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Mathematical Modelling of Longitudinal Vibration on Propulsion System 5200 DWT General Cargo Ship

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***Abstract*— The vibration level of the propulsion system will change during its operation. This vibration is caused by harmonic excitation forces produced by the rotation of the main engine and propeller shaft. Ship propulsion systems experience longitudinal, torsional and lateral vibrations. Excessive vibration will produce noise and reduce engine performance. Vibrations can also cause resonance in the system, which can be fatal and damage the structure. The excitation frequency value is close to or equal to the natural system frequency, which causes resonance. This paper has identified the vibration response of the propulsion system by using numerical software through mathematical modelling governed by ABS. In addition, the total vibration response was obtained using the modal analysis method by summing up the contributions of each mode. The excitation source generated is due to the rotation of the main engine. Ultimately, the response obtained will be adjusted to the standard class. The modelling results obtained a 3-Degree-of-Freedom forced vibration model consisting of three masses and three springs. The resulting response values are displacement and velocity, where the highest response occurs at 347 rpm with a deviation of ±0.1345 mm to ±0.3371 mm and a velocity value of ±4.8847 mm/s to ±12.2424 mm/s. The slightest response occurs at 459 rpm with a deviation range of ±0.0034 mm to ±0.0050 mm and velocity values of ±0.1634 mm/s to ±0.2382 mm/s. Based on all the results of adjusting the vibration response value with the ABS class vibration limit graph, the vibration is still below the permissible threshold line.**

Keywords—Propulsion, Mathematical Modelling, Modal Analysis, Displacement and Velocity Response, ABS.

I. Introduction

Propulsion systems that work for a certain period will experience changes in vibration levels per their operation. The harmonic excitation forces are due to the propeller shaft and the main engine's rotation [1]. Several vibrations occur in ship propulsion systems, such as lateral, torsional, and longitudinal. Vibrations that occur excessively will produce noise, decrease engine performance and damage the ship structure [2]. Therefore, the vibration level must be known to anticipate all possible failures of structures and components of the propulsion system [3].

Many studies on ship vibrations have been conducted, especially on propulsion systems. Putranto et al. [2] studied determining the vibration of patrol boat propulsion systems using the finite element method. The natural frequency values of several mode shapes are calculated by modal analysis to determine whether resonance occurs if the ship is operated at a service speed of 25 knots. Garudhea [4] analyzed the vibration response of the flexible coupling of the main engine. The study used a mathematical modelling method with the help of numerical computational engineering software. Data on the misalignment value, moment of inertia, stiffness coefficient, and mass inertia are processed to obtain the frequency value used to determine the occurrence of resonance and response. Irtiza [5] has researched the modelling, design and analysis of submarine propeller shafts using vibration theory and computer software. Mathematical simulations were also carried out to verify the results obtained from the software. The modelling and analysis were carried out using the finite element method. The expected outcome of this analysis is to get the least vibration to reduce the overall noise.

Sachin's study [6] discusses the numerical estimation of modal parameters such as natural frequencies and mode shapes using the basic vibration theory of two and three degrees of freedom (DOF) vibration systems. The principle of mode shape orthogonality is used in this study to transform the physical coordinate system into a generalized coordinate system that generates nonseparable differential equations. Heroe et al. [7] analyzed the vibration characteristics in the foundation of the 907 GT offshore supply ship's main engine under operating conditions. This study uses a numerical method

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based on mathematical modelling of the construction of the main engine foundation, where the vibration characteristics of the foundation are analyzed using the natural frequency value and the velocity response given by the main engine foundation due to the excitation generated by the rotation of the main engine when the ship is operating at service speed.

This study introduces a new approach by combining numerical modal analysis with ABS classification standards to assess the longitudinal vibration response of a 5200 DWT general cargo ship's propulsion system. Unlike previous research [2,4,5], which focused on natural frequency, flexible couplings, and submarine propeller shafts, this study emphasizes regulatory compliance in the modeling process. While past studies [6] analyzed undamped MDOF systems without real-world excitation, this research incorporates engine rotation as a primary vibration source. Using a 3-DOF forced vibration model, the total system vibration response is obtained through modal analysis by summing the contribution of each mode, providing a detailed assessment of displacement and velocity responses. Additionally, unlike studies that rely solely on simulations, this research validates results against ABS vibration limits, ensuring practical relevance and a more comprehensive approach to propulsion system vibration analysis.

II. Method

A. Longitudinal Vibration

The most interesting case regarding machine vibration, which often causes forced vibration, is the longitudinal vibration of the propulsion system. Longitudinal vibration in ship propulsion systems is one of the vibrations with motion coordinates parallel to the propeller shaft axis. This vibration arises from the propeller's rotation and the main engine's radial movement. The axial force of the propeller (thrust) is held by the thrust block, which is then forwarded to the ship's construction (main engine foundation). Due to the axial force of the propeller, the thrust block and its foundation will experience a longitudinal shift [8].

B. Propulsion System Modelling

Modelling of the system is done by determining the degrees of freedom or the number of coordinates of the propulsion system to determine the position or movement of the complete system in each part. Figure 1 is a mathematical modelling scheme of the propulsion system with a coordinate number of three degrees of freedom consisting of three springs, three masses and one pedestal.

Where M_1 is the mass of the propeller, increased by 60% for the additional hydrodynamic mass and added ½ the mass of the propeller shaft, M_2 is ½ the mass of the propeller shaft and the mass of the flange shaft, M_3 is the mass of the line shaft and the mass of the engine plus the mass of the thrust bearing with an additional 25% of the engine mass for structural, K_1 is the stiffness of the propeller shaft and flange shaft, K_2 is the stiffness of the line shaft and thrust bearing, K_3 is the stiffness of the thrust bearing and engine foundation, and $F(t)$ is the excitation force in the form of rotation of the main engine.

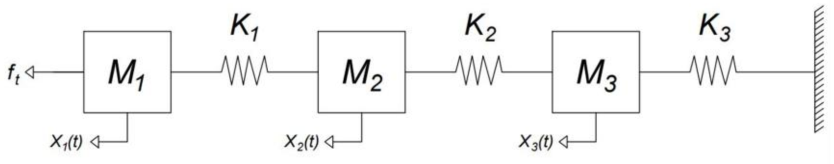

Figure. 1. Three-degree-of-freedom model of ship propulsion system.

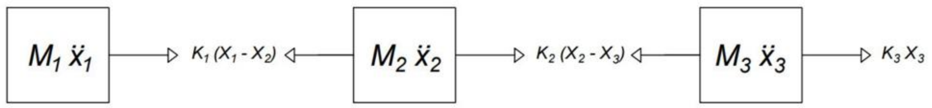

Figure. 2. Free body diagram of ship propulsion system.

C. Derivation of Equation of Motion

A free-body diagram of the ship propulsion system modelling is required to develop the system's motion equation each time. Figure 2. shows the free-body diagram of each mass in the ship's propulsion system.

Equation (1) is the form of the motion equation for M_1 as follows:

$$m_1\,\ddot{x}_1 = -k_1\,(x_1 - x_2)$$
$$m_1\,\ddot{x}_1 = -k_1\,x_1 + k_1\,x_2$$
$$m_1\,\ddot{x}_1 + k_1\,x_1 - k_1\,x_2 = F \tag{1}$$

Equation (2) is the form of the motion equation for M_2 as follows:

$$m_2\,\ddot{x}_2 = -k_2\,(x_2 - x_3) + k_1\,(x_1 - x_2)$$
$$m_2\,\ddot{x}_2 = -k_2\,x_2 + k_2\,x_3 + k_1\,x_1 - k_1\,x_2$$
$$\mathrm{m}_2\,\ddot{x}_2 + k_2\,x_2 + k_2\,x_3 + k_1\,x_2 = k_1\,x_1$$
$$m_2\,\ddot{x}_2 + (k_1 + k_2)x_2 + k_2\,x_3 - k_1\,x_1 = F$$
$$-k_1\,x_1 + m_2\,\ddot{x}_2 + (k_1 + k_2)x_2 - k_2\,x_3 = F \tag{2}$$

Equation (3) is the form of the motion equation for M_3 as follows:

$$m_3\,\ddot{x}_3 = -k_3\,x_3 + k_2\,(x_2 - x_3)$$
$$m_3\,\ddot{x}_3 = -k_3\,x_3 + k_2\,x_2 - k_2\,x_3$$
$$m_3\,\ddot{x}_3 + k_3\,x_3 + k_2\,x_3 = k_2\,x_2$$
$$m_3\,\ddot{x}_3 + (k_3 + k_2)x_3 - k_2\,x_2 = F$$
$$-k_2\,x_2 + m_3\,\ddot{x}_3 + (k_2 + k_3)x_3 = F \tag{3}$$

D. Derivation of Equation of Motion

Modal analysis is a method used to analyze vibration and can be applied to systems with multiple degrees of freedom multi-degree of freedom or MDOF. The vibration response of a multi-degree freedom system involves studying how the system reacts to external forces. The total vibration response of the system can be obtained by summing the contributions from each vibration mode [9][10].

For a system with sinusoidal force $F(t)$, the particular response of each mode can be calculated using the Equation (4) as follows:

$$x_p(t) = A_i \sin(\omega_f t) \tag{4}$$

ω_f is the frequency of the forced force, t is the time, and A_i is the amplitude of mode i. For amplitude, it can be calculated by Equation (5) as follows.

$$A_i = \frac{\phi_i^T F_0}{\omega_n^2 - \omega_f^2} \tag{5}$$

ω_n is the natural frequency, ϕ is the mode shape, and F_0 is the force amplitude. The total response can be found by summing up the particular responses in each mode, which is shown in Equation (6) [9][10].

$$x(t) = \sum_{i=1}^{n} \phi_i x_p(t) \tag{6}$$

Using this method, we can separate the contribution of each mode and understand how each mode contributes to the total response of the system.

E. Vibration Limits

The American Bureau of Shipping (ABS) recommends that the displacement for component response should be below 1 mm and the possibility of damage above 2 mm. As for the response size based on vibration velocity, ABS recommends below 30 mm/s and the possibility of damage occurring above 60 mm/s [11]. Figure 3. is a graph of vibration limits according to ABS.

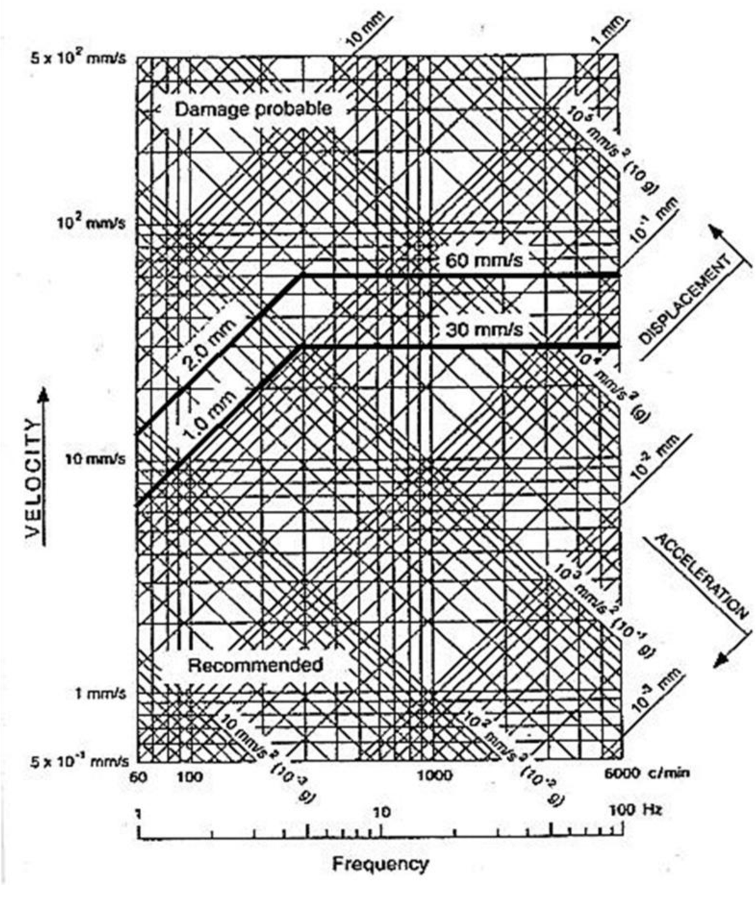

Figure. 3. Vibration limitation by ABS.

III. Results and Discussion

A. Research Data

Table 1 shows the principal dimensions of the 5200 DWT General Cargo Ship, while Table 2 shows the technical specification of the main engine. Table 3 shows the variation of the main engine rotation used in this study.

TABLE 1.
PRINCIPAL DIMENSIONS OF GENERAL CARGO SHIP 5200 DWT [12]

Main of Principal	Dimension
LOA	96.5 m
LPP	90.8 m
LWL	93.22 m
B	15.8 m
H	7.4 m
T	5.95 m
Vs	11.6 knot

TABLE 2.
MAIN ENGINE SPECIFICATIONS OF GENERAL CARGO SHIP 5200 DWT [12]

Descriptions	Specifications
Merk	ANTAI WUXI
Type	G8300ZC16B
Rate Power	1765 kW
Rate Speed	525 RPM
Cylinder Bore	300 mm
Piston Stroke	380 mm
Mass	20500 kg

TABLE 3.
MAIN ENGINE VARIATIONS OF GENERAL CARGO SHIP 5200 DWT [12]

Variations	RPM
1	240
2	310
3	347
4	360
5	459

B. *Propulsion System Vibration Equatio*

Referring to Figure 2 and the description of the equations of motion for each period in Equation (1), Equation (2) and Equation (3), the vibration equations of the 5200 DWT General Cargo propulsion system will be obtained as follows.

$$m_1\,\ddot{x}_1 + k_1\,x_1 - k_1\,x_2 = F$$
$$-k_1\,x_1 + m_2\,\ddot{x}_2 + (k_1\, + k_2)x_2 - k_2\,x_3 = F$$
$$-k_2\,x_2 + m_3\,\ddot{x}_3 + (k_2\, + k_3)x_3\; = F$$

The differential Equation can be converted into matrix form. The matrix form is as follows.

$$\begin{bmatrix} M1 & 0 & 0 \\ 0 & M2 & 0 \\ 0 & 0 & M3 \end{bmatrix}\begin{bmatrix} \ddot{x}1 \\ \ddot{x}2 \\ \ddot{x}3 \end{bmatrix} + \begin{bmatrix} K1 & -K1 & 0 \\ -K1 & (K1+K2) & -K2 \\ 0 & -K2 & (K2+K3) \end{bmatrix}\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} F1(t) \\ F2(t) \\ F3(t) \end{bmatrix}$$

C. *Displacement Response*

The 5200 DWT General Cargo propulsion system's vibration response was calculated at five engine speed variations, according to Table 3. The displacement response values are obtained by modal analysis according to Equation (6). The results of displacement response calculations are presented in Figures 4 to 8.

Figure 4 shows the displacement response at 240 rpm engine rotation. The displacement responses vary from –0.025 mm to 0.025 mm for Mass Component 1 and 2 and from –0.008 mm to 0.008 mm for Mass Component 3. The displacement amplitude is 0.025 mm for Mass Component 1 and 2 and 0.008 for Mass Component 3.

Figure 5 shows the displacement response at 310 rpm engine rotation. The displacement responses vary from –0.034 mm to 0.034 mm for Mass Component 1 and 2 and –0.004 mm to 0.004 mm for Mass Component 3. The displacement amplitude is 0.034 mm for Mass Component 1 and 2 and 0.004 for Mass Component 3.

Figure 6 shows the displacement response at 347 rpm engine rotation. The displacement responses vary from –0.337 mm to 0.337 mm for Mass Component 1, –0.335 mm to 0.335 mm for Mass Component 2, and –0.134 mm to 0.134 mm for Mass Component 3. The displacement amplitude is 0.337 mm for Mass Component 1, 0.335 for Mass Component 2, and 0.134 for Mass Component 3.

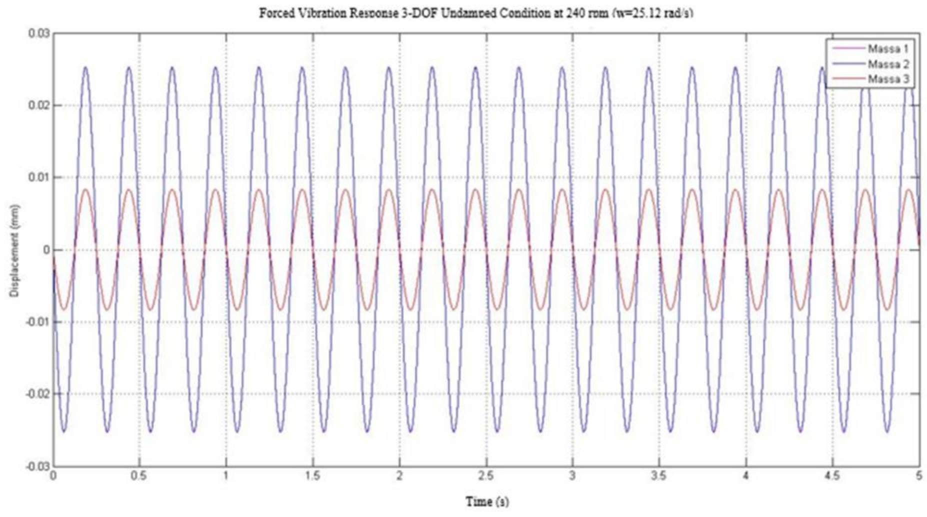

Figure. 4. Graph of displacement response at 240 RPM.

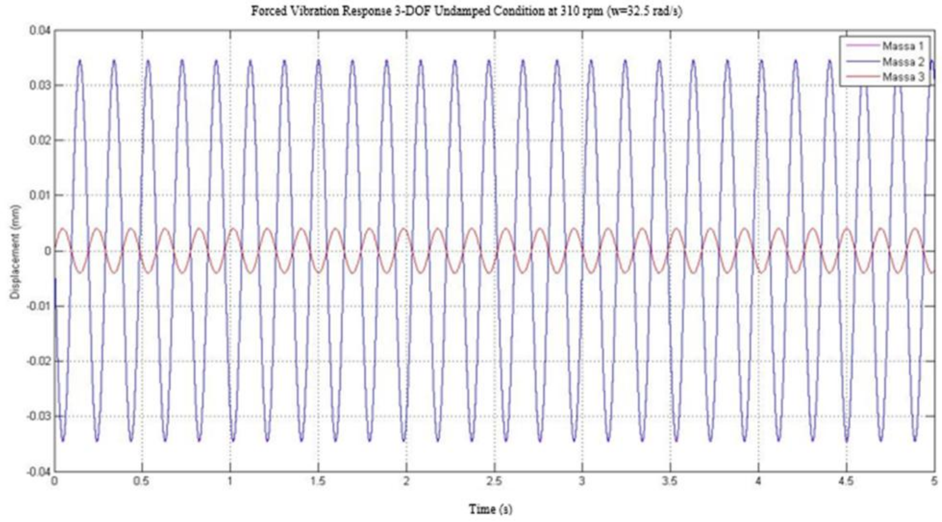

Figure. 5. Graph of displacement response at 310 RPM.

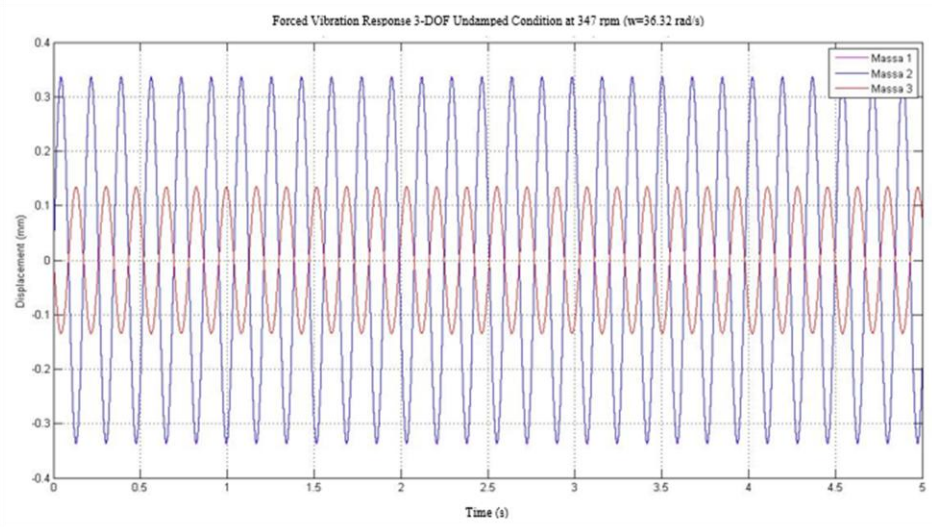

Figure. 6. Graph of displacement response at 347 RPM.

Figure 7 shows the displacement response at 360 rpm engine rotation. The displacement responses vary from –0.049 mm to 0.049 mm for Mass Component 1 and 2 and –0.025 mm to 0.025 mm for Mass Component 3. The displacement amplitude is 0.049 mm for Mass Component 1 and 2 and 0.025 for Mass Component 3.

Figure 8 shows the displacement response at 459 rpm engine rotation. The displacement responses vary from –0.003 mm to 0.003 mm for Mass Component 1 and 2 and –0.004 mm to 0.004 mm for Mass Component 3. The displacement amplitude is 0.003 mm for Mass Component 1 and 2 and 0.004 for Mass Component 3.

Figure 8 shows the displacement graph when the engine rotation condition is 459 rpm with a value range on Mass Component - 1 of -0.003 mm to 0.003 mm. The range of deviation values on Mass Component - 2 is -0.003 mm to 0.003 mm. The range of deviation values on Mass Component - 3 is -0.004 mm to 0.004 mm.

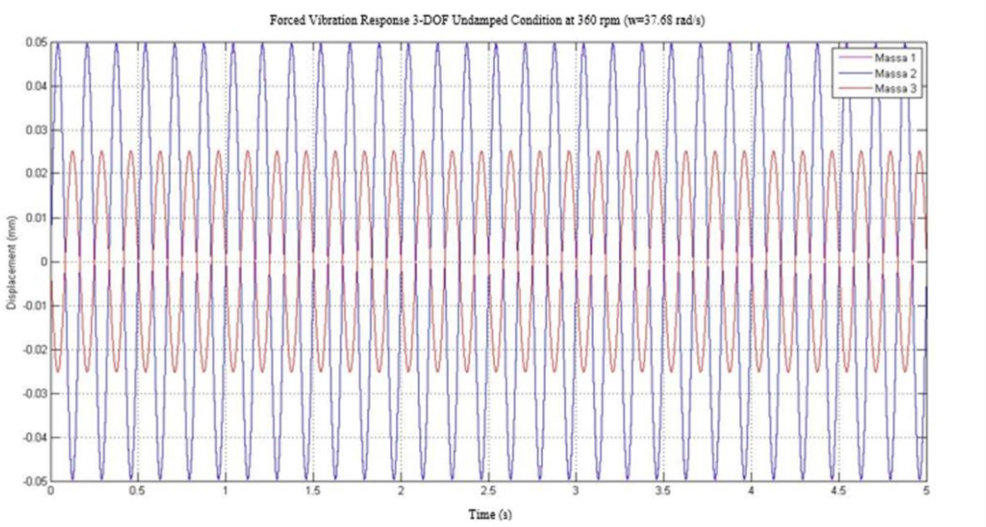

Figure. 7 . Graph of displacement response at 360 RPM.

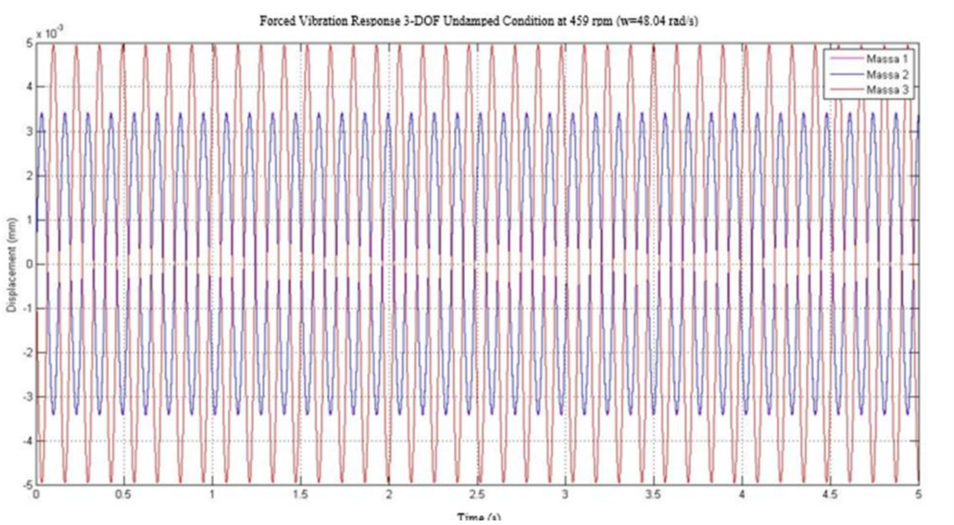

Figure. 8. Graph of displacement response at 459 RPM.

D. Velocity Response

The velocity response is obtained by differentiating the vibration response equation. The calculation results of vibration velocity are presented in Figures 9, 10, 11, 12 and 13.

Figure 9 illustrates the velocity response for an engine rotation of 240 rpm. A velocity response range of –0.635 mm/s to 0.635 mm/s for Mass Component 1, –0.633 mm/s to 0.633 mm/s for Mass Component 2, and –0.209 mm/s to 0.209 mm/s for Mass Component 3.

Figure 10 illustrates the velocity response for an engine rotation of 310 rpm. A velocity response range of –1.122 mm/s to 1.122 mm/s for Mass Component 1, –1.118 mm/s to 1.118 mm/s for Mass Component 2, and –0.131 mm/s to 0.131 mm/s on Mass Component 3.

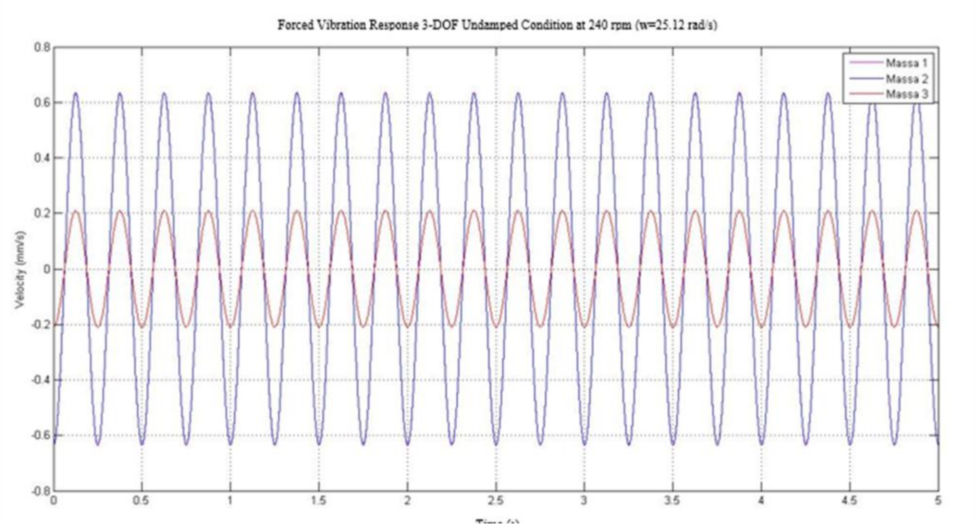

Figure. 9. Graph of velocity response at 240 RPM..

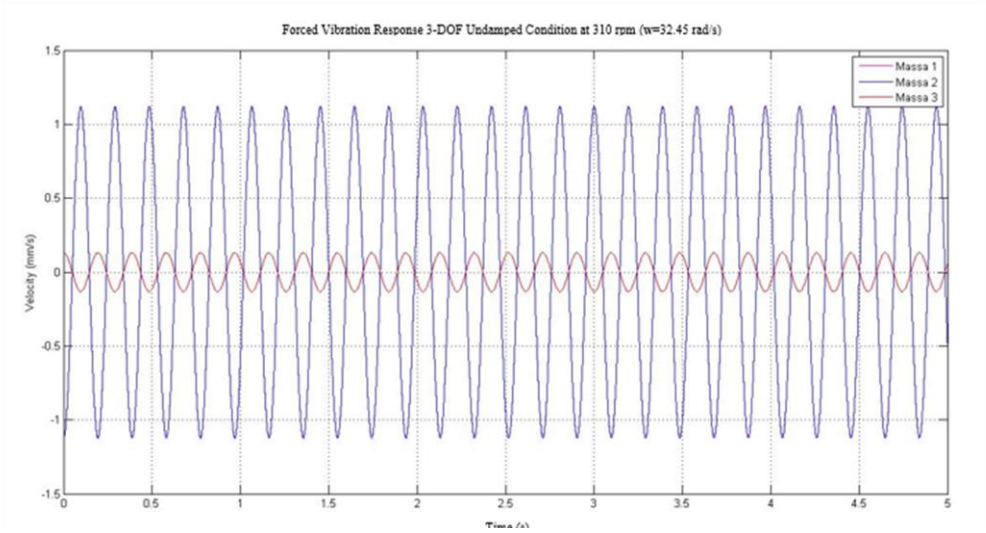

Figure. 10. Graph of velocity response at 310 RPM..

Figure 11 illustrates the velocity response for an engine rotation of 347 rpm. A velocity response range of –12.241 mm/s to 12.241 mm/s for Mass Component 1, –12.185 mm/s to 12.186 mm/s for Mass Component 2, and –4.885 mm/s to 4.884 mm/s for Mass Component 3.

Figure 12 illustrates the velocity response for an engine rotation of 360 rpm. A velocity response range of –1.875 mm/s to 1.875 mm/s for Mass Component 1, –1.886 mm/s to 1.886 mm/s for Mass Component 2, and –0.948 mm/s to 0.948 mm/s for Mass Component 3.

Figure 13 illustrates the velocity response for an engine rotation of 459 rpm. A velocity response range of –0.164 mm/s to 0.164 mm/s for Mass Component 1, –0.163 mm/s to 0.163 mm/s for Mass Component 2, and –0.238 mm/s to 0.238 mm/s for Mass Component 3.

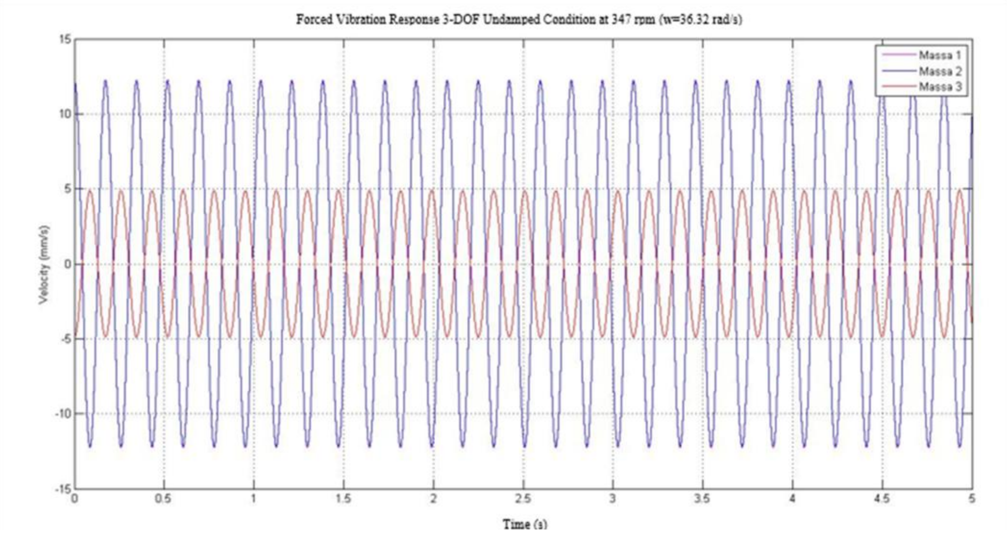

Figure. 11. Graph of velocity response at 347 RPM..

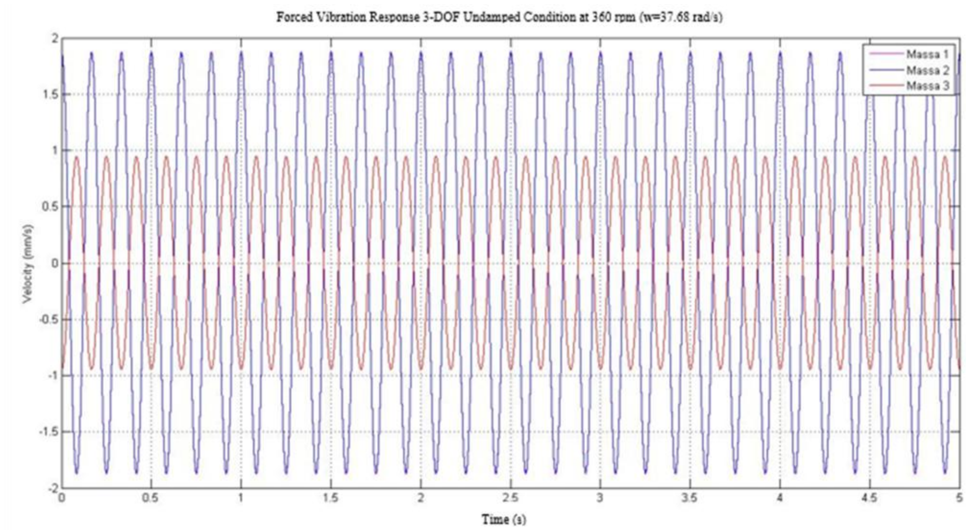

Figure. 12. Graph of velocity response at 360 RPM..

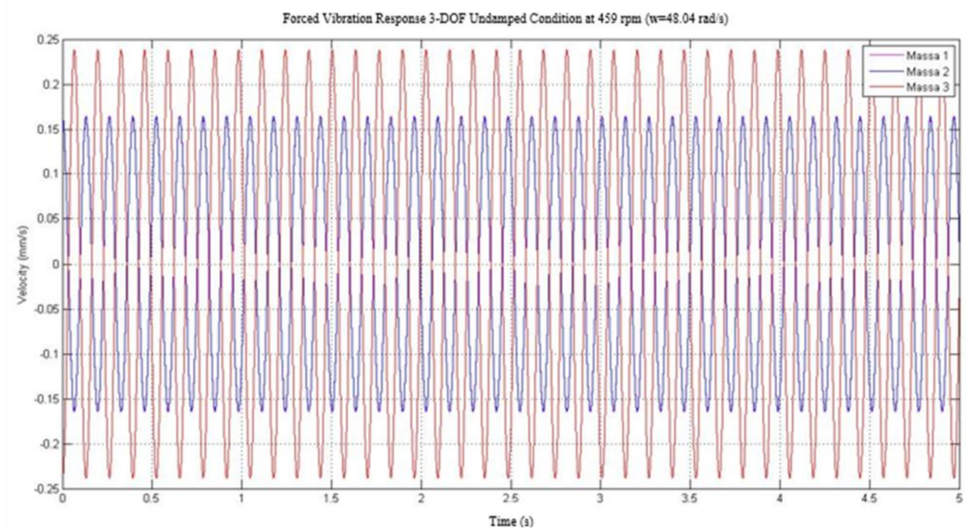

Figure. 13. Graph of velocity response at 459 Rpm.

E. Response Amplitude Analysis

From the five variations of the main engine rotation, it can be seen that the resulting deviation and velocity response graphs are classified as steady responses. The displacement amplitude and velocity at all engine speed variations also change. Table 4 shows the maximum amplitude value of displacement at all variations of the main engine rotation in the three masses. Table 5 shows the maximum amplitude value of the velocity at all variations of the main engine speed in the three masses.

TABLE 4.
AMPLITUDE OF DISPLACEMENT AT VARIATIONS OF THE MAIN ENGINE ROTATION

Displacement	240 Rpm	310 Rpm	347 Rpm	360 Rpm	459 Rpm
Mass 1	0.0253	0.0346	0.3371	0.0498	0.0034
Mass 2	0.0252	0.0345	0.3355	0.0495	0.0034
Mass 3	0.0084	0.0040	0.1345	0.0252	0.0050

TABLE 5.
AMPLITUDE OF VELOCITY AT VARIATIONS OF THE MAIN ENGINE ROTATION

Velocity	240 Rpm	310 Rpm	347 Rpm	360 Rpm	459 Rpm
Mass 1	0.6353	1.1226	12.2424	1.8756	0.1647
Mass 2	0.6339	1.1185	12.1870	1.8664	0.1634
Mass 3	0.2100	0.1313	4.8847	0.9484	0.2382

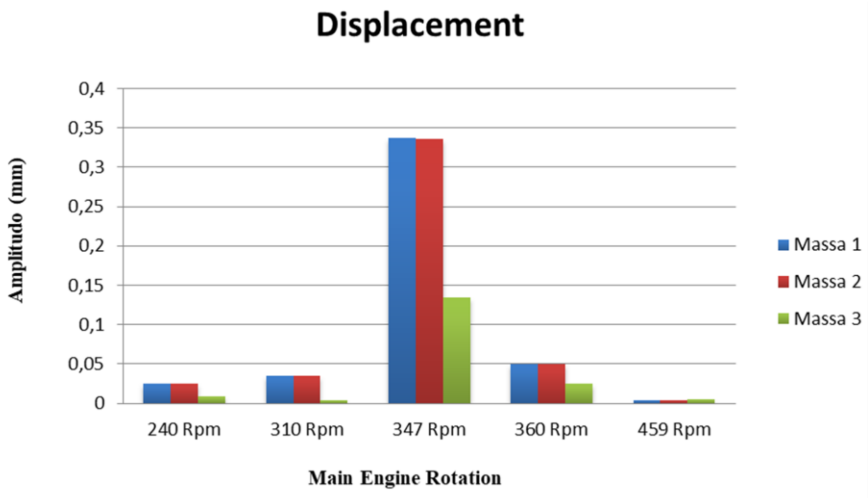

Figure. 14. Graph of displacement amplitude at variation of engine speed.

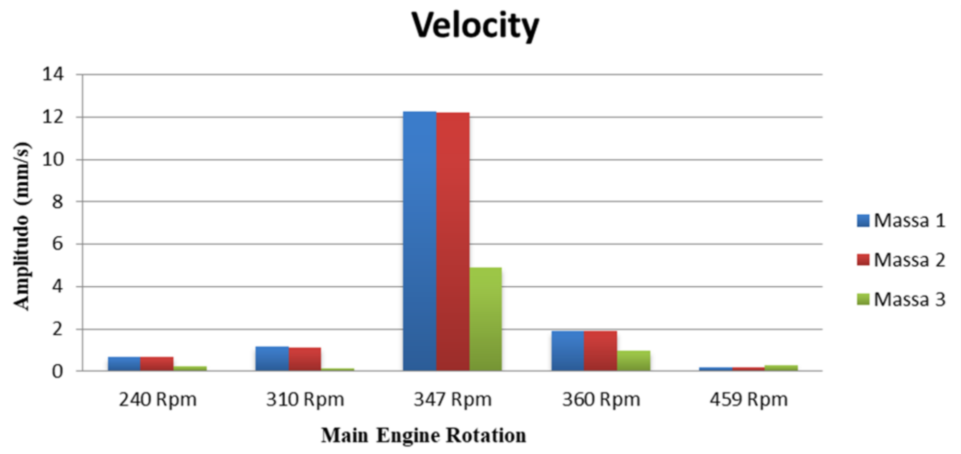

Figure. 15. Graph of velocity amplitude at variation of engine speed.

Figure 14 and Figure 15 show that the lowest vibration occurs at the highest engine speed condition of 459 rpm with a displacement value of 0.003 mm in Mass Component - 1, Mass Component - 2 of 0.003 mm and 0.004 mm in Mass Component - 3. As for the velocity response, a value of 0.164 mm / s was obtained in Mass

Component - 1, Mass Component - 2 of 0.163 mm / s and 0.238 mm / s in Mass Component – 3.

Conversely, the highest vibration occurs in the condition of engine rotation at 347 rpm with a displacement amplitude value of 0.337 mm at Mass Component 1, 0.335 mm at Mass Component 2, and 0.134 mm at Mass Component 3. The velocity responses recorded were 12.242 mm/s for Mass Component 1, 12.187 mm/s for Mass Component 2, and 4.884 mm/s for Mass Component 3. At 347 rpm, the engine experiences peak vibration due to its rotational speed nearing one of the system's natural frequencies, resulting in a substantial increase in vibration.

Based on the ABS provisions concerning the limitations of displacement value and vibration velocity, it can be concluded that the vibration response produced by the five conditions of the main engine rotation to the propulsion system falls within the safe category and complies with the ABS standard class.

IV. Conclusion

This reasearch analyzed the vibration response of a ship's propulsion system caused by harmonic excitation forces from the main engine and propeller shaft rotation. Using numerical software and mathematical modeling based on ABS standards, the research identified the system's longitudinal, torsional, and lateral vibrations. The results showed that the highest vibration response occurred at 347 rpm, while the lowest was at 459 rpm. Despite variations in displacement and velocity values, the vibration levels remained within ABS's permissible limits. Therefore, the propulsion system operates safely without exceeding acceptable vibration thresholds, ensuring structural integrity and engine efficiency. From the five variations of the main engine rotation, it can be concluded that the resulting vibration response is in the safe category and is still permitted by the ABS class standard.

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