the X-axis, vessel $j$ is overlapping with vessel $i$. In this situation, it can be seen that $X_{ij} = 0$ because vessel $j$ is not completely to the right, and $X_{ji} = 0$ because vessel $i$ is not to the right of vessel $j$. On the Y axis, the value $Y_{ij} = 0$ because vessel $j$ is not above vessel $i$, and $Y_{ji} = 0$ because vessel $i$ is not completely above vessel $j$. Therefore, the value of constraint (4) is 0 or $X_{ij} + X_{ji} + Y_{ij} + Y_{ji} = 0$. 

Figure 6. The main parameter and the decision variables of the mathematical model.
In Figure 7(b), conditions on the X-axis show that vessel \( j \) is completely on the right of vessel \( i \) \((X_{ij} = 1)\) and vessel \( i \) is not to the right of vessel \( j \) \((X_{ji} = 0)\). Meanwhile, on the Y-axis, vessel \( i \) is parallel (not above) vessel \( j \) \((Y_{ij} = 0)\) and vessel \( j \) is also parallel (not above) vessel \( i \) \((Y_{ji} = 0)\). Therefore, constraint (4) has a value of 2 or \(X_{ij} + X_{ji} + Y_{ij} + Y_{ji} = 1\) and is said to be feasible.

In Figure 7(c), conditions on the X-axis vessel \( j \) is not to the right of vessel \( i \) \((X_{ij} = 0)\), and vessel \( i \) is not to the right of vessel \( j \) \((X_{ji} = 0)\). Meanwhile, on the Y-axis, vessel \( i \) is completely above vessel \( j \) \((Y_{ij} = 1)\), and vessel \( j \) is not above vessel \( i \) \((Y_{ji} = 0)\). Therefore, constraint (4) has a value of 1 or \(X_{ij} + X_{ji} + Y_{ij} + Y_{ji} = 1\) and is said to be feasible.

In Figure 7(d), there is a different condition where on the X-axis, vessel \( j \) is completely on the right of vessel \( i \) \((X_{ij} = 1)\) and vessel \( i \) is not to the right of vessel \( j \) \((X_{ji} = 0)\). Meanwhile, on the Y-axis, vessel \( i \) is completely above vessel \( j \) \((Y_{ij} = 1)\), and vessel \( j \) is not above vessel \( i \) \((Y_{ji} = 0)\). Therefore, constraint (4) has a value of 2 or \(X_{ij} + X_{ji} + Y_{ij} + Y_{ji} = 2\) and is said to be feasible.

\[
bt_i + t_i = ub_i \quad \forall i \in N \tag{5}
\]
Constraint (5) ensures in the model that the unberthing time is the sum of the time berthing of the vessel with the operating time (unloading/loading) of the vessel at the pier.

\[
(a_i + a_j) \leq b_i \leq (TH - t_i) \quad \forall i \in N \tag{6}
\]
Constraint (6) ensures in the model that the position of the vessel's berthing time \((bt_i)\) is between the arrival time \((a_i)\) and the reduction of the planning period from the operating time (unloading/loading) of the ship at the berth \((TH - t_i)\). This also means that ships that anchorage outside the planning horizon cannot be included in the model and for other situations where a vessel cannot be included in the model if its operating time (unloading/loading) exceeds the planning horizon \((TH)\). In this way, the planning period must be enlarged to adapt to the optimization situation.
\[ \alpha_i \leq p_i \leq (\beta_i - l_i) \quad \forall i \in N \]  
Constraint (7) is to make the vessel \( i \in N \) berth at the berth that corresponds to the type of cargo (pier division).

The berthing position of the vessel is between the initial position of the berth (\( \alpha \)) and the final position of the berth (\( \beta \)) of the type of cargo handled at the berth \( c \in C \). Each vessel \( i \in N \) carries the type of cargo \( c_i \in C \). Meanwhile, constraints (8) and (9) explain that \( X_{ij} \) and \( Y_{ij} \) are binary constraints.

**TABLE. 2. AN EXAMPLE OF THE DATA INPUTS USED IN THE STUDY CASE’S MATHEMATICAL MODEL**

<table>
<thead>
<tr>
<th>Dataset No.3</th>
<th>Vessel No</th>
<th>Cargo Type</th>
<th>Payload (Ton)</th>
<th>( h_t ) (meters)</th>
<th>( t_t ) (hours)</th>
<th>Anchorage Time</th>
<th>( a_t ) (hours)</th>
<th>( a_i ) (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 Dry Bulk</td>
<td>2</td>
<td>800</td>
<td>195,8</td>
<td>27-Jul</td>
<td>1.03 PM</td>
<td>0,00</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1 Dry Bulk</td>
<td>2</td>
<td>8337</td>
<td>185,9</td>
<td>01-Aug</td>
<td>2.00 AM</td>
<td>0,00</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1 General Cargo</td>
<td>1</td>
<td>29102</td>
<td>209</td>
<td>29-Jul</td>
<td>1.43 AM</td>
<td>0,00</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1 General Cargo</td>
<td>1</td>
<td>2182</td>
<td>128,7</td>
<td>01-Aug</td>
<td>4.50 PM</td>
<td>0,00</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1 Dry Bulk</td>
<td>2</td>
<td>27500</td>
<td>198</td>
<td>31-Jul</td>
<td>2.17 PM</td>
<td>0,00</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1 Dry Bulk</td>
<td>2</td>
<td>33594</td>
<td>206,8</td>
<td>29-Jul</td>
<td>12.45 AM</td>
<td>1,25</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1 Dry Bulk</td>
<td>2</td>
<td>44497</td>
<td>247,5</td>
<td>28-Jul</td>
<td>12.45 AM</td>
<td>0,00</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>1 Liquid Bulk</td>
<td>1</td>
<td>1590</td>
<td>146,5</td>
<td>05-Aug</td>
<td>12.30 PM</td>
<td>84,50</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>1 General Cargo</td>
<td>1</td>
<td>49335</td>
<td>110</td>
<td>06-Aug</td>
<td>5.15 AM</td>
<td>101,25</td>
<td>1</td>
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<tr>
<td>10</td>
<td>1 General Cargo</td>
<td>1</td>
<td>18860</td>
<td>206,8</td>
<td>06-Aug</td>
<td>12.40 AM</td>
<td>96,67</td>
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</tr>
<tr>
<td>11</td>
<td>1 General Cargo</td>
<td>1</td>
<td>3236</td>
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<td>08-Aug</td>
<td>3.30 AM</td>
<td>147,50</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>1 General Cargo</td>
<td>1</td>
<td>2685</td>
<td>143</td>
<td>08-Aug</td>
<td>4.00 PM</td>
<td>160,00</td>
<td>1</td>
</tr>
</tbody>
</table>

**TABLE. 3. AN EXAMPLE OF THE DECISION VARIABLES RESULTING FROM THE DATASET**

<table>
<thead>
<tr>
<th>Dataset No.3</th>
<th>Vessel No</th>
<th>X-Axis</th>
<th>Berthing Time</th>
<th>Unberthing Time</th>
<th>Y-Axis</th>
<th>Real Data Optimization</th>
<th>Total Time Spent At Port</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-9 Aug</td>
<td>1</td>
<td>1.0</td>
<td>62.8</td>
<td>02-Aug</td>
<td>1.0</td>
<td>1208</td>
<td>132,4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.0</td>
<td>20.7</td>
<td>02-Aug</td>
<td>1.0</td>
<td>4.93 PM</td>
<td>157.8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.0</td>
<td>46.6</td>
<td>02-Aug</td>
<td>1.0</td>
<td>8.54 PM</td>
<td>174.5</td>
</tr>
<tr>
<td>9-15 Aug</td>
<td>1</td>
<td>1.0</td>
<td>10.3</td>
<td>02-Aug</td>
<td>1.0</td>
<td>10.38 PM</td>
<td>179.7</td>
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<td></td>
<td>2</td>
<td>1.0</td>
<td>59.2</td>
<td>02-Aug</td>
<td>1.0</td>
<td>11.11 PM</td>
<td>231.6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.0</td>
<td>126.1</td>
<td>02-Aug</td>
<td>1.0</td>
<td>6.06 AM</td>
<td>287.1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.0</td>
<td>201.8</td>
<td>04-Aug</td>
<td>1.0</td>
<td>9.46 AM</td>
<td>343.9</td>
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**TABLE 2. AN EXAMPLE OF THE DATA INPUTS USED IN THE STUDY CASE’S MATHEMATICAL MODEL**

**TABLE 3. AN EXAMPLE OF THE DECISION VARIABLES RESULTING FROM THE DATASET**
III. RESULTS AND DISCUSSION

A. Data Inputs

Table 2 shows the example from one of the datasets used in the study case. Dataset no. 3 above was based on the situation in the North/West pier. At first, the planning was divided into four weeks (not necessarily 7 days) starting from August 2nd until August 31st and the planning horizon is equal to 720 hours. Later, the vessels are clustered based on their arrival time. The type of the vessel was then categorized based on the pier’s cargo zone (see Figure 5).

It is known that every vessel has a distinguished payload, length ($l_t$), berthing time ($t_b$), and arrival/anchorage time ($a_t$). Notice that for the $a_t$, some vessels already arrived from the previous time horizon $a_t = 0$ and for the rest of the vessels, their $a_t$ will depend on their arrival date is subtracted by the initial date of the planning horizon and then converted into hours.

The total time spent at port as explained previously in Figure 6, is the total time of a vessel starting from the arrival time to the unberthing time. Meanwhile, the waiting time to berth ($w_t b$) will only consider the time spent by a vessel waiting to be berthed caused by the planning of berth allocation. The summary of all vessels’ waiting times to berth from all datasets used in this paper is shown in Table 4.

Several datasets are used in the mathematical model as shown in Table 4. Each pier category has three datasets that consist of different vessel populations and cargo types. Looking by the cargo type, the North/West and South pier predominantly consists of general cargo vessels while the second population of most vessels are dry bulk for the North/West pier and containerships in the South pier.

It is known here from Table 4 that the mathematical model is giving better results in reducing the total waiting time at berth from all datasets.

<table>
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<th>TABLE 4. SUMMARY OF ALL DATASETS</th>
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Figure 8. Results comparison between the real data and the optimization of waiting time to berth

B. Results

Table 3 shows an example of the decision variables from the datasets where each vessel will be allocated in the X-axis for the berthing time position ($b_t$), and unberthing time position ($u_b$). Meanwhile, in the Y-axis, the vessel will be allocated for the berthing position at the pier ($b_p$) and where it ends.

In the end, the total time spent at the port from the real data (realization) will be compared with the optimized one caused by the mathematical model. Notice that there are possibly some reductions yielded in negative numbers which indicate that the optimized time spent at the port is worse than the real data’s time spent at the port, thus making it negative (no reduction).

Dataset 1, located in the North/West pier, is found to be the busiest of all datasets with a population of 44 vessels with a total of 3143 total hours of waiting time to berth while getting optimized to 2740 hours (12.82%). Datasets 2 and 3, which have similar populations of 27 vessels, are found to have similar percentages of reduction (26.18% and 30.74%).

The rest of the datasets are in the South pier. Some extreme findings are shown in the datasets 4 and 5. In Dataset 4, although it has the largest population of vessels among the others (58 vessels), it has the lowest waiting time to berth (207 hours). It may indicate the actual process of berthing and allocating the berth.

One of the most extreme differences is shown in dataset no.5 where there is an 88.13% reduction in the
waiting time to berth coming from only 43 vessels. Based on the actual reflection of the real situation at the terminal, there are some factors aside from berth allocation that may have a major impact on the inefficiency of the waiting time to berth for a vessel.

A mathematical model is developed to model the real condition based on the situation at the Jamrud Terminal Tanjung Perak Indonesia, which serves as a multipurpose cargo terminal. The objective of the model is to find the minimum total waiting time of all vessels that are served at the terminal. Later, the model’s results will be compared with the data from the terminal’s actual record of the vessels.

The results yielded from 6 datasets show that the waiting time from current berth allocation planning activity could be improved as the efficiency ranges vary. Some factors to be considered in the planning are the number of vessels in a planning horizon and the type of cargo of the vessels. Both of those factors will hugely determine the current piers’ capacity.

The mathematical model tested in this paper, however, is used to evaluate a whole month of planning, which is less suitable with the current practice of the planning done by the terminal planner. The planning itself is dynamic planning that is constantly updated by the terminal planner. It makes more sense since, on the seaside, new vessels will keep arriving, and on the land side, there are possible changes in the crane or other operational performance at the terminal.

Therefore, a rolling horizon needs to be considered in the future so that the planning can be done in a shorter time horizon since the bigger the dataset, the longer the computation will take to solve the optimization.

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