

Implementation of Spatial Autoregressive with Autoregressive Disturbance (SARAR) using GMM to Identify Factors Caused Poverty in West Java

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Received: 21 April 2024

Revised: 19 June 2024

Accepted: 1 July 2024

ABSTRACT – Poverty is one of the crucial problems that has a negative impact on all sectors. As a developing country, Indonesia has a fairly high poverty rate. The government's efforts to overcome the problem of poverty can be circumvented by detecting the factors that influence it to determine the policies taken by using statistical modeling. There is a spatial effect on poverty in West Java Province. Spatial Data Analysis is the only statistical model that can explain the relationship between an area and the surrounding area. If the response variable contains a lag that correlates with each other, it is called a Spatial Autoregressive with Autoregressive Disturbances (SARAR) model. The Generalized Method of Moment (GMM) approach is used to get an estimator from the model. This method is applied to obtain the factors that influence poverty in West Java Province. The results of this study indicate that the GMM SARAR poverty modeling with customized weights provides relatively better estimation results. In addition, the relationship between locations (spatial lag dependence) is positive and significant. Expected Years of Schooling and Per capita Expenditure have a negative and significant effect on the increase in the percentage of poor people in West Java.

Keywords – Spatial Data Analysis, SARAR, GMM, Poverty

I. INTRODUCTION

The Millennium Development Goals (MDGs) are a global development paradigm which was declared by the Millennium Summit by 189 member countries of the United Nations (UN) in New York in September 2000. One of the points of agreement of all nations in the world contained in the MDGs is reducing poverty and hunger. In various countries, efforts to reduce poverty have not shown satisfactory results. In September 2015, world leaders agreed on 17 sustainable development goals set out in the Sustainable Development Goals (SDGs). Poverty remains one of the important points in the SDGs [1]. The SDGs is part of the 2030 Agenda, which is a global action plan with universal, integrated and inclusive principles to ensure that "No-one Left Behind" [2].

Since 2015, Indonesia has supported the implementation of the SDGs. The government has shown a high commitment to the SDGs through Presidential Regulation No. 59 of 2017 concerning the Implementation of the Achievement of Sustainable Development Goals, which also mandates the Regional Government to jointly achieve the TPB/SDGs Agenda 2030. In response, the West Java Provincial Government, through Bappeda, wishes to demonstrate the same commitment to achieving the TPB/SDGs agenda. The priority to be achieved by West Java Province refers to the 2018–2023 Rencana Pembangunan Jangka Menengah Daerah (RPJMD) document [3].

In 2020, West Java Province is the province with the largest population in Indonesia, which is around 49.56 million. The first target of the first goal of the SDGs is "Eliminating extreme poverty (people below the poverty line) with a purchasing power of less than \$1.25 PPP (Purchasing Power Parity)". Badan Pusat Statistik (BPS) noted that the percentage of the extremely poor in West Java continued to increase during the March 2020–March 2021 period. In March 2020, 1.35 million people or 2.71% were in the extreme poor category. This figure makes West Java ranked third nationally. Then, in March 2021, the extreme poor population in West Java became the highest nationally, reaching 1.79 million people or 3.57 percent of the total population. Thus, the West Java Provincial Government makes poverty a major problem in the 2018–2023 RPJMD and it is a strategic development issue in the next five years [4].

The government's efforts to overcome the problem of poverty can be circumvented by detecting the factors that influence it to determine the policies taken. What can be done is to find out the factors that influence poverty using statistical modeling.

However, in reality simple regression analysis is not appropriate to use because it contains spatial aspects because the observation unit in the research is the district/city area in West Java. If this is ignored, it can lead to biased and inconsistent estimates because the assumption of independence between observation units is violated. Spatial Data Analysis is the statistical model that can explain the relationship between an area and the surrounding area. In Spatial Data Analysis, a variable in the model is not only influenced by explanatory variables, but is also influenced by spatial interactions between spatial units [5].

Several researchers have carried out modeling on poverty. Isfahani et al. [6] conducted research on Regency/City data in West Java Province using the Fixed Effect Spatial Error Model method to examine the factors that influence poverty in West Java Province. The data used is panel data covering a 5 years time period, namely 2015-2019. The research results show that there are error dependencies from one region to another. Apart from that, population density, school enrollment rates, life expectancy and per capita expenditure have a significant effect on the percentage of poor people in West Java Province. The per capita expenditure variable has the greatest influence on poverty. Yulianto [7] conducted research on the factors that influence poverty in West Java Province. In this research, the Spatial Autoregressive (SAR) method was used. The results of the analysis show that the average length of schooling, per capita expenditure, education, and population have a significant effect on the percentage of poor people in West Java Province. Meanwhile, economic growth does not have a significant effect on the percentage of poor people in West Java Province.

These studies only involve one spatial dependency, namely on the response variable or on the error only. Poverty is a multidimensional problem that will affect many sectors and, of course, many factors that affect the level of poverty. Due to data limitations, not all variables that are considered to affect poverty are included in the model so that it will be considered as an error that is correlated between regions. In addition to the errors that are correlated between regions, it is suspected that there is a spatial dependence on the response variable, namely the percentage of the poor. Therefore, in this study, we used a spatial method that has a spatial dependence on the response variable as well as a spatial dependence on the error value. The method is called Spatial Autoregressive Model with Autoregressive Disturbances (SARAR), also known as Spatial Autocorrelation (SAC). This model is a combination of the Spatial Autoregressive (SAR) and the Spatial Error Model (SEM).

Explanatory variables that are considered to affect poverty but are not included in the model are called omitted variables. Omitted variables are one of the main causes of endogeneity [8]. Endogeneity occurs when there is a correlation between explanatory variables and residuals, or $Cov(x, u) \neq 0$. Generalized Methods of Moment (GMM) is an estimation method that can handle endogeneity problems that are difficult for other methods [9].

Based on the explanation in the background, the percentage of poor people in West Java Province has different variations due to spatial effects and it is suspected that there are correlated errors between regions. Based on this, modeling is needed that is able to accommodate the spatial effects contained in the response variable and also the error and is able to handle endogeneity problems. The purpose of this research is to apply Spatial Autoregressive with Autoregressive Disturbance (SARAR) with the Generalized Method of Moment (GMM) approach in modeling the percentage of poor people. Meanwhile, the aim of this research is to identify factors that influence the percentage of poor people in West Java Province in 2020.

II. LITERATURE REVIEW

A. SARAR Model

First-order autoregressive spatial model with first-order autoregressive disturbances ($n \in \mathbf{N}$) or referred to as a model SARAR (1,1) defined as [10]:

$$\begin{aligned} \mathbf{y} &= \mathbf{X}\boldsymbol{\beta} + \rho\mathbf{W}\mathbf{y} + \mathbf{u} \\ &= \mathbf{Z}\boldsymbol{\delta} + \mathbf{u} \\ \mathbf{u} &= \lambda\mathbf{M}\mathbf{u} + \boldsymbol{\varepsilon} \end{aligned} \tag{1}$$

Where $\mathbf{Z} = [\mathbf{X}, \mathbf{W}\mathbf{y}]$, $\boldsymbol{\delta} = [\boldsymbol{\beta}^T, \rho]^T$; \mathbf{y} is the $n \times 1$ vector of observations of the dependent variable; \mathbf{W} and \mathbf{M} are $n \times n$ spatial weighting matrices; \mathbf{X} is the $n \times k$ matrix of observations on k independent variables; ρ and λ are spatial autoregressive parameters in the dependent variable y and the disturbance term u respectively; $\boldsymbol{\beta}$ is the $k \times 1$ vector of regression parameters; \mathbf{u} is the $n \times 1$ vector of regression disturbances; $\boldsymbol{\varepsilon}$ is an $n \times 1$ vector of innovations.

The vectors $\mathbf{W}\mathbf{y}$ and $\mathbf{M}\mathbf{u}$ are usually referred to as the spatial lags \mathbf{y} and \mathbf{u} , respectively. All elements (\mathbf{y} , \mathbf{X} , \mathbf{W} , \mathbf{M} , \mathbf{u} and $\boldsymbol{\varepsilon}$) depend on the sample size (n) for general reasons. The analysis allows for $\mathbf{W} = \mathbf{M}$, which will happen frequently in applications [11].

In practice, the spatial weighting matrices \mathbf{W} and \mathbf{M} are row normalized so that $\sum_{j=1}^n w_{ij} = 1$ and $\sum_{j=1}^n m_{ij} = 1$ [11]. Therefore, for $|\rho| < 1$ and $|\lambda| < 1$ the matrices $\mathbf{I} - \rho\mathbf{W}$ and $\mathbf{I} - \lambda\mathbf{M}$ are nonsingular [10].

Then we obtain the reduced form of the SARAR model as [10]:

$$\begin{aligned} \mathbf{y} &= (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{X}\boldsymbol{\beta} + \mathbf{u}. \\ \mathbf{u} &= (\mathbf{I} - \lambda\mathbf{M})^{-1}\boldsymbol{\varepsilon} \end{aligned} \tag{2}$$

Applying a Cochrane-Orcutt type transformation to the equation (1) yields [12]:

$$\mathbf{y}_s(\lambda) = \mathbf{Z}_s(\lambda)\boldsymbol{\delta} + \boldsymbol{\varepsilon} \tag{3}$$

where $\mathbf{y}_s(\lambda) = \mathbf{y} - \lambda\mathbf{M}\mathbf{y}$ and $\mathbf{Z}_s(\lambda) = \mathbf{Z} - \lambda\mathbf{M}\mathbf{Z}$. $\mathbf{y}_s(\lambda)$ and $\mathbf{Z}_s(\lambda)$ to indicate the dependence of the transformed variable on λ .

B. Generalized Methods of Momen (GMM) Estimation

The multi-step estimation procedure is described below [13]. In the first step, instruments are needed for \mathbf{Z} , and in the next step, instruments are needed for \mathbf{MZ} . The ideal instruments in this case would be:

$$\begin{aligned} E(\mathbf{Z}) &= E[\mathbf{X}, \mathbf{WE}(\mathbf{y})] \\ E(\mathbf{MZ}) &= E[\mathbf{MX}, \mathbf{MWE}(\mathbf{y})] \end{aligned}$$

All the columns of $E(\mathbf{Z})$ and $E(\mathbf{MZ})$ are linear in

$$\mathbf{X}, \mathbf{WX}, \mathbf{W}^2\mathbf{X}, \dots, \mathbf{MX}, \mathbf{MWX}, \mathbf{MW}^2\mathbf{X}, \dots \tag{4}$$

Let the instrument matrix \mathbf{H} , be a subset of the column in (4), say [14]:

$$\mathbf{H} = [\mathbf{X}, \mathbf{WX}, \dots, \mathbf{W}^q\mathbf{X}, \mathbf{MX}, \mathbf{MWX}, \dots, \mathbf{MW}^q\mathbf{X}], \tag{5}$$

where typically, $q \leq 2$.

Then:

$$\begin{aligned} \mathbf{PZ} &= (\mathbf{X}, \mathbf{PWy}) \\ \mathbf{PMZ} &= (\mathbf{MX}, \mathbf{PMWy}) . \end{aligned}$$

Moment Condition

Let $\tilde{\mathbf{u}}$ denote some predictor of \mathbf{u} . Furthermore, for the convenience of notation let $\tilde{\mathbf{u}} = \mathbf{Mu}$, $\tilde{\mathbf{u}} = \mathbf{M}\tilde{\mathbf{u}} = \mathbf{M}^2\mathbf{u}$, $\tilde{\tilde{\mathbf{u}}} = \mathbf{M}\tilde{\mathbf{u}}$, and $\tilde{\tilde{\tilde{\mathbf{u}}}} = \mathbf{M}^2\tilde{\mathbf{u}}$. Similarly, let $\tilde{\boldsymbol{\varepsilon}} = \mathbf{M}\boldsymbol{\varepsilon}$.

According to [15] the estimator for λ will be an GMM estimators which corresponds to the following moment conditions:

$$\begin{aligned} n^{-1}E[\tilde{\boldsymbol{\varepsilon}}^T\tilde{\boldsymbol{\varepsilon}}] &= n^{-1}tr\{\mathbf{M} [diag(E[\boldsymbol{\varepsilon}_i^2])]\mathbf{M}^T\}, \\ n^{-1}E[\tilde{\boldsymbol{\varepsilon}}^T\tilde{\boldsymbol{\varepsilon}}] &= 0, \end{aligned}$$

Using the new notation, the moment condition can also be written as:

$$\begin{aligned} n^{-1}E[\boldsymbol{\varepsilon}^T\mathbf{A}_1\boldsymbol{\varepsilon}] &= 0, \\ n^{-1}E[\boldsymbol{\varepsilon}^T\mathbf{A}_2\boldsymbol{\varepsilon}] &= 0, \end{aligned} \tag{6}$$

with

$$\begin{aligned} \mathbf{A}_1 &= \mathbf{M}^T\mathbf{M} - diag(\mathbf{m}_i^T \mathbf{m}_i) \\ \mathbf{A}_2 &= \mathbf{M}, \end{aligned}$$

where \mathbf{m}_i is the i -th column of the weights matrix \mathbf{M} .

The first condition in the equation (6) allows innovation to become heteroskedastic of unknown form. If the innovations are homoscedastic with finite variance σ^2 , this condition simplifies to [16]:

$$n^{-1}E[\tilde{\boldsymbol{\varepsilon}}^T\tilde{\boldsymbol{\varepsilon}}] = \sigma^2 n^{-1}tr\{\mathbf{MM}^T\}. \tag{7}$$

Estimation Procedure

The suggested estimation procedure has two steps. Each step consists of substeps which include the estimation of λ and δ . In step 1, estimates are calculated from the original model (1). These estimates are used in step 2 to calculate the estimates from the transformed model (3), with λ replaced by the estimator. Now, we'll expand on each step in detail:

Step 1.1 - 2SLS Estimator

In the first step, $\boldsymbol{\delta}$ is estimated by 2SLS applied to the untransformed model (1) using the matrix of instruments \mathbf{H} , yielding to [17]:

$$\tilde{\boldsymbol{\delta}}_{2SLS} = (\tilde{\mathbf{Z}}^T\mathbf{Z})^{-1}\tilde{\mathbf{Z}}^T\mathbf{y} \tag{8}$$

with $\tilde{\mathbf{Z}} = \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{Z} = \mathbf{P}_H\mathbf{Z} = (\mathbf{X}, \tilde{\mathbf{W}}\mathbf{y})$. The estimates $\tilde{\boldsymbol{\delta}}_{2SLS}$ yield an initial vector of residuals, $\tilde{\mathbf{u}}_{2SLS}$ as follows :

$$\tilde{\mathbf{u}}_{2SLS} = \mathbf{y} - \mathbf{Z}\tilde{\boldsymbol{\delta}}_{2SLS} \tag{9}$$

Step 1.2 - Initial GMM estimator of λ based on 2SLS residuals

Let $\tilde{\boldsymbol{\delta}}_{2SLS}$ is the initial estimator $\boldsymbol{\delta}$ obtained from Step 1.1, let $\tilde{\mathbf{u}}_s = \mathbf{M}\tilde{\mathbf{u}}$. Then, we can operationalize the sample moments corresponding to (6), and they are as follows: [12]:

$$\begin{aligned} \mathbf{m}(\lambda, \tilde{\boldsymbol{\delta}}_{2SLS}) &= \frac{1}{n} \left(\tilde{\mathbf{u}}_{2SLS}^T (\mathbf{I} - \lambda\mathbf{M}^T)\mathbf{A}_1 (\mathbf{I} - \lambda\mathbf{M}) \tilde{\mathbf{u}}_{2SLS} \right) \\ &= \tilde{\mathbf{G}} \left(\frac{\lambda}{\lambda^2} \right) - \tilde{\mathbf{g}} \end{aligned} \tag{10}$$

where,

$$\tilde{\mathbf{G}} = \frac{1}{n} \begin{pmatrix} \tilde{\mathbf{u}}^T(\mathbf{A}_1 + \mathbf{A}_1^T) \tilde{\mathbf{u}}_s & -\tilde{\mathbf{u}}_s^T \mathbf{A}_1 \tilde{\mathbf{u}}_s^T \\ \vdots & \vdots \\ \tilde{\mathbf{u}}^T(\mathbf{A}_q + \mathbf{A}_q^T) \tilde{\mathbf{u}}_s & -\tilde{\mathbf{u}}_s^T \mathbf{A}_q \tilde{\mathbf{u}}_s^T \end{pmatrix}$$

$$\tilde{\mathbf{g}} = \frac{1}{n} \begin{pmatrix} \tilde{\mathbf{u}}^T \mathbf{A}_1 \tilde{\mathbf{u}} \\ \vdots \\ \tilde{\mathbf{u}}^T \mathbf{A}_q \tilde{\mathbf{u}} \end{pmatrix}$$

for $q = 1, 2$.

Therefore, the initial GMM estimator for λ is obtained simply by minimizing the following relationship:

$$\check{\lambda}_{GMM} = \operatorname{argmin}_{\lambda} \left\{ \left[\tilde{\mathbf{G}}(\lambda) - \tilde{\mathbf{g}} \right]^T \left[\tilde{\mathbf{G}}(\lambda) - \tilde{\mathbf{g}} \right] \right\} \tag{11}$$

Furthermore, the above expression can be interpreted as a nonlinear least squares system of equations. As a result, the above system's solution yields the initial estimate.

Drukker et al. [18] suggest an expression for the \mathbf{A}_s matrix in the homoscedastic case as follows:

$$\mathbf{A}_1 = \{1 + [n^{-1} \operatorname{tr}(\mathbf{M}^T \mathbf{M})]^2\}^{-1} [\mathbf{M}^T \mathbf{M} - n^{-1} \operatorname{tr}(\mathbf{M}^T \mathbf{M}) \mathbf{I}]$$

$$\mathbf{A}_2 = \mathbf{M}$$

On the other hand, when heteroskedasticity is assumed, the following expression for \mathbf{A}_1 and \mathbf{A}_2 are recommended [15]:

$$\mathbf{A}_1 = \mathbf{M}^T \mathbf{M} - \operatorname{diag}(\mathbf{M}^T \mathbf{M})$$

$$\mathbf{A}_2 = \mathbf{M}$$

Step 1.3 - Efficient GMM Estimator of ϱ Based on 2SLS Residuals

The efficient GMM estimator of λ is a weighted nonlinear least squares estimator. In particular, this estimator is $\check{\lambda}$ where [13]:

$$\check{\lambda}_{OGMM} = \operatorname{argmin}_{\lambda} \left[\mathbf{m}(\lambda, \check{\boldsymbol{\delta}}_{2SLS})^T \check{\boldsymbol{\Psi}}^{-1} \mathbf{m}(\lambda, \check{\boldsymbol{\delta}}_{2SLS}) \right] \tag{12}$$

where the weighting matrix is $\check{\boldsymbol{\Psi}}_n^{-1}$. The matrix $\boldsymbol{\Psi}$ is the variance of the moment conditions $\mathbf{m}(\lambda, \check{\boldsymbol{\delta}}_{2SLS})$. The matrix $\check{\boldsymbol{\Psi}}^{-1} = \check{\boldsymbol{\Psi}}^{-1}(\check{\lambda}_{gmm})$ defined as follows. Let $\check{\boldsymbol{\Psi}} = [\check{\Psi}_{rs}]_{r,s=1,2}$ with

$$\check{\Psi}_{rs} = (2n)^{-1} \operatorname{tr}[(\mathbf{A}_r + \mathbf{A}_r^T) \check{\boldsymbol{\Sigma}}(\mathbf{A}_s + \mathbf{A}_s^T) \check{\boldsymbol{\Sigma}}] + n^{-1} \check{\boldsymbol{\alpha}}_r^T \check{\boldsymbol{\Sigma}} \check{\boldsymbol{\alpha}}_s,$$

where,

$$\check{\boldsymbol{\Sigma}} = \operatorname{diag}_{i=1, \dots, n}(\check{\epsilon}_i^2)$$

$$\check{\boldsymbol{\epsilon}} = (\mathbf{I} - \check{\lambda}_{gmm} \mathbf{M}) \tilde{\mathbf{u}}$$

The expression for $\check{\boldsymbol{\alpha}}_r$, with $r = 1, 2$, for the case where the estimates obtained from 2SLS is given as follows:

$$\check{\boldsymbol{\alpha}}_r = (\mathbf{I} - \check{\lambda}_{gmm} \mathbf{M}) \mathbf{H} \check{\mathbf{P}} \check{\boldsymbol{\alpha}}_r$$

with \mathbf{H} as a $n \times p$ instrument matrix,

$$\check{\mathbf{P}} = \left(\frac{1}{n} \mathbf{H}^T \mathbf{H} \right)^{-1} \left(\frac{1}{n} \mathbf{H}_n^T \mathbf{Z}_n \right) \left[\left(\frac{1}{n} \mathbf{H}^T \mathbf{Z} \right)^T \left(\frac{1}{n} \mathbf{H}^T \mathbf{H} \right)^{-1} \left(\frac{1}{n} \mathbf{H}^T \mathbf{Z} \right) \right]^{-1}$$

as a matrix of dimension $p \times k$, and

$$\check{\boldsymbol{\alpha}}_r = -n^{-1} [\mathbf{Z}^T (\mathbf{I} - \check{\lambda}_{gmm} \mathbf{M}) (\mathbf{A}_r + \mathbf{A}_r^T) (\mathbf{I} - \check{\lambda}_{gmm} \mathbf{M}) \tilde{\mathbf{u}}]$$

Step 2.1 - GS2SLS Estimator

Consider the Cochrane-Orcutt spatial transformation model in (3). Using $\check{\lambda}_{OGMM}$ from step 1.3 (or a consistent estimator from step 1.2), Arraiz et al. [13] define the GS2SLS estimator as

$$\check{\boldsymbol{\delta}}_{GS2SLS}(\check{\lambda}_{ogmm}) = [\check{\mathbf{Z}}_s^T(\check{\lambda}_{ogmm}) \mathbf{Z}(\check{\lambda}_{ogmm})]^{-1} \check{\mathbf{Z}}_s^T(\check{\lambda}_{ogmm}) \mathbf{y}_s(\check{\lambda}_{ogmm}) \tag{13}$$

where

$$\mathbf{y}_s = \mathbf{y} - \check{\lambda}_{ogmm} \mathbf{M} \mathbf{y}$$

$$\mathbf{Z}_s = \mathbf{Z} - \check{\lambda}_{ogmm} \mathbf{M} \mathbf{Z}$$

$$\check{\mathbf{Z}}_s = \mathbf{P}_H \mathbf{Z}_s$$

$$\mathbf{P}_H = \mathbf{H}(\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T$$

Step 2.2 - Efficient GMM estimator of λ based on GS2SLS residual

In the final step, according to Bivand & Piras [12], the efficient GMM estimator of λ based on the GS2SLS residuals is obtained by minimizing the following expression:

$$\hat{\lambda}_{OGMM} = \underset{\lambda}{\operatorname{argmin}} \left\{ \left[\hat{\mathbf{G}} \left(\frac{\lambda}{\lambda^2} \right) - \hat{\mathbf{g}} \right]^T \left(\hat{\Psi}^{\lambda\lambda} \right) \left[\hat{\mathbf{G}} \left(\frac{\lambda}{\lambda^2} \right) - \hat{\mathbf{g}} \right] \right\} \tag{14}$$

where $\hat{\Psi}^{\lambda\lambda}$ is an estimator for the variance-covariance matrix of the (normalized) sample moment vector based on the GS2SLS residuals. This estimator is different in the case of homoscedastic and heteroscedastic errors.

For the homoscedastic case, the r, s (with $r, s = 1, 2$) element of $\hat{\Psi}^{\lambda\lambda}$ is given by:

$$\begin{aligned} \hat{\Psi}^{\lambda\lambda} = & [\hat{\sigma}^2]^2 (2n)^{-1} \operatorname{tr}[(\mathbf{A}_r + \mathbf{A}_r^T) (\mathbf{A}_s + \mathbf{A}_s^T)] \\ & + \hat{\sigma}^2 n^{-1} \tilde{\mathbf{a}}_r^T \tilde{\mathbf{a}}_s^T \\ & + n^{-1} (\hat{\mu}^{(4)} - 3[\hat{\sigma}^2]^2) \operatorname{vec}_D (\mathbf{A}_r)^T \operatorname{vec}_D (\mathbf{A}_s) \\ & + n^{-1} \hat{\mu}^{(3)} [\tilde{\mathbf{a}}_r^T \operatorname{vec}_D (\mathbf{A}_s) + \tilde{\mathbf{a}}_s^T \operatorname{vec}_D (\mathbf{A}_r)], \end{aligned} \tag{15}$$

where

$$\begin{aligned} \tilde{\mathbf{a}}_r &= \hat{\mathbf{T}} \tilde{a}_r \\ \hat{\mathbf{T}} &= \mathbf{H} \hat{\mathbf{P}}, \\ \hat{\mathbf{P}} &= \hat{\mathbf{Q}}_{HH}^{-1} \hat{\mathbf{Q}}_{HZ} [\hat{\mathbf{Q}}_{HZ}^T \hat{\mathbf{Q}}_{HH}^{-1} \hat{\mathbf{Q}}_{HZ}^T]^{-1} \\ \hat{\mathbf{Q}}_{HH}^{-1} &= (n^{-1} \mathbf{H}^T \mathbf{H}), \\ \hat{\mathbf{Q}}_{HZ} &= (n^{-1} \mathbf{H}^T \mathbf{Z}), \\ \mathbf{Z} &= (\mathbf{I} - \hat{\lambda} \mathbf{M}) \mathbf{Z}, \\ \tilde{a}_r &= -n^{-1} [\mathbf{Z}^T (\mathbf{A}_r + \mathbf{A}_r^T) \hat{\boldsymbol{\varepsilon}}] \\ \hat{\sigma}^2 &= n^{-1} \hat{\boldsymbol{\varepsilon}} \hat{\boldsymbol{\varepsilon}}, \\ \hat{\mu}^{(3)} &= n^{-1} \sum_{i=1}^n \hat{\varepsilon}_i^3, \\ \hat{\mu}^{(4)} &= n^{-1} \sum_{i=1}^n \hat{\varepsilon}_i^4, \end{aligned}$$

For the homoscedastic case, the r, s (with $r, s = 1, 2$) element of $\hat{\Psi}^{\lambda\lambda}$ is given by:

$$\hat{\Psi}^{\lambda\lambda} = (2n)^{-1} \operatorname{tr}[(\mathbf{A}_r + \mathbf{A}_r^T) \hat{\boldsymbol{\Sigma}} (\mathbf{A}_s + \mathbf{A}_s^T) \hat{\boldsymbol{\Sigma}}] + n^{-1} \tilde{\mathbf{a}}_r^T \hat{\boldsymbol{\Sigma}} \tilde{\mathbf{a}}_s^T, \tag{16}$$

where, $\hat{\boldsymbol{\Sigma}}$ is a diagonal matrix whose i -th diagonal element is $\hat{\varepsilon}_i^2$.

C. Factors that Influence Poverty

Poverty is a multidimensional problem that is influenced by other variables. There are various theories regarding the factors that influence poverty.

Expected Years of Schooling

According to [19], education is a way to save oneself from poverty. Education plays an important role in shaping a country's ability to absorb modern technology and increase its capacity for sustainable growth and development. The higher a person's education, the greater their ability and opportunity to earn income and a good job, so that they are further away from poverty.

The new growth theory emphasizes the important role of government, especially in developing human capital and encouraging research and development to increase human productivity. In fact, it can be seen that investing in education will be able to improve the quality of human resources as indicated by increasing a person's knowledge and skills.

Life expectancy

A population with good health is an important input for reducing poverty. According to [20] this factor is important because a person cannot develop his capacity to the maximum if he does not have optimal health status. Life Expectancy Rate is a tool for evaluating the government's performance in improving the welfare of the population in general, and improving health status in particular. Government intervention to improve health is an important policy tool for reducing poverty.

Per Capita Expenditure

Per capita expenditure can reveal general household consumption patterns by using indicators of the proportion of expenditure on food and non-food. The composition of household expenditure can be used as a measure to assess the level of economic welfare of the population. The lower the percentage of expenditure on food to total expenditure, the better the level of welfare [21]. Per capita expenditure influences the poverty level. The higher per capita expenditure can be interpreted as an improvement in the community's economy in meeting its needs.

Labor Force Participation Rate

The Labor Force Participation Rate is an illustration of how big a role the population is involved in the economy of a region. Through The Labor Force Participation figures, we can see the high or low proportion or ratio of the working age population involved in the economy of a region [22]. The Labor Force Participation is one of the factors causing poverty. The Labor Force Participation influences the amount of output of an economic activity, so that the more productive people are, the higher the output will be, which will influence economic growth. An increase in The Labor Force Participation in a region means an increase in per capita income and consumption levels which can influence a reduction in poverty levels [23].

III. METHODOLOGY

A. Data

This study uses secondary data obtained from the Badan Pusat Statistik (BPS) of West Java in 2020. The unit of observation used is all regencies/cities in West Java Province, which consists of 27 regencies/cities. The data period used is 2020 data.

Table 1 Research Variable

Symbol	Variable Name
Y	Percentage of the Poor Population
X_1	Expected Years of Schooling
X_2	Life Expectancy
X_3	Per capita Expenditure
X_4	Labour Force Participation Rate

B. Methods

To examine the problem of poverty in districts/cities in West Java Province, taking into account the existence of spatial effects on each model, it is necessary to model Spatial Autoregressive with Autoregressive Disturbance (SARAR).

The analysis stages carried out in this research are as follows:

1. Examine the characteristics of the variables studied in the model in order to obtain a general picture of poverty conditions that is useful as a basis for further analysis.
2. Develop a spatial weighting matrix using the Contiguity matrix and Customized weighted spatial matrix.
3. Carry out the Moran's I test to detect spatial autocorrelation.
4. Carry out a spatial dependency test with the Lagrange Multiplier test to detect spatial dependencies more specifically with three test statistics, namely LMerror, LMLag, and SARMA.
5. Estimate parameters using the Generalized method of moments (GMM).
6. Determine the best model from the parameter estimation results using GMM with two different spatial weighting matrices. Determining the best model is based on two criteria, namely the significant spatial lag coefficient of the response variable (ρ) and the largest value of the coefficient of determination (R^2).
7. Carry out analysis and interpretation based on the best model formed.
8. Formulate conclusions based on the variables arranged in the best model formed.

IV. RESULTS AND DISCUSSIONS

A. Data Exploration

The distribution of poverty rates in districts/cities in West Java in 2020 can be seen in the picture below:

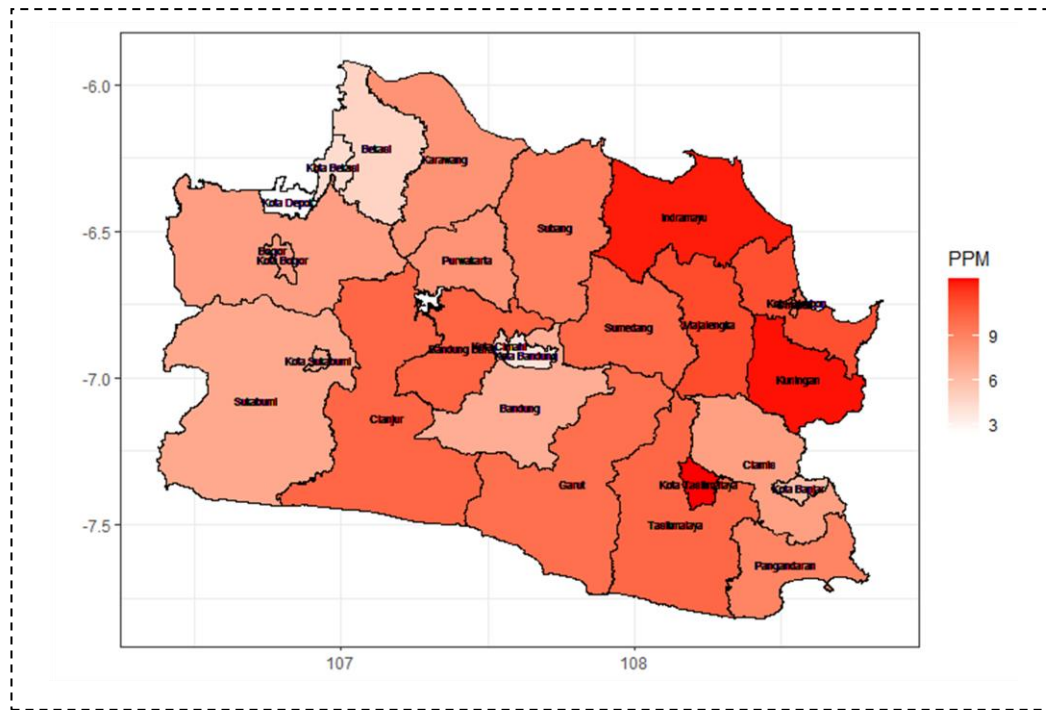


Figure 1 Distribution of the Percentage of the Poor Population Regency/City Population in West Java in 2020

Figure 1 above illustrates the distribution of the percentage of the poor population by districts and cities in West Java. Color degradation shows the value of the percentage of poverty. The darker the red indicates, the higher the percentage of poverty. On the contrary, the lighter the red indicates, the lower the percentage of poverty.

The distribution of the poor population in West Java is quite varied between districts/cities, with Depok City having the smallest percentage of the population at 2.44% and Tasikmalaya City having the largest proportion of 12.97%. Depok City has a fairly low poverty rating. This is because Depok City is the capital buffer where most of the population works in DKI Jakarta Province. Likewise, geographically close areas to DKI Jakarta Province, such as Bekasi City and Bekasi Regency, have a fairly low poverty rate of 4.38% and 4.82%, respectively. In addition, Bandung City also has a relatively low poverty rate, which is 3.99%. Bandung City is the capital of West Java Province, which is the economic heart of the province itself. And also, the surrounding area, which is geographically close to the city of Bandung, such as the city of Cimahi, has a low percentage of 5.11%. Districts or cities with a high percentage of the poor population tend to be located around districts or cities that have a percentage of the poor population as well (clustered). This shows that there is a spatial dependence, which will be tested by looking at the Moran Index. Descriptive statistics of the response variables and explanatory variables for 27 districts/cities in West Java are presented in Table 2 below:

Table 2 Descriptive Statistics of Regency/City Research Variables in West Java in 2020

Research variable	Min	Mean	Max.	Standard deviation
Percentage of the Poor Population (Y)	2,450	8,425	12,970	2,808
Expected Years of Schooling (X_1)	11,70	12,79	14,20	0,777
Life Expectancy (X_2)	69,45	72,22	74,97	1,404
Per capita Expenditure (X_3)	7852	10717	16887	2249,819
Labor Force Participation Rate (X_4)	55,74	65,01	76,79	4,355

B. Relationship between Variables

In this section, we will identify the relationship between response variables and explanatory variables in the model. The relationship between variables can be done by looking at the scatterplot between the Percentage of the Poor Population variable and the determinant variable.

The scatterplot in question is presented in Figure 2. In this picture it can be seen that of the four Explanatory variables tested, all of them have a negative relationship with the Percentage of the Poor Population variable. The increase that occurred in the Percentage of the Poor Population variable was followed by a decrease in the explanatory variables. And if the Percentage of the Poor Population variable decreases, the explanatory variables will increase.

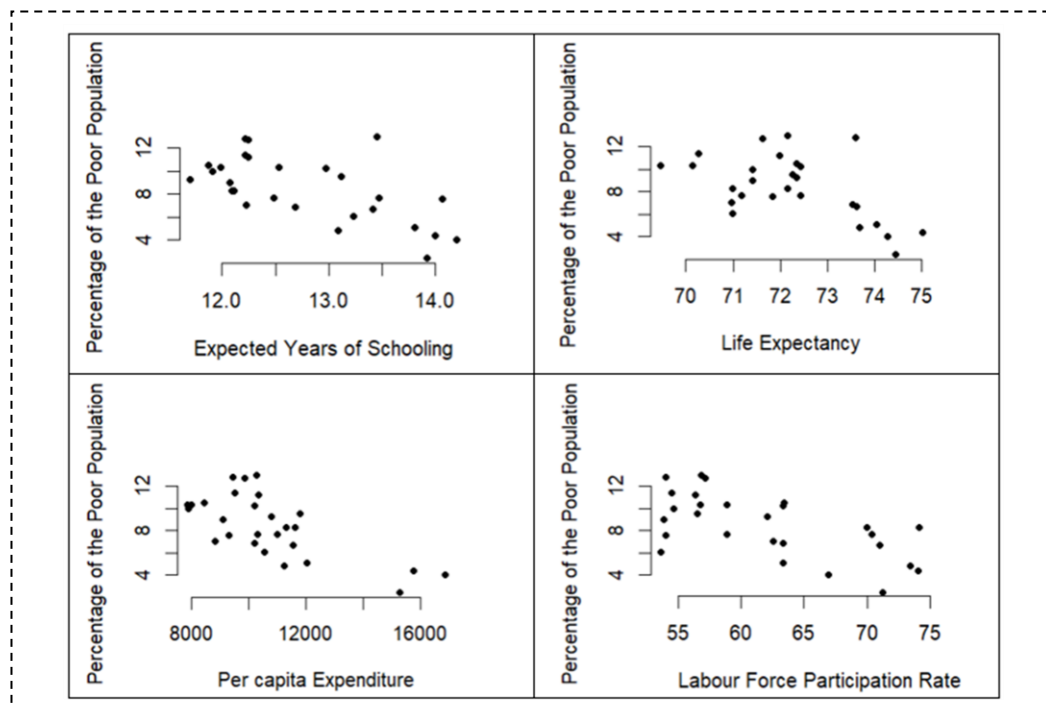


Figure 2 Scatterplot of the relationship between Percentage of the Poor Population and the explanatory variables

C. Spatial Weight Matrix

In this study, two types of spatial weighting matrices were used, namely the contiguity matrix and the customized social and economic linkage matrix. The contiguity matrix was chosen to represent the relationship according to the intersection of the sides/angles between regions. Areas that intersect regionally are considered to have close characteristics. Meanwhile, the customized matrix was chosen to represent the social and economic relationships between locations. Areas that do not contain side intersections, may be related or related to other areas because of economic relations or close social characteristics.

Contiguity Matrix

The spatial *contiguity* weight matrix used in this study is rook contiguity, queen contiguity, and bishop contiguity.

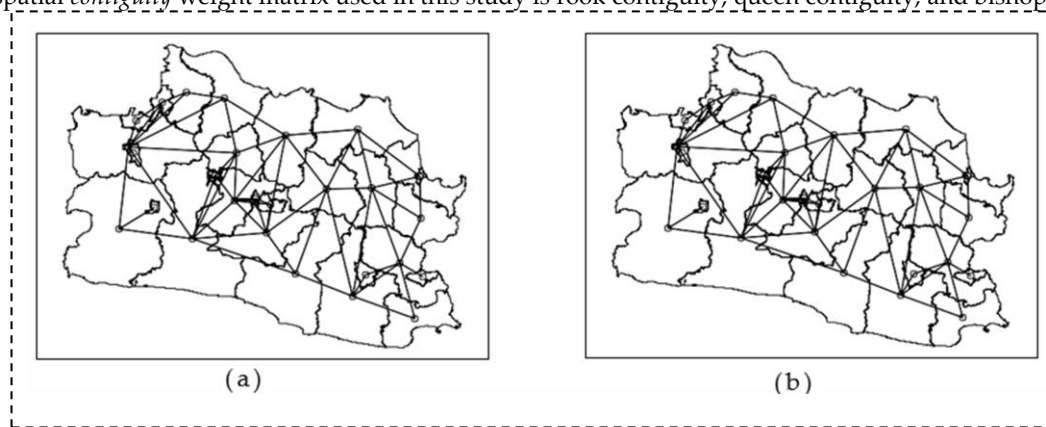


Figure 3 Plot (a) Rook Contiguity dan (b) Queen Contiguity

Figure 3 is a rook contiguity and queen contiguity spatial weight matrix plot. The point on the map shows the center of the district/city, while the line connecting one point to another shows the intersection between regions. From the results of the analysis, it can be seen that the rook contiguity and queen contiguity produce the same spatial weight matrix. That is, on the map of West Java, there are only side tangents, no angular tangents. So, the spatial weight matrix used is rook contiguity.

Customized Matrix

In this study, customized weights are also used because of the special characteristics of the research study, so that it does not only consider the geographical proximity/offensive relationship. Therefore, it is possible with customized weights to be closer to the closeness of the relationship between regions. The customized weighting approach is based

on aspects of the distribution system in the 4 Development Coordinating Boards (Bakor PP). With the division of the region as follows (www.jabarprov.go.id):

Table 3 The Regional Division of the Development Government Coordinating Board (Bakor PP)

Region	Regency/City
I	Bogor Regency, Bogor City, Depok City, Sukabumi Regency, Sukabumi City, and Cianjur Regency.
II	Purwakarta Regency, Subang Regency, Karawang Regency, Bekasi Regency, and Bekasi City.
III	Cirebon Regency, Cirebon City, Indramayu Regency, Majalengka Regency, and Kuningan Regency.
IV	Bandung Regency, Bandung City, Cimahi City, Bandung Barat Regency, Sumedang Regency, Garut Regency, Tasikmalaya Regency, Tasikmalaya City, Ciamis Regency, and Banjar City.

Regencies/cities that are in the same regional category will be given a value of 1, while regencies/cities that are in different regional categories will be given a value of 0. For example, Bogor Regency and Bogor City are considered to have a spatial relationship. It is because the two cities are in the same regional category, namely Region I.

D. Spatial Autocorrelation and Spatial Effect Test

The initial identification in the analysis process is the spatial autocorrelation test and the spatial effect test. The spatial autocorrelation test was carried out using Moran's I, while the spatial effect test was carried out using the Lagrange Multiplier, which can be seen in Table 4.

Table 4 Spatial autocorrelation test results and spatial effects test with Rook Contiguity Weights

Test	Statistic	<i>p</i> – value
Moran's I	0,4461	0,0003
LMerr	2,1474	0,1428
LMlag	6,6836	0,0097 **
SARMA	10,5192	0,0052 **

Based on Table 4, it can be seen that the statistical results of the Moran's I test are 0.4461 and the *p* – value is 0.00029, which shows a value smaller than 0.05, meaning that there is a spatial dependence.

The results of the spatial dependency test can also be seen in Table 4. The test results obtained from the 3 test statistics used were obtained by testing with a significant SARMA $\alpha = 5\%$. This shows that there are spatial dependencies both on the lag and on the error model. So that the Spatial Autoregressive with Autoregressive Disturbances (SARAR) model can be used.

Table 5 Spatial autocorrelation test results and spatial effects test with Customized Weights

Test	Statistic	<i>p</i> – value
Moran's I	0,2442	0,0029
LMerr	7,2347	0,0072 **
LMlag	13,7923	0,0002 ***
SARMA	16,0713	0,0003 ***

Based on Table 5, it can be seen that the statistical results of the Moran's I test are 0.2442 and the *p* – value is 0.0029, which shows a value smaller than 0.05, meaning that there is a spatial dependence.

The results of the spatial dependency test can also be seen in Table 5. The test results obtained from the 3 test statistics used were obtained by testing with a significant SARMA $\alpha = 5\%$. This shows that there are spatial dependencies both on the lag and on the error model. So, the Spatial Autoregressive with Autoregressive Disturbances (SARAR) model can be used.

E. Estimation of Model Parameters

The modeling was carried out using the Spatial Autoregressive with Autoregressive Disturbances (SARAR) method, and the parameter estimation was carried out using the Generalized Method of Moment (GMM) procedure. The spatial weights used are Rook Contiguity weights and Customized weights. The results of parameter estimation using Rook Contiguity weights can be seen in Table 6.

Table 6 Parameter Estimation of SARAR Model and GMM Method with Rook Contiguity weight

	Estimate	<i>z</i> – value	<i>p</i> – value
Intercept	1,6007	0,0687	0,9452
X_1	-0,8995	-2,0820	0,0374
X_2	0,1880	0,5634	0,5731
X_3	-0,0003	-1,4323	0,1521
X_4	-0,0137	-0,2077	0,8355
Rho	1,0373	6,2459	$4,22 \times 10^{-10}$
Lambda	-0,5970		

Based on the results of the parameter estimation by the GMM method in the SARAR equation using Rook Contiguity weight in Table 6, the SARAR equation can be written as follows:

$$y = 1,6007 - 0,8995 X_1 + 0,1880 X_2 - 0,0003 X_3 - 0,0137 X_4 + 1,0373 \sum_{i=1}^n \sum_{j=1}^n w_{ij}y_j - 0,5970 \sum_{i=1}^n \sum_{j=1}^n w_{ij}u_j \tag{17}$$

The results of parameter estimation using Customized weights can be seen in Table 7.

Table 7 Parameter Estimation of SARAR Model and GMM Method with Customized weights

	Estimate	<i>z</i> - value	<i>p</i> - value
Intercept	25,7409	1,0122	0,3114
X_1	-1,3301	-3,5932	0,0003
X_2	-0,0651	-0,1684	0,8662
X_3	-0,0004	-2,1030	0,0355
X_4	0,0406	0,7128	0,4759
Rho	0,7487	9,6827	$2,2 \times 10^{-16}$
Lambda	-1,8749		

Based on the results of the parameter estimation by the GMM method in the SARAR equation using Customized weights in Table 7, the SARAR equation can be written as follows:

$$y = 25,7409 - 1,3301 X_1 - 0,0651 X_2 - 0,0004 X_3 + 0,0406 X_4 + 0,7487 \sum_{i=1}^n \sum_{j=1}^n w_{ij}y_j - 1,8749 \sum_{i=1}^n \sum_{j=1}^n w_{ij}u_j \tag{18}$$

F. Best Model Selection

Based on the two alternative model parameter estimation results, the best model will then be determined, which will be analyzed further. The determination of the best model is based on two criteria, namely the spatial lag coefficient of the response variable (ρ_j) and the coefficient of determination (R^2). The model with the highest coefficient of determination (R^2) and the most significant response variable spatial lag coefficient (ρ_j) is the best and will be studied further. The two criteria are presented in Table 8.

Table 8 Criteria for Choosing the Best Model

Model	<i>p</i> - value	R^2
GMM SARAR Rook Contiguity weight	$3,94 \times 10^{-10}$	0,7806
GMM SARAR Customized weight	$2,2 \times 10^{-16}$	0,8116

Based on the above criteria, it can be said that the estimation results of model parameters with customized weights are relatively better than the results of parameter estimates using the rook contiguity weighting approach. This is because the estimation results of GMM parameters with customized weights produce a significant response variable spatial lag coefficient (ρ_j) and the largest coefficient of determination (R^2).

G. Interpretation

The SARAR model with a customized weight matrix produces the model in equation (18), with significant explanatory variables being Expected Years of Schooling (X_1) and Per capita Expenditure (X_3).

In the equation, it can be seen that the explanatory variable Expected Years of Schooling (X_1) significantly affects the response variable of the Percentage of the Poor Population with a coefficient of -1.3301. A negative Expected Years of Schooling coefficient means that the lower the Expected Years of Schooling will result in an increase in the percentage of the poor population. This is in line with the research conducted by Siregar & Wahyuniarti [24], which says that education has a significant influence on the level of poverty. The higher the education, the lower the poverty rate in an area. With a high education, a person will have the expertise, abilities, and skills necessary to increase their productivity and get better welfare.

The explanatory variable Per capita Expenditure (X_3) also significantly affects the response variable of the percentage of the poor population with a coefficient of -0.0004. The coefficient of per capita expenditure, which is negative, means that the lower the per capita expenditure, the poorer people will be. This result is in accordance with the theory put forward by Hasanah, Syaparuddin, & Rosmeli [25], which states that per capita expenditure has a negative and significant effect on poverty levels. Because the higher the per capita expenditure can be interpreted as the improvement of the community's economy in meeting their needs. The level of welfare is said to increase if there is an increase in real income, which causes an increase in household per capita expenditure and lowers the poverty level.

The significant value of spatial lag (ρ) indicates that there is a correlation between the Percentage of the Poor Population in one area and the surrounding area. Because the coefficient is positive, the increase in the Percentage of the Poor in an

area is accompanied by an increase in the Percentage of the Poor Population in the surrounding area. While the value of spatial error (λ) indicates that there is a relationship between the error in one region and the error in the surrounding area, even though it is negative.

V. CONCLUSIONS AND SUGGESTIONS

Conclusions

Based on the results and discussion, the conclusions obtained are as follows:

1. Through testing the effects of spatial dependence using the Moran's I test, the results obtained show that there is a spatial dependence on the percentage of poor people in districts or cities in West Java in 2020. The model used is a combined model between the spatial lag model and the spatial error model or the autoregressive model with autoregressive disturbances based on the calculation results from the Lagrange Multiplier (LM) test, and parameter estimation is carried out using the Generalized Method of Moment (GMM).
2. The GMM SARAR model with Customized weights produces better estimation results than the GMM SARAR model with Rook Contiguity weights in poverty modeling in West Java.
3. Expected Years of Schooling and per capita expenditure are significant in influencing the percentage of poor people in districts/cities in West Java in 2020. Expected Years of Schooling has a negative effect on the percentage of poor people in West Java, meaning that the higher the Expected Years of Schooling in an area, the lower the percentage of poor people in that area. Per capita expenditure also has a negative effect on the percentage of poor people in West Java, meaning that the higher the per capita expenditure in an area, the lower the percentage of poor people in that area.

Suggestions

Suggestions that can be given from this research are as follows:

1. There is a potential that the data we have is also a time series. So, we need a similar model (SARAR), which was developed using panel data, so that the influence between regions and over time can be seen.
2. The spatial model formed in poverty modeling does not only take into account the spatial interaction of the response and error variables but also takes into account or includes the spatial interaction of the explanatory variables.
3. Based on the results obtained, the Expected Years of Schooling and per capita expenditure have an important role in reducing poverty. In poverty alleviation efforts, the government is expected to pay more attention to the education factor and per capita expenditure so that the poverty rate can be controlled or suppressed.

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