Texture Dirac Mass Matrices and Lepton Asymmetry in the Minimal Seesaw Model with Tri-Bimaximal Mixing

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Abstract

We examined the minimal seesaw mechanism of 3×2 Dirac matrix by starting our analysis with the masses of light neutrinos with tri/bi-maximal mixing in the basis where the charged-lepton Yukawa matrix and heavy Majorana neutrino mass matrix are diagonal. We found all possible Dirac mass textures which contain one zero entry or two in the matrix.

KEYWORDS: tri-bimaximal mixing, minimal seesaw model, mass matrices and lepton asymmetry

I. INTRODUCTION

The recent neutrino oscillation experiments have provided us with robust evidence that neutrinos have tiny masses and their flavor mixing involves two large angles and one small angle [x]. A global analysis of current neutrino oscillation data yields $7.2 \times 10^{-5}V^2 \leq \Delta m^2_{sol} \leq 8.9 \times 10^{-5}V^2$ and $1.7 \times 10^{-3}V^2 \leq \Delta m^2_{atm} \leq 3.3 \times 10^{-3}V^2$ for the squared mass differences of solar and atmospheric neutrinos and $30^o \leq \theta_{12} \leq 38^o$, $36^o \leq \theta_{23} \leq 54^o$ and $0^o \leq \theta_{13} \leq 10^o$ for the flavor mixing angles at the 99% confidence level (the best-fit values are $\Delta m^2_{sol} = 8.0 \times 10^{-5}V^2$, $\Delta m^2_{atm} = 2.5 \times 10^{-3}V^2$, $\theta_{12} = 34^o$, $\theta_{23} = 45^o$ and $\theta_{13} = 0^o$[y]. Where we define $\Delta m^2_{sol} = m^2_2 - m^2_1$ and $\Delta m^2_{atm} = \left|m^2_3 - m^2_2\right|$ with the neutrino mass eigenvalues m_1, m_2 and m_3. The on-going and forthcoming neutrino oscillation experiments will shed light on the sign of Δ_{atm} and the magnitude of θ_{13} and even the CP-violating phase.

The seesaw mechanism is arguably the most attractive way to explain the smallnes of neutrino masses. In its conventional form, the seesaw invokes three heavy singlet right-handed neutrinos ν_R [1]. However, the number three is not sacret. The two non-zero light neutrino mass difference required by experiment could be explained with just two heavy right-handed neutrinos [2]. The seesaw mechanism with two right-handed neutrinos predicts one of the physical light neutrino mass to be exactly zero; which is permissible within the current knowledge of neutrino masses and mixings.

On the other hand, the observed neutrino mixing matrix is compatible with the so called tri-bimaximal form, introduced by Harrison, Perkins and Scott [3]. This tri-bimaximal neutrino mixing is based on the idea that there are both bi-maximal (0, 1, 1)/$\sqrt{2}$ as well as tri-maximal (1, 1, 1)/$\sqrt{3}$ mixings in the lepton sector.

The results of the LEP experiments on the measurement of the invisible width of the Z boson imply that only three flavor neutrinos exist in nature (see Ref.[4]). The simplest one to give 3×3 mass matrix m_ν is both Dirac mass matrix m_D and right-handed massive Majorana mass matrix M_R are 3×3 matrices.

II. THE MINIMAL SEESAW MODEL

The most economical seesaw model which compatible with solar and atmospheric neutrinos is satisfied by two right-handed neutrinos. The leptonic part of the Yukawa interactions in presence of three left-handed and two right-handed neutrinos can be written as

$$-\mathcal{L} = \overline{\psi_{Li}}\,(Y_\nu)_{ij}\,\tilde{\phi} N_{Rj} + \overline{\psi_{Li}}\,(Y_\ell)_{ij}\,\phi e_{Rj} + \frac{1}{2}\overline{N^c_{Ri}}\,(M_R)_{ij}\,N_{Rj} + h.c. \quad (1)$$

where N_R denote the right-handed neutrino fields which are singlet under the standard model gauge group, ϕ is SU(2) higgs doublet with $\tilde{\phi} = i\sigma_2\phi^*$, ψ_{Li} is the lepton doublet of flavor i, and E_{Ri} are the right-handed charged leton singlet. the Yukawa coupling constants Y_ν and Y_ℓ are comlex-valued matrices. After the electroweak symmetry breaking one gets the charged mass matrix $m_\ell = vY_\ell$ and the Dirac mass matrix for the neutrino as $m_D = vY_\nu$ where v is the vacuum exectation value of the neutral component of the higgs doublet ϕ. The Majorana mass matrix M_R is 2×2 comlex symmetric matrix. The mass matrix for the neutral fermions can be written as

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \quad (2)$$

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The light neutrino mass matrix m_ν after the seesaw diagonalization is given by the seesaw formula

$$m_\nu = -m_D M_R^{-1} m_D^T \tag{3}$$

This master formula (3) is valid when the eigenvalues of M_R are much larger than the elements of m_D and in such a case the eigenvalues of m_ν come out very small with respect to those of m_D.

In general the Majorana mass matrix M_R is non-diagonal form in the basis where the charged current is flavor diagonal. In this form one can make a basis rotation so that the right-handed Majorana mass matrix becomes diagonal by the unitary matrix. All possible heavy Majorana mass matrices M_R, their inverse and their diagonal form are given in the Table I, with the values

$$d_\pm = \frac{M_2 \pm \sqrt{4M_1^2 + M_2^2}}{2} \tag{4}$$

and

$$D_\pm = \frac{M_1 + M_3 \pm \sqrt{M_1^2 + 4M_2^2 - 2M_1M_3 + M_3^2}}{2} \tag{5}$$

TABLE I: The Heavy Majorana Mass Matrices

M_R	$(M_R)_{diag}$	M_R^{-1}
$\begin{pmatrix} 0 & 0 \\ 0 & M_1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & M_1 \end{pmatrix}$	singular
$\begin{pmatrix} 0 & M_1 \\ M_1 & 0 \end{pmatrix}$	$\begin{pmatrix} -M_1 & 0 \\ 0 & M_1 \end{pmatrix}$	$\begin{pmatrix} 0 & \frac{1}{M_1} \\ \frac{1}{M_1} & 0 \end{pmatrix}$
$\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$	$\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{M_1} & 0 \\ 0 & \frac{1}{M_2} \end{pmatrix}$
$\begin{pmatrix} 0 & M_1 \\ M_1 & M_2 \end{pmatrix}$	$\begin{pmatrix} d_- & 0 \\ 0 & d_+ \end{pmatrix}$	$\begin{pmatrix} -\frac{M_2}{M_1^2} & \frac{1}{M_1} \\ \frac{1}{M_1} & 0 \end{pmatrix}$
$\begin{pmatrix} M_1 & M_2 \\ M_2 & M_1 \end{pmatrix}$	$\begin{pmatrix} M_2 - M_1 & 0 \\ 0 & M_2 + M_1 \end{pmatrix}$	$\frac{\begin{pmatrix} M_1 & -M_2 \\ -M_2 & M_1 \end{pmatrix}}{M_2^2 - M_1^2}$
$\begin{pmatrix} M_1 & M_2 \\ M_2 & M_3 \end{pmatrix}$	$\begin{pmatrix} D_- & 0 \\ 0 & D_+ \end{pmatrix}$	$\frac{\begin{pmatrix} M_3 & -M_2 \\ -M_2 & M_1 \end{pmatrix}}{M_1 M_3 - M_2^2}$

In this form the Dirac mass matrix must be 3×2 form. It implies the the determinant of light massive neutrino matrix is zero

$$\det m_\nu = 0 \tag{6}$$

Since

$$U \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} V = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix} \tag{7}$$

it gives at least one of the eigenvalues of m_ν is exactly zero; i.e, either $m_1 = 0$ for normal neutrino mass hierarchy ($m_1 < m_2 < m_3$) or $m_3 = 0$ for inverted neutrino mass hierarchy ($m_1 > m_2 > m_3$).

III. TRI-BIMAXIMAL MIXING AND DIRAC MASS TEXTURE

It is an experimental fact that within measurement errors the observed neutrino mixing matrix is compatible with the so called tri-bimaximal form, introduced by Harrison, Perkins and Scott (HPS). The matrix is given by

$$V = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \tag{8}$$

One purpose of this paper is just to find the Dirac neutrino mass matrix for either m1 = 0 or m3 = 0. Looking back to Eqs. (1), we have 3×2 matrix $m_D = vY_\nu$, we can write it in the form

$$m_D = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{pmatrix} \tag{9}$$

Using the diagonal inverse matrix M_R^{-1} we obtain

$$\begin{aligned} m_\nu &= \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{pmatrix} \begin{pmatrix} \frac{1}{M_1} & 0 \\ 0 & \frac{1}{M_2} \end{pmatrix} \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{pmatrix} \\ &= \begin{pmatrix} \frac{x_1^2}{M_1} + \frac{y_1^2}{M_2} & \frac{x_1x_2}{M_1} + \frac{y_1y_2}{M_2} & \frac{x_1x_3}{M_1} + \frac{y_1y_3}{M_2} \\ \frac{x_1x_2}{M_1} + \frac{y_1y_2}{M_2} & \frac{x_2^2}{M_1} + \frac{y_2^2}{M_2} & \frac{x_2x_3}{M_1} + \frac{y_2y_3}{M_2} \\ \frac{x_1x_3}{M_1} + \frac{y_1y_3}{M_2} & \frac{x_2x_3}{M_1} + \frac{y_2y_3}{M_2} & \frac{x_3^2}{M_1} + \frac{y_3^2}{M_2} \end{pmatrix} \end{aligned} \tag{10}$$

Then applying the HPS matrix to diagonalize the neutrino mass matrix m_ν

$$V^T m_\nu V = \begin{pmatrix} & m_{ij} & \end{pmatrix} \tag{11}$$

we have

$$\begin{aligned}
m_{11} &= \frac{\left(4y_1^2+y_2^2+y_3^2-4y_1\left(y_2+y_3\right)+2y_2y_3\right)M_1+\left(4x_1^2+x_2^2+x_3^2-4x_1\left(x_2+x_3\right)+2x_2x_3\right)M_2}{6M_1M_2}\\
m_{12} &= \frac{\left(2y_1^2-y_2^2-y_3^2+y_1\left(y_2+y_3\right)-2y_2y_3\right)M_1+\left(2x_1^2-x_2^2-x_3^2+x_1\left(x_2+x_3\right)-2x_2x_3\right)M_2}{3\sqrt{2}M_1M_2}\\
m_{13} &= \frac{\left(2y_1-y_2-y_3\right)\left(y_2-y_3\right)M_1+\left(2x_1-x_2-x_3\right)\left(x_2-x_3\right)M_2}{2\sqrt{3}M_1M_2}\\
m_{23} &= \frac{\left(y_1+y_2+y_3\right)\left(y_2-y_3\right)M_1+\left(x_1+x_2+x_3\right)\left(x_2-x_3\right)M_2}{\sqrt{6}M_1M_2}\\
m_{22} &= \frac{\left(y_1+y_2+y_3\right)^2M_1+\left(x_1+x_2+x_3\right)^2M_2}{3M_1M_2}\\
m_{33} &= \frac{\left(y_2-y_3\right)^2M_1+\left(x_2-x_3\right)^2M_2}{2M_1M_2}
\end{aligned}\tag{12}$$

$$V^Tm_\nu V \equiv \hat{m}_\nu = \begin{pmatrix} & & \\ & d_i\delta_{ij} & \\ & & \end{pmatrix}\tag{13}$$

The lightest neutrino is allowed to be massless; i.e., either $m_1 = 0$ (normal neutrino mass hierarchy, NH) or $m_3 = 0$ (inverted neutrino mass hierarchy, IH) has no conflict with the present neutrino oscillation measurements. In both cases, the non-vanishing neutrino masses can be determined in terms of m^2_{sol} anad m^2_{atm}:

$$m_1 = 0 \Rightarrow \begin{cases} m_2 = \sqrt{m^2_{sol}} \approx 8.97\times10^{-3}eV, \\ m_3 = \sqrt{m^2_{atm}+m^2_{sol}} \approx 5.08\times10^{-2}eV. \end{cases}\tag{14}$$

$$m_3 = 0 \Rightarrow \begin{cases} m_1 = \sqrt{m^2_{atm}-m^2_{sol}} \approx 4.92\times10^{-2}eV, \\ m_2 = \sqrt{m^2_{atm}} \approx 5.00\times10^{-2}eV. \end{cases}\tag{15}$$

These light neutrinos give heavy masses; for normal hierarchy

$$\begin{aligned} M_1 &= 8.98\times10^{12}GeV \approx 10^{13}GeV\\ M_2 &= 1.07\times10^{14}GeV \approx 10^{14}GeV \end{aligned}\tag{16}$$

for normal hierarchy, and

$$\begin{aligned} M_1 &= 9.12\times10^{12}GeV \approx 10^{13}GeV\\ M_2 &= 9.07\times10^{13}GeV \approx 10^{14}GeV \end{aligned}\tag{17}$$

for inverse hierarchy.

IV. LEPTON ASYMMETRY

When the Majorana right-handed neutrinos decay into leptons and Higgs scalars, they violate the lepton number since right-handed neutrino fermionic lines do not have any preferred arrow

$$\begin{aligned} N_R &\rightarrow \ell + H^*\\ N_R &\rightarrow \bar{\ell} + H \end{aligned}\tag{18}$$

The interference between the tree-level decay amplitude and the absorptive part of the one-loop vertex leads to a lepton asymmetry

$$\epsilon = \sum_{i=1,2}\frac{\Gamma\left(N_i\rightarrow \ell H^*\right)-\Gamma\left(N_i\rightarrow \bar{\ell}H\right)}{\Gamma\left(N_i\rightarrow \ell H^*\right)+\Gamma\left(N_i\rightarrow \bar{\ell}H\right)}\tag{19}$$

We assume a hierarchical mass pattern of the heavy neutrinos $M_1 << M_2$. in this case, the interactions of N_1 can be in thermal equilibrium when N_2 decay is washed-out by the lepton number violating processes with $_1$. Thus only the decays of N_1 are relevant for generation of the final lepton asymmetry $\epsilon \approx \epsilon_1$. in this case, the CP asymmetry paarameter in the

TABLE II: Texture Dirac mass matrices and their light neutrino mass matrices

m_D	$\hat{m}_\nu$ (diag)	Notes
$\begin{pmatrix} x & x & x \\ y & y & y \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3y^2}{M_2}+\frac{3x^2}{M_1} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	two zeros
$\begin{pmatrix} -2x & x & x \\ -2y & y & y \end{pmatrix}$	$\begin{pmatrix} \frac{6y^2}{M_2}+\frac{6x^2}{M_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	two zeros
$\begin{pmatrix} 0 & x & -x \\ 0 & y & -y \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{2y^2}{M_2}+\frac{2x^2}{M_1} \end{pmatrix}$	two zeros
$\begin{pmatrix} 0 & x & -x \\ y & y & y \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3y^2}{M_2} & 0 \\ 0 & 0 & \frac{2x^2}{M_1} \end{pmatrix}$	NH
$\begin{pmatrix} x & x & x \\ 0 & y & -y \end{pmatrix}$	$\begin{pmatrix} \frac{6x^2}{M_1} & 0 & 0 \\ 0 & \frac{3y^2}{M_2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	IH
$\begin{pmatrix} -2x & x & x \\ y & y & y \end{pmatrix}$	$\begin{pmatrix} \frac{6x^2}{M_1} & 0 & 0 \\ 0 & \frac{3y^2}{M_2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	IH
$\begin{pmatrix} x & x & x \\ -2y & y & y \end{pmatrix}$	$\begin{pmatrix} \frac{6y^2}{M_2} & 0 & 0 \\ 0 & \frac{3x^2}{M_1} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	IH
$\begin{pmatrix} 0 & x & x \\ y & z & z \end{pmatrix}$	$\begin{pmatrix} \frac{y(y-z)}{M_2} & 0 & 0 \\ 0 & \frac{y(y+2z)}{M_2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	IH, $x^{*)}$
$\begin{pmatrix} t & x & x \\ 0 & y & y \end{pmatrix}$	$\begin{pmatrix} \frac{t(t-x)}{M_1} & 0 & 0 \\ 0 & \frac{t(t+2x)}{M_1} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	IH, $y^{**)}$

$*)x=\sqrt{\frac{\left(y^2+yz-2z^2\right)M_1}{2M_2}}$

$**)y=\sqrt{\frac{\left(t^2+tx-2x^2\right)M_2}{2M_1}}$

minimal seesaw model is given by

$$\epsilon=\frac{1}{8\pi v^2}\frac{\Im\left[\left(m_D^\dagger m_D\right)_{12}^2\right]}{\left(m_D^\dagger m_D\right)_{11}}f\left(\frac{M_2^2}{M_1^2}\right) \quad (20)$$

where

$$f(x)=\sqrt{x}\left[1-(1+x)\ln\left(\frac{1+x}{x}\right)+\frac{1}{1-x}\right] \quad (21)$$

This asymmetry is valid in the basis where the right handed neutrino is diagonal. $f(x)\approx 3/\sqrt{x}$ for $x>>1$. Sphaleron processes will convert this lepton asymmetry into baryon asymmetry, as a result, the baryon asymmetry is approximately described as

$$\eta_B=0.96\times 10^{-2}\left(-\epsilon\right)\kappa \quad (22)$$

where κ is the efficiency factor, that parameterizes dilution effects for generated lepton asymmetry through washing-out processes.

V. CONCLUSION

In this paper we analyze the texture zeros in the neutrino Yukawa Coupling matrix m_D and the heavy Majorana neutrino mass matrix M_R in the context of the minimal seesaw model including 2 heavy right-handed neutrinos. We illustrate which textures are compatible with the present neutrino oscillation data and discuss their implications for the future neutrino experiments. We do not make the assumption that M_R is diagonal. We examined the minimal seesaw mechanism of 3×2 Dirac matrix by starting our analysis with the masses of light neutrinos with tri/bi-maximal mixing in the basis where the charged-lepton Yukawa matrix and heavy Majorana neutrino mass matrix are diagonal. We found all possible Dirac mass textures which contain one zero entry or two in the matrix.

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