Strength reduction factor evaluation of the circular reinforced concrete column with varying eccentricity ratio (e/h)

Wahyuniarsih Sutrisno[a*], Mudji Irmawan[a], Dwi Prasetya[a]

***Abstract**:This paper presents strength reduction factor evaluation of circular reinforced concrete column with varying eccentricity ratio (e/h) using the first-order-reliability-methods. The resistance properties of the reinforced concrete column is estimated using the monte-carlo simulation with random normally distributed material properties. Only dead and live load combination considered in the analysis. The parameters being investigated when evaluating the resistance of the reinforced concrete column are the concrete compressive strength, steel yield strength, coefficient of variation for both the concrete and steel materials, reinforced concrete column size, and the longitudinal reinforcement ratio. When evaluating the strength-reduction factor, the safety index values are 3.0, 3.5, and 4.0. From the analysis, it was found out that the strength reduction factor, for e/h higher than one and with safety index equal to 3.0, was equal to 0.9 which agrees well with the ACI 318 strength reduction factor for tension-controlled region. However, for e/h lower than one and safety index equal to 3.0, the strength reduction factor was equal to 0.6 which was lower than the ACI 318 strength reduction factor for compression-compression controlled region.*

Keywords: *Reduction factor, monte-carlo simulation, random material properties, eccentricity ratio*

INTRODUCTION

The use of unified design theory for reinforced concrete element have been used as a standard design method since ACI 318-02 due to more robust application in design. By using this unified design theory, the reinforced concrete (RC) member can be distinguished into compression- and tension-controlled section. This classification also eliminates the necessary to check the maximum reinforcement ratio for beam element. Hence, it can have over-reinforced concrete beam design but with lower reduction factor as in reinforced concrete column. Over-reinforced concrete beam will fall into the category of compression-controlled section.

UpToDate, the use of high-strength material become more widely used. The behavior of high-strength material was significantly different than the normal-strength ones [1-4]. Hence, it would be required to have the safety of the design to be evaluated by means of reliability analysis [5]. The reliability analysis of RC column often involved structural reliability where the probability of failure of the objective function (G) being evaluated with a certain value of safety index (β). This objective function G also known as the "limit state function" [5]. The limit state function can be further described as the resistance (R) deducted with the loads (S). Since the closed form expression for the resistance is not possible to derived, due to complexity of the materials model, the probability distribution of the resistance can be estimated using the Monte-Carlo simulation analysis. The probability failure of the limit state function is then can be computed using the first-order-reliability-methods (FORM) [6].

To compute the resistance (R), a probabilistic resistance model is required. Stewart and Attard [5] use explicit mathematical equation to determine the axial and bending moment capacity of rectangular reinforced concrete column with two faces reinforcement. In this paper, a more robust analysis to estimate the axial and moment capacity is used by utilizing the cross-sectional analysis with fiber-based method [7, 8]. With the used cross-sectional analysis method, a linear and nonlinear material behavior including irregular section of RC column can be investigated easily.

In this paper, the strength reduction factor of circular RC column is being investigated. The strength reduction factor is computed by solving the limit state function which involves the FORM. The concrete compressive strength being investigated are 40, 50, and 60 MPa and the bar yield strength varies from 320 to 500 MPa. The selection of the material strengths was based on the availability of materials in Indonesia. In the analysis, only axial and flexure behavior are considered which was generated by taking different ratio of eccentricity over the cross-section width.

RESEARCH SIGNIFICANCE

A reliability analysis based which involves Monte-Carlo simulation (MCS) and FORM is presented in this paper. The analysis of the cross-section is carried out using fiber-based element. Variation for both concrete and rebar material plus the geometric properties are being investigated with varying ratio of load eccentricity over the cross-section width. The main objective is to evaluate the strength reduction factor used in design with varying value of the safety index (β).

METHODOLOGY

In this paper, the research methodology is divided into three steps. In the first step, random controlled input data using Box and Muller method [9] are initiated. This random input data consisted of random normally distributed concrete compressive strength (f_{ci}) and random normally distributed steel yield strength (f_{yi}). In addition to random input data, deterministic input data (which is a constant value) are also prepared. This input data consisted of the column width (b), column height (h), ratio of live load to dead load ($R_{L/D}$), longitudinal reinforcement ratio (ρ), and concrete decking of the reinforced concrete column.

In the second step, the objective function G or the limit state function which is function of the resistance (R) and loads (S) should be evaluated by:

$$G(R,S) = R - S \tag{1}$$

$$R = \left[P^2 + \left(\frac{M}{h} \right)^2 \right]^{0.5} \tag{2}$$

Table 1 Input data for parametric study

No.	Parameter identification	Diameter (mm)	Num. of bars	Reinf. ratio (%)	f_c (MPa)	f_y (MPa)	$\Omega_{concrete}$ (%)	Ω_{steel} (%)
1	Global variation of RC column	500	8	2	40	400	20	8
2	Variation in concrete strength	500	8	2	40 50 60	400	20	8
3	Variation in steel rebar yield strength	500	8	2	40	320 400 500	20	8
4	Variation in steel rebar yield strength with zero variation in concrete material	500	8	2	40	320 400 500	0	8
5	Effect of variation in steel rebar material quality (zero variation in concrete material)	500	8	2	40	400	0	6 8 10
6	Effect of variation in concrete material quality (zero variation in steel rebar material)	500	8	2	40	400	10 20 30	0
7	Effect of longitudinal reinf. Ratio	500	8	3 4 5	40	400	20	8
8	Effect of column dimension	500 600 700	8	2	40	400	20	8

$$S = \left\{ (D+L)^2 + \left[\frac{(D+L)e}{h} \right]^2 \right\}^{0.5} \qquad (3)$$

where P is the axial load capacity, M is the bending moment capacity, h is the column height, D is the dead load, L is the live load, and e is the load eccentricity. The P and M are obtained from the cross-sectional analysis with fiber-based model with specific eccentricity ratio value. If both R and S were known, the safety index (β) of the system can be computed as:

$$\sqrt{\sigma_R^2 + \sigma_S^2}\,\beta = \bar{R} - \bar{S} \qquad (4)$$

where $\bar{R}$ is the mean resistance, $\bar{S}$ is the mean acting load, σ_R is the standard deviation of the resistance, and σ_S is the standard deviation of the loads. In Eqn.(4) the load notation (S) can be further decomposed into dead (D) and live (L) loads and therefore Eqn.(4) becomes:

$$\sqrt{\sigma_R^2 + \sigma_D^2 + \sigma_L^2}\,\beta = \bar{R} - \bar{D} - \bar{L} \qquad (5)$$

where σ_D and σ_L are the standard deviation for dead and live loads, respectively. Since the safety index is being given as an input and the loads can be set to unity for one type load, say for only the dead load (D = 1 and L = $R_{L/D}$D = $R_{L/D}$), the mean resistance can be back calculated to be used as an input to compute the required strength reduction factor of the resistance. Hence, the mean resistance $\bar{R}$ can be obtained by rearranging Eqn.(4) and is:

$$R_{1,2} = \frac{-2\left[R_{L/D}+1\right] \pm \sqrt{x}}{2\left(\left[\beta\Omega_R\right]^2 - 1\right)} \qquad (6)$$

$$x = \left(2\left[R_{L/D}+1\right]\right)^2 - 4\left(\left[\beta\Omega_R\right]^2 - 1\right)\ldots \\ \left(\left[\Omega_D\beta\right]^2 + \left[\Omega_D R_{L/D}\beta\right]^2 - \left[R_{L/D}+1\right]^2\right) \qquad (7)$$

In the third step, the strength reduction factor (ϕ) of the system can be computed by:

$$\phi = \frac{1-\left(\alpha_R\beta\Omega_R\right)}{v_R} \qquad (8)$$

where V_R is the ratio of nominal value to mean value of the strength reduction factor and α_R is the direction cosines for the strength reduction factor. Here, the value for V_R is taken as 0.95 and the expression for the reduction factor direction cosines α_R is:

$$\alpha_R = \frac{\sigma_R}{\sqrt{\left(\sigma_R^2 + \sigma_D^2 + \sigma_L^2\right)}} \qquad (9)$$

For each input random data accompanied with the specified deterministic input in MCS, the calculation of the reduction factor follows step one to step three. The MCS is used to compute the mean resistance $\bar{R}$ and standard deviation of the resistance σ_R. In the MCS, ten thousand random data being generated for the concrete compressive strength and the steel yield strength.

ANALYSIS AND DISCUSSIONS

A. COEFFICIENT OF VARIATION OF THE CIRCULAR REINFORCED CONCRETE COLUMN

Table 1 shows the input parameter data to be use in the parametric study analysis using MCS. There are eight input group parameters being investigated. To gain an insight on the coefficient of variation of the resistance (Ω_R), a reference input as outlined Table 1 (see number one) was used. Figure 1 shows the reference for the Ω_R of the RC column. There are three series presented in Figure 1. The black line represents Ω_R with $\Omega_{concrete}$ = 20% and Ω_{steel} = 8%. The red line represents Ω_R with $\Omega_{concrete}$ = 20% and Ω_{steel} = 0%. The green line represents Ω_R with $\Omega_{concrete}$ = 0% and Ω_{steel} = 8%. The purpose was to gain an insight on the effect of $\Omega_{concrete}$ and Ω_{steel} to the Ω_R of the RC column. For a small ratio of eccentricity, the variation in concrete strength is prominent in affecting the Ω_R as shown in Figure 1. However, as the ratio of eccentricity becomes higher, the RC cross-section may fall in tension-controlled region. Since the concrete strength in tension is neglected, the steel reinforcing bar handles all the tension forces in the cross section. This makes the variation in the steel yield strength become more prominent in affecting the Ω_R.

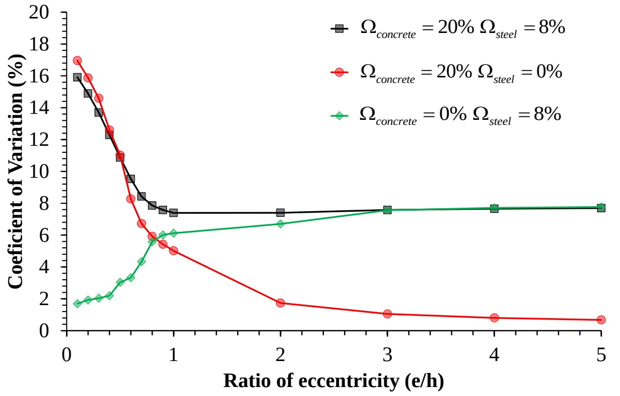

Figure 1 Coefficient of variation of the Resistance (Ω_R)
($\Omega_{concrete}$ = 20%, Ω_{steel} = 8%, f_c = 40 MPa, f_y = 400 MPa)

The concrete compressive strength may also affect the Ω_R. For that purpose, the concrete compressive strength inputs are set to 40, 50, and 60 MPa. Figure 2 shows the response for Ω_R with varying concrete compressive strength. As shown in Figure 2, the variation in Ω_R only varies slightly, and higher concrete strength shows does seem to give higher value of Ω_R when e/h is less than one. It should be noted that the premature cover spalling behavior [10-13] for the concrete cover is not yet being considered which may greatly affect the value of Ω_R due to higher uncertainties (concrete cover strength which was affected by the tensile confining pressure in the concrete core-to-cover interface).

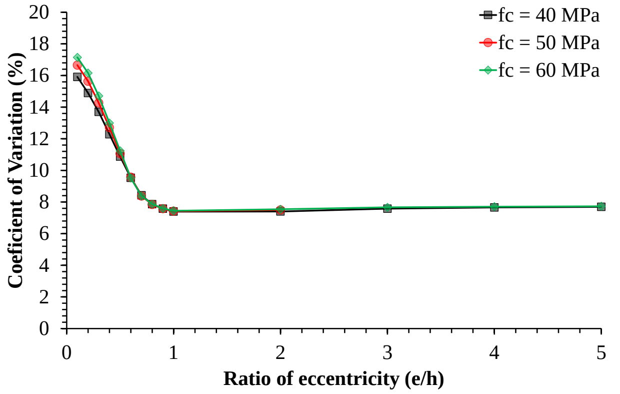

Figure 2 Effect of concrete compressive strength to Ω_R
($\Omega_{concrete}$ = 20%, Ω_{steel} = 8%, f_y = 400 MPa)

To investigate the effect of steel yield strength, three input data are being investigated. These input data for the yield strength are 320, 400, and 500 MPa. Figure 3 and 4 shows the effect of steel yield strength to the Ω_R. As shown in Figure 3, the steel yield strength affects the variation in Ω_R when the ratio of eccentricity was lower than two. For ratio of eccentricity higher than two the steel yields strength effect to the variation of Ω_R was found to be negligible. To eliminates the effect of variation in concrete material, the coefficient of the concrete material is set to zero and the result is shown in Figure 4. In Figure 4, the value for Ω_R was found to be higher for RC column with lower yield strength.

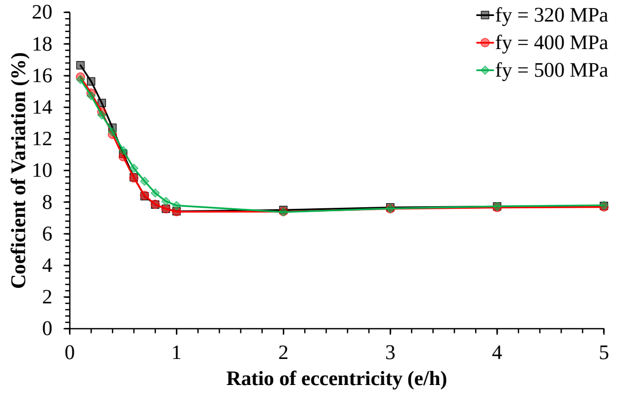

Figure 3 Effect of steel yield strength to Ω_R
($\Omega_{concrete}$ = 20%, Ω_{steel} = 8%, f_c = 40 MPa)

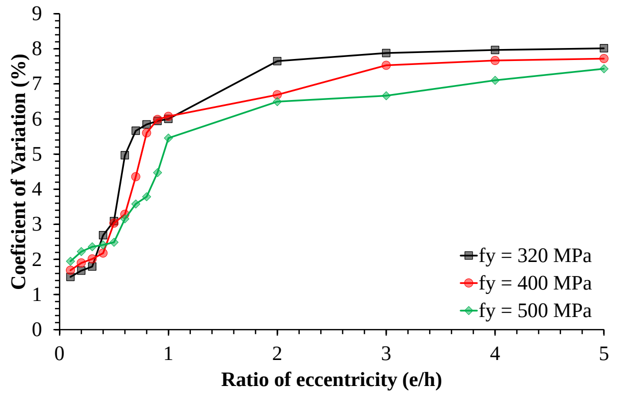

Figure 4 Effect of steel yield strength to Ω_R
($\Omega_{concrete}$ = 0%, Ω_{steel} = 8%, f_c = 40 MPa)

To study the effect of variation in the coefficient of variation for both the concrete and steel reinforcement ($\Omega_{concrete}$, Ω_{steel}), the value for the material coefficient of variations are varied. For that purpose, the coefficient variations for concrete material are set to 10, 20, and 30%. On the other hand, the coefficient variation for the steel rebar material are set to 6, 8, and 10 %.

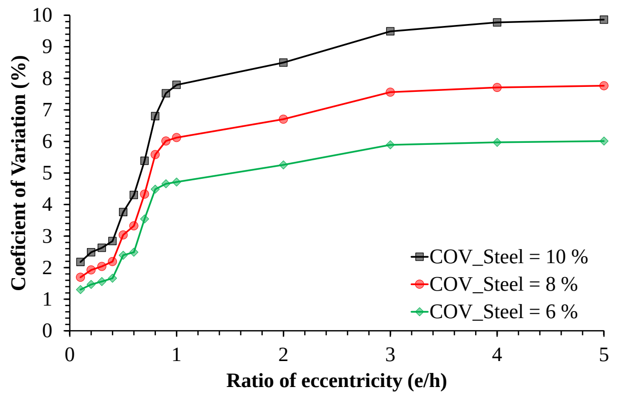

Figure 5 Effect of steel rebar material quality to Ω_R
($\Omega_{concrete}$ = 0%, f_c = 40 MPa, f_y = 400 MPa)

Figure 5 shows the effect of the variation in Ω_{steel} to the Ω_R. As shown in Figure 5, when the ratio of eccentricity is higher than one, the variation of Ω_{steel} significantly affects the value of Ω_R. For a small ratio of eccentricity (e/h less than one), the variation in Ω_{steel} also affect the value for Ω_R

but not as significant as when the eccentricity ratio is higher than one.

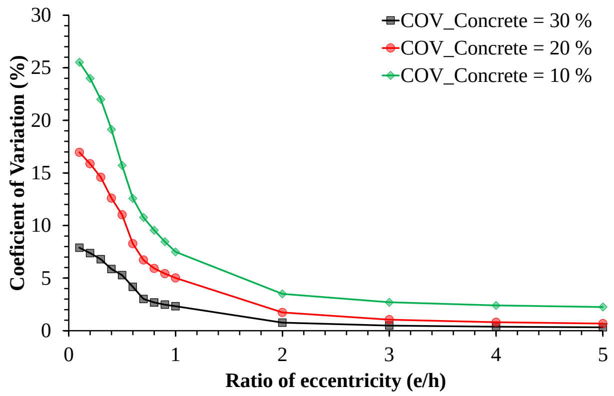

Figure 6 Effect of concrete material quality to Ω_R (Ω_{steel} = 0%, f_c = 40 MPa, f_y = 400 MPa)

Figure 6 shows the effect of the variation in $\Omega_{concrete}$ to the Ω_R. As shown in Figure 6, the variation in $\Omega_{concrete}$ affects the value of Ω_R for all the eccentricity ratio but it does seem to be more prominent when the ratio of eccentricity is less than one.

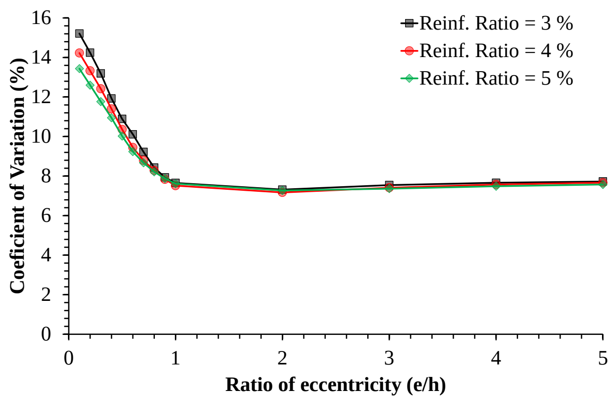

Figure 7 Effect of variation in longitudinal reinforcement ratio to Ω_R ($\Omega_{concrete}$ = 20%, Ω_{steel} = 8%,)

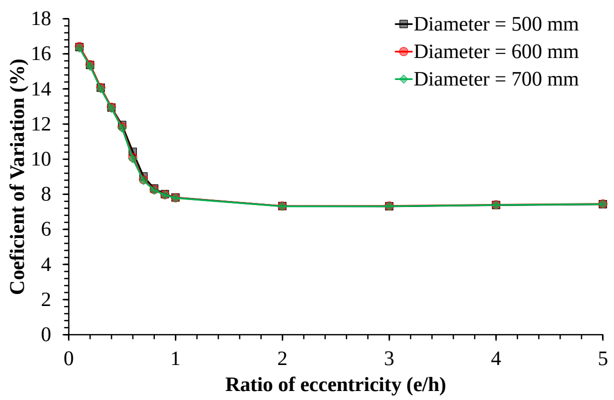

Figure 8 Effect of variation in column dimension to Ω_R

Figure 7 shows the effect of longitudinal reinforcement ratio to the value of Ω_R. As shown in Figure 7, when the ratio of eccentricity is higher than unity, the effect of longitudinal reinforcement ratio was barely noticeable. This can be caused by the increase in the bending moment capacity was somewhat has linear relationship with the longitudinal reinforcement ratio. On the other hand, the effect of longitudinal reinforcement ratio can alter the value of Ω_R when the ratio of eccentricity less than unity. This can be caused by the contribution of the longitudinal steel reinforcement in carrying the axial compression load acting on the RC column become higher in comparison to the concrete. As noticed in Figure 7, as the longitudinal reinforcement ratio increased, for e/h = 0.1, the value for Ω_R was decreased which supports the previous statement.
Figure 8 shows the effect of variation in the column dimension to the value of Ω_R. As shown in Figure 8, it was seen that there are no effects of changing the column dimension to the variation of Ω_R.

B. STRENGTH REDUCTION FACTOR OF CIRCULAR REINFORCED CONCRETE COLUMN

In the previous section, coefficient variation of RC column can be affected by many parameters. In this section, the strength reduction factor of the reference input data as shown in Table 1 is being investigated with varying value of the safety index (β). The investigated values for the safety index are 3, 3.5, and 4. The value for safety index equal to three have been used widely as the minimum safety index for design.

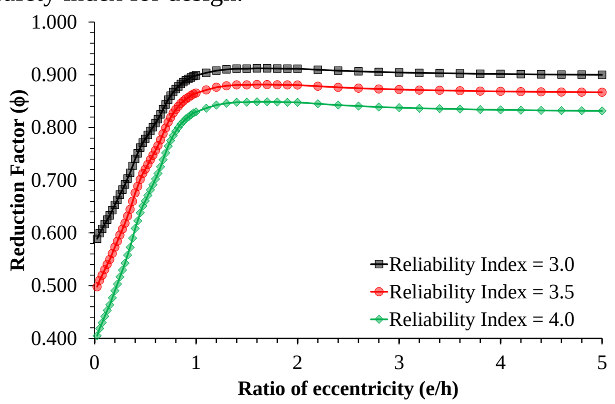

Figure 9 Reduction factor of the RC column with varying safety index value

Figure 9 shows the strength reduction factor of the RC column, which was computed using Eqn.(8). As shown in Figure 9, when the ratio of eccentricity higher than unity, the reduction factor was found to be almost similar. This kind of behavior was controlled by the coefficient variation of the steel reinforcing bars. For safety index equal to three, the strength reduction factor was close to 0.9 which agrees well with the ACI 318 building code [14]. On the other hand, when the ratio of eccentricity lower than unity, the reduction factor drops to its lowest value equal to 0.6. This shows that the concrete quality plays a very important aspect for the safety of concrete under compression. In ACI 318, the reduction factor for circular section is 0.7 which was higher than the one obtained from this study. This tells that the coefficient variation of the concrete used in ACI 318 was somewhat lower than the used input in this study. Hence, further investigation is required as to the exact input for the concrete material parameter.

CONCLUSIONS

This paper has presented strength reduction factor evaluation of circular RC column with varying ratio of eccentricity. From the analysis, it was found out that the variation in the resistance coefficient of variation (Ω_R) of the RC column was affected by many factors such as the concrete compressive strength, the steel rebar yield strength, the coefficient of variation in the materials, and the longitudinal reinforcement ratio. On the other hand, the column dimension did not affect the variation in Ω_R. From the study, the concrete material can significantly affect the Ω_R when the ratio of eccentricity is lower than two. On the other hand, the steel material can significantly affect Ω_R when the ratio of eccentricity is higher than one.

As for the strength reduction factor, for RC column under dominated flexure, the reduction factor value with safety index equal to three was found equal to 0.9. The obtained value was in close agreement with the ACI 318 building code. However, for RC column under dominated compression, the lowest reduction factor was found equal to 0.6 and was lower than the ACI 318 building code. Hence, further investigation is required as to the exact input for the concrete material parameter. For future research, it is important to look on other cross-sectional shape, other loading condition, and also using a more realistic stress-strain curve to better model RC column with high-strength materials.

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