

# Efficient Auxiliary Information Based Exponentially Weighted Moving Coefficient of Variation Control Chart using Hybrid Estimator: An Application to Monitor NPK Fertilizer

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**ABSTRACT** – In this era, manufacturing sectors should ensure the quality of their production process and products. They must reduce the variability that occurs in their operation. Coefficient variation control charts have become important statistical Process Control (SPC) tools for monitoring processes when the process mean linear function with the standard deviation. In recent years, auxiliary information-based-CV control charts using memory type structure have been investigated to enhance the sensitivity of control charts. Auxiliary information is selected when the variable remains stable during the monitoring period. Nevertheless, the AIB statistic is constructed based on lognormal transformation, and no research investigated the memory type CV chart using estimator of AIB-CV from the combination of ratio and regression form called hybrid form. This research proposes a hybrid auxiliary information-based exponentially weighted moving coefficient of variation (Hybrid AIB-EWMCV) control chart for detecting small to moderate shifts in the CV process. The Average Run Length (ARL) simulation shows that increasing the level of correlation and sample sizes enhances the detection ability of the control chart. Also, the proposed chart performs well than existing chart. A real dataset from fertilizer manufacturing is implemented to explain the condition of the process by using a Hybrid AIB-EWMCV control chart.

**Keywords**– Auxiliary Information, Average Run Length, Coefficient of Variation, Control Chart, NPK Fertilizer.

## I. INTRODUCTION

Quality control has become an essential part of the production process. It helps check the condition of specification products and whether it is as expected. In recent years, practitioners considered Statistical Process Control (SPC) method for process monitoring. Control charts are powerful tools to detect the source of variations. Mainly, the control chart is used to monitor the shift of process location and dispersion. However, there are several situations where control charts based on the mean or standard deviation cannot be applied. For example, the drug company operates one line for producing different drug levels in the same type of drug. Another case is when the bread company makes some bread with various weights in one line. This condition ensures the process mean changes from time to time but is considered in control, and the standard deviation is a function of the process mean. Hence, the coefficient of variation (CV) control chart is the best choice for the monitoring process.

Shewhart and memory type CV was first initiated by Kang et al. and Hong et al. [1]–[4]. It gives better performance than existing charts. Recently, Auxiliary information-based (AIB) control charts have been favored to enhance the detection ability of control charts. For monitoring process mean and process dispersion, Haq et al. [5] presented adaptive CUSUM and EWMA with auxiliary information using the VSI strategy, increasing the sensitivity of the FSI chart. Anwar et al. [6] proposed a mixed EWMA-CUSUM chart based on auxiliary information for simultaneously monitoring of process parameters. Concerning process CV, Noor-ul-Amin et al. [7] studied the simultaneous monitoring of process mean and process coefficient of variation with auxiliary information (AIB-Max EWMAQ). It outperforms the *Max EWMAQ* in terms of detection ability. Abbasi [8] developed AIB-chart Shewhart based using various CV estimators such as regression form [9], ratio form [9], and hybrid form [10]. The study concluded that the hybrid form estimator performs better than the competing estimator. Afshan et al. [11] investigated the performance of auxiliary information based exponentially weighted moving coefficient of variation control chart (EWMCV) using a lognormal estimator [12], which performs better than EWMCV without auxiliary information [13]. Nuriman et al. [14] also proposed the auxiliary information chart based on a generally weighted moving coefficient of variation (AIB-GWMCV). They proved that auxiliary information significantly affects the performance of the chart. Recently, Cahyono et al. [15] suggested the regression form of CV estimator to construct auxiliary information based on EWMA chart for monitoring CV (AIB-EMWCV<sub>Reg</sub>).

Previous studies have utilized regression form estimator [15] in memory-type CV charts with auxiliary information for monitoring moderate to large shifts. This paper aims to propose a Hybrid AIB-EWMCV control chart and to compare the performance of the proposed chart using levels of correlation and sample sizes. Further, the

comparison performance of the proposed control chart and the existing chart for monitoring small to moderate shifts is served in section III. The Monte Carlo procedure simulates the Average Run Length (ARL) in various correlations and samples.

**II. MATERIAL AND METHODS**

**A. Exponentially Weighted Moving Coefficient of Variation (EWMCV) Control Chart**

Let  $\{Y_{i,1}, Y_{i,2}, \dots, Y_{i,n}\}$  is a sample random variable  $\sigma_x$  when the in-control process. Let the parameter of CV ( $\gamma$ ) at the time  $i$  is defined as  $\gamma_i = \frac{\sigma_i}{\mu_i}$  and the CV statistic is  $\gamma_i = \frac{S_i}{\bar{Y}_i}$ ,  $-\infty < \gamma_i < \infty$ .  $S_i$  and  $\bar{Y}_i$  is sample standard deviation and mean at the time  $i$  with  $\gamma_0$  for in-control process CV. In the case of monitoring CV,  $S_i$  and  $\bar{Y}_i$  can change between time but  $\gamma_i$  ought to reach the  $\gamma_0$ . Hong et al [2] presented EWMCV statistics,

$$Z_i = \lambda\gamma_i + (1-\lambda)Z_{i-1}, \quad i \geq 1, \quad 0 < \lambda \leq 1, \tag{1}$$

The control limits are defined as

$$UCL_i = \mu_{\gamma_i} + L \sqrt{\frac{\lambda}{(2-\lambda)} [1 - (1-\lambda)^{2i}]} \sigma_{\gamma_i}, \tag{2}$$

$$CL_i = \mu_{\gamma_i}, \tag{3}$$

$$LCL_i = \mu_{\gamma_i} - L \sqrt{\frac{\lambda}{(2-\lambda)} [1 - (1-\lambda)^{2i}]} \sigma_{\gamma_i}, \tag{4}$$

$L$  is a control limit value. The  $\mu_{\gamma_i}$  and  $\sigma_{\gamma_i}^2$  are suggested by [16] in equations (5) and (6)

$$\mu_{\gamma_i} \approx \gamma \left( 1 - \frac{0.25 - \gamma^2}{n} \right), \tag{5}$$

$$\sigma_{\gamma_i}^2 \approx \frac{\gamma^2}{n} \left( 0.5 + \frac{0.4375}{n} + \gamma^2 \left( 1 + \frac{1 + 9\gamma^2}{n} \right) \right) - (\mu_{\gamma_i} - \gamma)^2 \tag{6}$$

**B. Hybrid Estimator of Auxiliary Information Based CV**

Let  $Y$  and  $X$  represent the main characteristic and auxiliary variable,  $(Y_{i,t}, X_{i,t}) \sim N_2(\mu, \Sigma)$ , where  $\mu = \begin{pmatrix} \mu_Y \\ \mu_X \end{pmatrix}$  and  $\Sigma = \begin{pmatrix} \sigma_Y^2 & \rho\sigma_Y\sigma_X \\ \rho\sigma_Y\sigma_X & \sigma_X^2 \end{pmatrix}$ . A hybrid estimator was proposed by Tripathi et al. [10]. It is based on the combination of ratio and regression estimators. The estimator is defined as,

$$\gamma_{Hybrid} = \frac{S_Y - \alpha_2(S_X - \sigma_X)}{\bar{Y}}, \tag{7}$$

$$\alpha_2 = 1.32105$$

where  $S_x$  and  $\sigma_x$  is defined as standard deviation of sample and population and  $\bar{Y}$  is mean sample of main characteristic . A standardized CV statistic can be determined as  $V_{\gamma_{Hybrid}} = \frac{\gamma_{Hybrid}}{\gamma}$  where  $\gamma$  is known parameter CV. By taking the expectation on both sides,

$$E(V_{\gamma_{Hybrid}}) = \frac{E(\gamma_{Hybrid})}{\gamma} = d_{2,\gamma_{Hybrid},n,\rho}, \tag{8}$$

$$\sigma_{V_{\gamma_{Hybrid}}} = \frac{\sigma_{\gamma_{Hybrid}}}{\gamma} = d_{3,\gamma_{Hybrid},n,\rho}, \tag{9}$$

$d_2$  and  $d_3$  are computed according to sample size  $n$  and coefficient correlation  $\rho_{XY}$  [8].  $\overline{\gamma_{Hybrid}} = \sum_{i=1}^t \frac{\gamma_{Hybrid}}{t}$  can be calculated to replace  $E(\gamma_{Hybrid})$  when the process CV is unknown.

**C. The Proposed Hybrid AIB-EWMCV Control Chart**

Let  $\{Y_{i,1}, Y_{i,2}, \dots, Y_{i,n}\}$  be a random sample for the main characteristic and  $\{X_{i,1}, X_{i,2}, \dots, X_{i,n}\}$  represent the auxiliary variable from the in-control process bivariate normal distribution, i.e.,  $(Y_{i,t}, X_{i,t}) \sim N_2(\mu_Y, \mu_X, \sigma_Y, \sigma_X, \rho_{XY})$  where  $\mu_Y, \mu_X$  are the means and  $\sigma_Y, \sigma_X$  are the standard deviation of main characteristic and auxiliary variable. In this case, it is assumed that the process parameters are known. The statistic is defined as

$$A_i = \lambda \gamma_{Hybrid,i} + (1 - \lambda) A_{i-1}, \tag{10}$$

where  $A_0 = E(\gamma_{Hybrid})$ .

Two-sided control limits for the statistic in (10) are defined as

$$LCL_i = \mu_{\gamma_{Hybrid,i}} - L \sqrt{\frac{\lambda}{(2-\lambda)} [1 - (1-\lambda)^{2i}]} \sigma_{\gamma_{Hybrid,i}}, \tag{11}$$

$$UCL_i = \mu_{\gamma_{Hybrid,i}} + L \sqrt{\frac{\lambda}{(2-\lambda)} [1 - (1-\lambda)^{2i}]} \sigma_{\gamma_{Hybrid,i}}, \tag{12}$$

where  $L$  and  $\lambda$  are the constant chosen based on the required in-control ARL. The process is stated to be out-of-control when the  $A_i < LCL_i$  or  $A_i > UCL_i$ .

**D. Simulation Algorithms**

Generally, the control chart performance is based on Average Run Length (ARL), classified as in-control ARL ( $ARL_0$ ) and out-of-control ARL ( $ARL_s$ ). The smaller ( $ARL_s$ ) means faster detection of the out-of-control signal. Meanwhile, the in-control ARL ( $ARL_0$ ) is expected to be larger, which means slower detection of false alarms. This paper utilizes the Monte Carlo procedure to determine the ARL.

1. Determine the shift of the process  $\tau = \frac{\gamma_1}{\gamma_0}$  with  $\gamma_0$  as in-control CV and  $\gamma_1$  as out-of-control CV. Note that  $\tau = 1$  indicates no shift in the process,
2. Decide constant parameters  $\lambda$  and  $L$ ,
3. Determine the desired in-control ARL ( $ARL_0$ )=370,
4. Generate and set random samples from normally bivariate distribution with each time has  $n$  samples with process parameters are  $\mu_0 = 10, \sigma_0 = 1, \gamma_0 = 0.1$  with various  $\rho_{XY}$  (0.9,0.7,0.5,0.3),  $n = 5, n = 10$ , and  $n = 15$
5. Generate 1,000 subgroups,
6. Compute mean and standard deviation of each time for  $Y$  and  $X$  respectively,
7. Compute Hybrid AIB-EWMCV statistics ( $A_i$ ),
8. Compute two-sided control limits  $LCL_i$  and  $UCL_i$ ,
9. Plot the statistic  $A_i$  and control limits. Observe the first out-of-control that exceeds control limits, and note as run length,
10. Repeat all of the steps 1,000 times to estimate  $ARL_0$  and  $ARL_s$ .

**E. The Charting Procedures**

The steps are required to use Hybrid AIB-GWMCV for monitoring the CV control chart:

1. Determine the CV in-control process  $\gamma_0$  or estimate from historical data if it is unknown,
2. Determine the constants  $\lambda$  and  $L$ .  $L$  is simulated for  $ARL_0 = 370$ ,
3. Compute  $\gamma_{Hybrid}$  of each time,
4. Compute  $A$  of each time,
5. Compute two-sided control limits  $LCL_i$  and  $UCL_i$ ,
7. Plot the statistic  $A_i$  and control limits. Note  $A_i$  values that exceed control limits,
8. Identify the cause of the out-of-control, and remove the assignable cause to get the in-control process.

III. RESULTS AND DISCUSSION

A. Simulated Data

The performance Hybrid AIB-EWMCV control chart is evaluated on different levels of correlation between  $Y$  and  $X$  ( $\rho_{xy}$ ), that are, 0.9, 0.7, 0.5, and 0.3, two different sample size  $n = 5$ , and  $n = 10$ , and  $\lambda = 0.001$ .

Table 1 L Constant Value for Control Limits

Sample	$\rho_{xy}$			
	0.9	0.7	0.5	0.3
5	2.2	2.1	1.9	2.1
10	2.1	1.9	1.9	2.1
15	2.1	1.9	2	2.1

Table 1 present the simulation result of  $L$  constant value for  $ARL_0 = 370$ .

Table 2 ARL Values of Hybrid AIB-EWMCV on Different Levels of Correlation and Sample Size

$\tau$	$n$	$\rho_{xy}$			
		0.9	0.7	0.5	0.3
1	5	372.714	376.852	370.617	372.922
	10	364.121	372.013	364.698	384.477
	15	368.393	375.741	378.148	383.627
1.2	5	70.944	100.926	108.284	116.698
	10	45.130	61.964	73.440	88.372
	15	35.806	48.416	60.748	69.874
1.4	5	35.774	51.012	54.890	60.788
	10	22.922	31.596	36.724	45.044
	15	18.078	25.036	31.156	35.672
1.6	5	24.018	34.860	37.920	40.936
	10	15.344	21.146	24.924	30.396
	15	12.356	16.800	20.816	23.798
1.8	5	18.072	25.876	28.330	30.464
	10	11.634	15.870	18.874	22.718
	15	9.312	12.634	16.038	17.954
2	5	14.678	21.156	22.938	24.700
	10	9.398	12.886	15.198	18.274
	15	7.598	10.290	12.736	14.672
2.2	5	12.352	17.564	18.874	20.604
	10	7.916	10.762	12.986	15.258
	15	6.430	8.662	10.818	12.276
2.4	5	10.508	15.170	16.276	17.858
	10	6.892	9.222	10.956	13.166
	15	5.572	7.492	9.398	10.652
2.6	5	9.404	13.428	14.350	15.462
	10	6.092	8.192	9.688	11.610
	15	4.924	6.610	8.252	9.324
2.8	5	8.374	11.932	12.886	13.852
	10	5.482	7.368	8.638	10.402
	15	4.458	6.008	7.336	8.408
3	5	7.570	10.886	11.608	12.462
	10	4.928	6.728	7.838	9.382
	15	4.060	5.418	6.664	7.652
4	5	5.160	7.396	7.896	8.434
	10	3.456	4.598	5.716	6.436
	15	2.890	3.782	4.606	5.196
5	5	3.984	5.422	5.904	5.940
	10	2.746	3.608	4.368	4.54
	15	2.226	2.950	3.574	3.988

Table 1 shows the ARL value's comparison on different correlations ( $\rho_{xy}$ ) and sample sizes ( $n$ ). Based on the simulation result, the proposed control chart reaches the best performance for detecting small to moderate shifts when the correlation of variables is high. Furthermore, the bigger sample size also affects the efficiency of the control chart. For  $\tau = \frac{\gamma_1}{\gamma_0} = 1.2$

and  $n = 5$ , the control chart has an out-of-control ARL of 70.944 when the correlation between two variables is 0.9 and 100.926, 108.284, 116.698 when the correlation is 0.7, 0.5, and 0.3, respectively. It indicate that the higher correlation of study variable and auxiliary variable enhance the detection ability of proposed control chart because the out-of-control ARL is decrease.

Another example is observed from the sample size. In  $\tau = \frac{\gamma_1}{\gamma_0} = 1.2$ ,  $\rho_{xy} = 0.9$ , and  $n = 15$  the proposed control chart has  $ARL_s = 35.806$  that smaller than  $n = 10$  and  $n = 5$  which has  $ARL_s = 45.130$  and  $ARL_s = 70.944$ , respectively. This result proves that the control chart performs well in big sample sizes each time. This phenomenon also occurs when the

chart monitors the moderate shift, for example, in  $\tau = \frac{\gamma_1}{\gamma_0} = 1.8$  and  $\rho_{XY} = 0.9$ . The proposed chart has  $ARL_s = 9.312$  by using  $n = 15$ . Otherwise, it has  $ARL_s = 11.634$  and  $ARL_s = 18.072$  by using  $n = 10$  and  $n = 5$ .

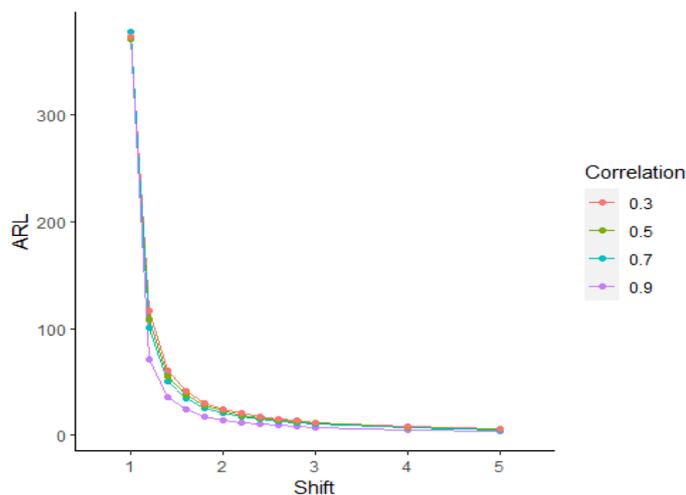


Figure 1 ARL Values of Hybrid AIB-EWMCV Control Chart with Various Correlation Levels and  $n = 5$

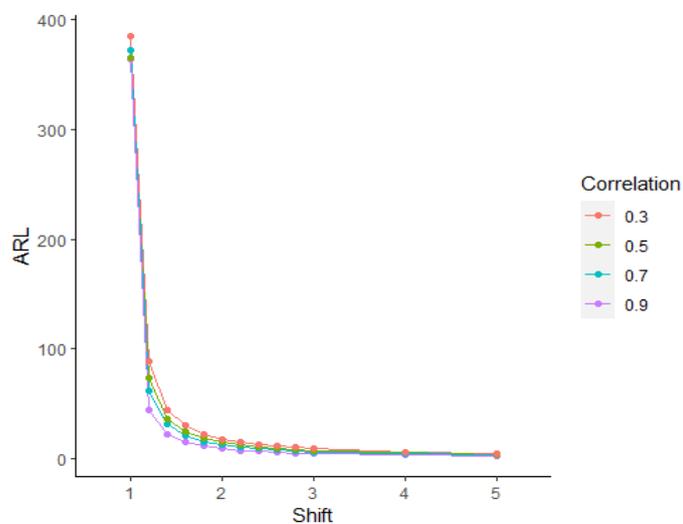


Figure 2 ARL Values of Hybrid AIB-EWMCV Control Chart with Various Correlation Levels and  $n = 10$

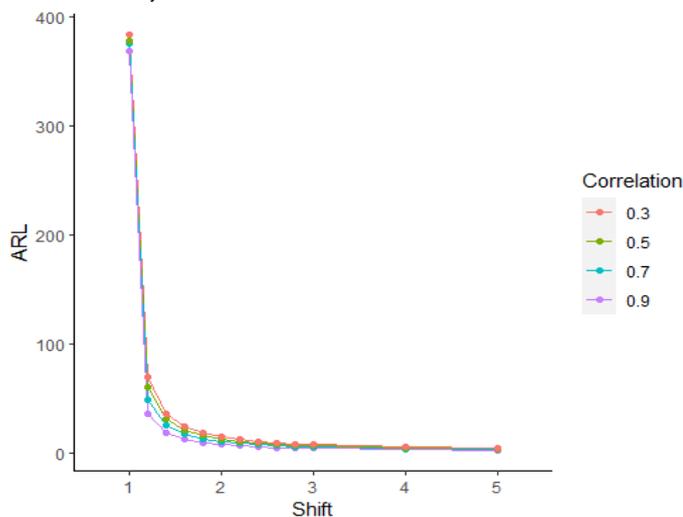


Figure 3 ARL Curves of Hybrid AIB-EWMCV Control Chart with Various Correlation Levels and  $n = 15$

Figure 1, Figure 2, and Figure 3 represent the ARL Curves for three sample sizes, which are 5,10, and 15, on four different correlation levels, such as 0.9, 0.7, 0.5, and 0.3. It can be viewed the proposed chart is more efficient when the correlation is high for all sample sizes. The higher correlation between study variable and auxiliary information decreases the  $ARL_s$ .

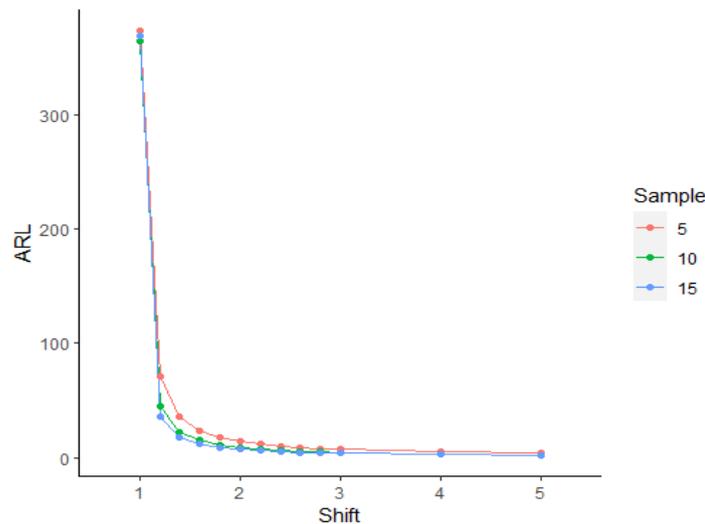


Figure 4 ARL Curves of Hybrid AIB-EWMCV Control Chart with Various Sample Sizes and  $\rho = 0.9$

From Figure 4, it can be seen that the increasing sample sizes decrease the  $ARL_s$ .

**B. Comparison with Existing Chart**

In this section, the performance of the Hybrid AIB-EWMCV control chart is compared with AIB-EWMCV<sub>Reg</sub> control chart. It will highlight the proposed chart's better performance in detecting small to moderate shifts. For a nondiscriminatory comparison, all charts are evaluated at fixed in-control ARL ( $ARL_0 = 370$ ),  $\rho_{XY} = 0.9$ , and  $n = 15$ . The control chart with a smaller  $ARL_s$  will be decided better than the others.

Table 3 ARL Comparison of Hybrid AIB-EWMCV and AIB-EWMCV<sub>Reg</sub>, using  $\rho_{xy} = 0.9$ , and  $n = 15$

$\tau$	Hybrid AIB-EWMCV	AIB-EWMCV <sub>Reg</sub>
1.0	368.393	368.540
1.2	35.806	68.848
1.4	18.078	37.282
1.6	12.356	25.646
1.8	9.312	19.582
2.0	7.598	15.950
2.2	6.430	13.432
2.4	5.572	11.606
2.6	4.924	10.148
2.8	4.458	9.190
3	4.060	8.258
4	2.890	5.434
5	2.226	4.242

In  $\tau = 1.2$  and  $\tau = 2.2$ , Hybrid AIB-EWMCV chart has  $ARL_s = 35.806$  and  $ARL_s = 6.430$ . Meanwhile,  $ARL_s$  of AIB-EWMCV<sub>Reg</sub> is 68.848 and 13.432. This result shows that our proposed control chart performs well than the existing chart for monitoring small to moderate shifts.

**C. Real Application**

This section shows the application of the proposed chart for monitoring the production process of NPK Fertilizer at Fertilizer manufacturing. There are 20 subgroups which each have five samples. The process parameter is assumed known, that is,  $\gamma_0 = 0.1$ . We take Nitrogen as a main characteristic and Phosphor as auxiliary information with  $n = 5$ ,  $\gamma_Y = 0.09$ ,  $\gamma_X = 0.1$ . The real dataset is shown in Table 4.

Figure 5 illustrates the out-of-control process when monitoring the production process of NPK Fertilizer. The first out-of-control is first detected at the 6<sup>th</sup> observation. Meanwhile, Figure 6 depicts the out-of-control signal that exceeds the upper control limit at the 15<sup>th</sup> observation. From Figure 5 and Figure 6, it can be seen that the proposed control chart detects the out-of-control rapidly than the AIB-EWMCV<sub>Reg</sub> control chart.

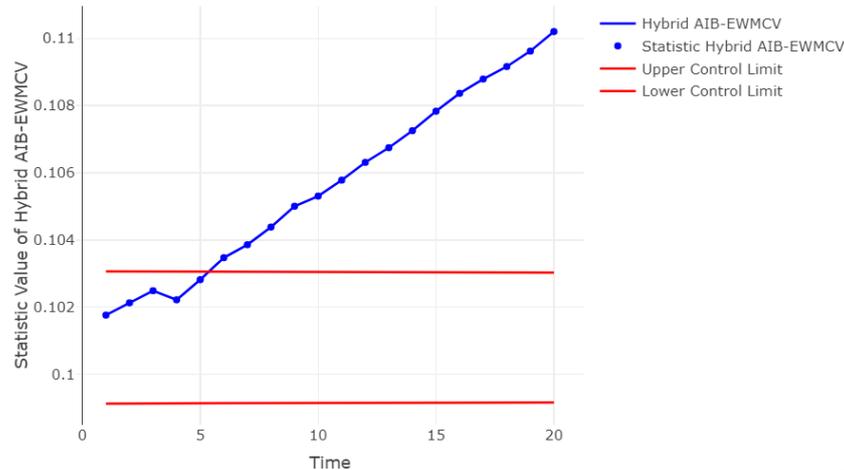


Figure 5 Hybrid AIB-EWMCV Control Chart using Real Dataset

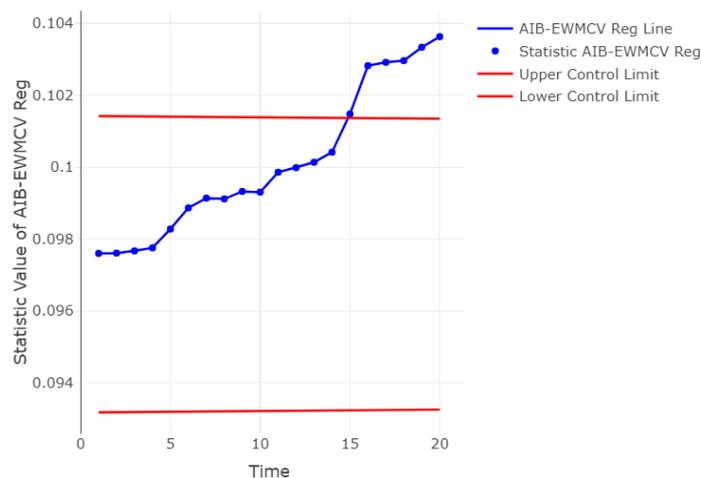


Figure 6 AIB-EWMCV<sub>Reg</sub> Control Chart using Real Dataset

#### IV. CONCLUSION

Overall, the increasing performance of the Hybrid AIB-EWMCV control chart is affected by correlation levels between the main characteristic and auxiliary variable, also sample sizes. When the correlation levels are high, and sample sizes rise, the proposed chart is more sensitive to detecting out-of-control signals. Moreover, the simulation result shows that the proposed control chart efficiently detects small to moderate shifts of CV. The Hybrid AIB-EWMCV is also applied using a real dataset to monitor the production process of NPK Fertilizer. From the monitoring result, the company should identify the variation source to get in control. The performance comparison, simulation, and real case, with the existing chart show that the proposed control chart is more efficient in detecting small to moderate shifts. For further study, it should be considered that the auxiliary variable may shift when the correlation between two variables is high.

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