Texture Dirac Mass Matrices and Lepton Asymmetry in the Minimal Seesaw Model with Tri-Bimaximal Mixing

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Abstract

We examined the minimal seesaw mechanism of $3 \times 2$ Dirac matrix by starting our analysis with the masses of light neutrinos with tri/bi-maximal mixing in the basis where the charged-lepton Yukawa matrix and heavy Majorana neutrino mass matrix are diagonal. We found all possible Dirac mass textures which contain one zero entry or two in the matrix.

KEYWORDS: tri-bimaximal mixing, minimal seesaw model, mass matrices and lepton asymmetry

I. INTRODUCTION

The recent neutrino oscillation experiments have provided us with robust evidence that neutrinos have tiny masses and their flavor mixing involves two large angles and one small angle [x]. A global analysis of current neutrino oscillation data yields $7.2 \times 10^{-5} V^2 \leq \Delta m^2_{sol} \leq 8.9 \times 10^{-5} V^2$ and $1.7 \times 10^{-3} V^2 \leq \Delta m^2_{atm} \leq 3.3 \times 10^{-3} V^2$ for the squared mass differences of solar and atmospheric neutrinos and $30^o \leq \theta_{12} \leq 38^o$, $36^o \leq \theta_{23} \leq 54^o$ and $0^o \leq \theta_{13} \leq 10^o$ for the flavor mixing angles at the 90% confidence level (the best-fit values are $\Delta m^2_{sol} = 8.0 \times 10^{-5} V^2$, $\Delta m^2_{atm} = 2.5 \times 10^{-3} V^2$, $\theta_{12} = 34^o$, $\theta_{23} = 45^o$ and $\theta_{13} = 0^o$) [y]. Where we define $\Delta m^2_{sol} = m^2_2 - m^2_1$ and $\Delta m^2_{atm} = |m^2_3 - m^2_2|$ with the neutrino mass eigenvalues $m_1$, $m_2$ and $m_3$. The ongoing and forthcoming neutrino oscillation experiments will shed light on the sign of $\Delta m^2_{atm}$ and the magnitude of $\theta_{13}$ and even the CP-violating phase.

The seesaw mechanism is arguably the most attractive way to explain the smallness of neutrino masses. In its conventional form, the seesaw invokes three heavy singlet right-handed neutrinos $\nu_R$ [1]. However, the number three is not sacret. The two non-zero light neutrino mass difference required by experiment could be explained with just two heavy right-handed neutrinos [2]. The seesaw mechanism with two right-handed neutrinos predicts one of the physical light neutrino mass to be exactly zero; which is permissible within the current knowledge of neutrino masses and mixings.

On the other hand, the observed neutrino mixing matrix is compatible with the so called tri-bimaximal form, introduced by Harrison, Perkins and Scott [3]. This tri-bimaximal neutrino mixing is based on the idea that there are both bi-maximal $(0, 1, 1)/\sqrt{2}$ as well as tri-maximal $(1, 1, 1)/\sqrt{3}$ mixings in the lepton sector.

The results of the LEP experiments on the measurement of the invisible width of the $Z$ boson imply that only three flavor neutrinos exist in nature (see Ref.[4]). The simplest one to give $3 \times 3$ mass matrix $m_{\nu}$ is both Dirac mass matrix $m_D$ and right-handed massive Majorana mass matrix $M_R$ are $3 \times 3$ matrices.

II. THE MINIMAL SEESAW MODEL

The most economical seesaw model which compatible with solar and atmospheric neutrinos is satisfied by two right-handed neutrinos. The leptonic part of the Yukawa interactions in presence of three left-handed and two right-handed neutrinos can be written as

$$-\mathcal{L} = \bar{\nu}_{Li}(Y_{Li})_{ij} \tilde{\phi} N_{Rj} + \bar{\nu}_{L1}(Y_{L1})_{ij} \phi c_{Rj} + \frac{1}{2} N_{Ri}(M_{R})_{ij} N_{Rj} + h.c.$$ (1)

where $N_{Ri}$ denote the right-handed neutrino fields which are singlet under the standard model gauge group, $\phi$ is SU(2) higgs doublet with $\phi = i\sigma_2 \phi^*$, $\bar{\psi}_{Li}$ is the lepton doublet of flavor $i$, and $E_{Ri}$ are the right-handed charged lepton singlet. The Yukawa coupling constants $Y_{Li}$ and $Y_{L1}$ are complex-valued matrices. After the electroweak symmetry breaking one gets the charged mass matrix $m_{e} = vY_{e}$ and the Dirac mass matrix for the neutrino as $m_{D} = vY_{\nu}$ where $v$ is the vacuum expectation value of the neutral component of the higgs doublet $\phi$. The Majorana mass matrix $M_{R}$ is $2 \times 2$ complex symmetric matrix. The mass matrix for the neutral fermions can be written as

$$M_{\nu} = \begin{pmatrix} 0 & m_{D} \\ m_{T} & M_{R} \end{pmatrix}$$ (2)
The light neutrino mass matrix $m_\nu$ after the seesaw diagonalization is given by the seesaw formula

$$m_\nu = -m_D M_R^{-1} m_D^T$$

(3)

This master formula (3) is valid when the eigenvalues of $M_R$ are much larger than the elements of $m_D$ and in such a case the eigenvalues of $m_\nu$ come out very small with respect to those of $m_D$.

In general the Majorana mass matrix $M_R$ is non-diagonal form in the basis where the charged current is flavor diagonal. In this form one can make a basis rotation so that the right-handed Majorana mass matrix becomes diagonal by the unitary matrix. All possible heavy Majorana mass matrices $M_R$, their inverse and their diagonal form are given in the Table I, with the values

$$d_\pm = \frac{M_2 \pm \sqrt{4M_1^2 + M_2^2}}{2}$$

(4)

and

$$D_\pm = \frac{M_1 + M_3 \pm \sqrt{M_1^2 + 4M_2^2 - 2M_1M_3 + M_3^2}}{2}$$

(5)

<table>
<thead>
<tr>
<th>TABLE I: The Heavy Majorana Mass Matrices</th>
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<tbody>
<tr>
<td>$M_R$</td>
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<tr>
<td>-------------------------</td>
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<tr>
<td>$\begin{pmatrix} 0 &amp; 0 \ 0 &amp; M_1 \end{pmatrix}$</td>
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<tr>
<td>$\begin{pmatrix} 0 &amp; M_1 \ M_1 &amp; 0 \end{pmatrix}$</td>
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<tr>
<td>$\begin{pmatrix} 0 &amp; M_2 \ M_1 &amp; M_2 \end{pmatrix}$</td>
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<td>$\begin{pmatrix} 0 &amp; M_1 \ M_1 &amp; M_2 \end{pmatrix}$</td>
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<td>$\begin{pmatrix} M_1 &amp; M_2 \ M_2 &amp; M_1 \end{pmatrix}$</td>
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<tr>
<td>$\begin{pmatrix} M_1 &amp; M_2 \ M_2 &amp; M_3 \end{pmatrix}$</td>
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</tbody>
</table>

In this form the Dirac mass matrix must be $3 \times 2$ form. It implies the determinant of light massive neutrino matrix is zero

$$\det m_\nu = 0$$

(6)

Since

$$U \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} V = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix}$$

(7)

it gives at least one of the eigenvalues of $m_\nu$ is exactly zero; i.e., either $m_1 = 0$ for normal neutrino mass hierarchy ($m_1 < m_2 < m_3$) or $m_3 = 0$ for inverted neutrino mass hierarchy ($m_1 > m_2 > m_3$).

### III. TRI-BIMAXIMAL MIXING AND DIRAC MASS TEXTURE

It is an experimental fact that within measurement errors the observed neutrino mixing matrix is compatible with the so called tri-bimaximal form, introduced by Harrison, Perkins and Scott (HPS). The matrix is given by

$$V = \begin{pmatrix} \sqrt{2}/3 & -1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix}$$

(8)

One purpose of this paper is just to find the Dirac neutrino mass matrix for either $m_1 = 0$ or $m_3 = 0$. Looking back to Eqs. (1), we have $3 \times 2$ matrix $m_D = vY$, we can write it in the form

$$m_D = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{pmatrix}$$

(9)

Using the diagonal inverse matrix $M_R^{-1}$ we obtain

$$m_\nu = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{pmatrix} \begin{pmatrix} 1/M_1 & 0 \\ 0 & 1/M_2 \end{pmatrix} \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{pmatrix} = \begin{pmatrix} \frac{x_1}{M_1} + \frac{x_2}{M_2} & \frac{x_1}{M_1} & \frac{x_2}{M_2} \\ \frac{y_1}{M_1} & \frac{y_2}{M_2} & \frac{y_3}{M_2} \end{pmatrix}$$

(10)

Then applying the HPS matrix to diagonalize the neutrino mass matrix $m_\nu$,

$$V^T m_\nu V = \begin{pmatrix} m_{ij} \end{pmatrix}$$

(11)

we have
tons and Higgs scalars, they violate the lepton number since for inverse hierarchy. For normal hierarchy, and

\[ m_{11} = \frac{(4y_1^2 + y_2^2 + y_3^2 - 4y_1(y_2 + y_3) + 2y_2y_3)M_1 + (4x_1^2 + x_2^2 + x_3^2 - 4x_1(x_2 + x_3) + 2x_2x_3)M_2}{6M_1M_2} \]

\[ m_{12} = \frac{(2y_1^2 - y_2^2 - y_3^2 + y_1(y_2 + y_3) - 2y_2y_3)M_1 + (2x_1^2 - x_2^2 - x_3^2 + x_1(x_2 + x_3) - 2x_2x_3)M_2}{3\sqrt{2}M_1M_2} \]

\[ m_{13} = \frac{(2y_1 - y_2 - y_3)(y_2 - y_3)M_1 + (2x_1 - x_2 - x_3)(x_2 - x_3)M_2}{2\sqrt{3}M_1M_2} \]

\[ m_{23} = \frac{(y_1 + y_2 + y_3)(y_2 - y_3)M_1 + (x_1 + x_2 + x_3)(x_2 - x_3)M_2}{\sqrt{6}M_1M_2} \]

\[ m_{22} = \frac{(y_1 + y_2 + y_3)^2M_1 + (x_1 + x_2 + x_3)^2M_2}{3M_1M_2} \]

\[ m_{33} = \frac{(y_2 - y_3)^2M_1 + (x_2 - x_3)^2M_2}{2M_1M_2} \]

The lightest neutrino is allowed to be massless; i.e., either \( m_1 = 0 \) (normal neutrino mass hierarchy, NH) or \( m_3 = 0 \) (inverted neutrino mass hierarchy, IH) has no conflict with the present neutrino oscillation measurements. In both cases, the non-vanishing neutrino masses can be determined in terms of \( m_{sol}^2 \) and \( m_{atm} \):

\[ V^T_m \nu \equiv \hat{\nu}_\nu = \left( d_i \delta_{ij} \right) \] (13)

\[ m_1 = 0 \Rightarrow \begin{cases} m_2 = \sqrt{m_{sol}^2} \approx 8.97 \times 10^{-3} eV, \\ m_3 = \sqrt{m_{atm}^2 + m_{sol}^2} \approx 5.08 \times 10^{-2} eV. \end{cases} \] (14)

\[ m_3 = 0 \Rightarrow \begin{cases} m_1 = \sqrt{m_{atm}^2 - m_{sol}^2} \approx 4.92 \times 10^{-2} eV, \\ m_2 = \sqrt{m_{atm}^2} \approx 5.00 \times 10^{-2} eV. \end{cases} \] (15)

These light neutrinos give heavy masses; for normal hierarchy

\[ M_1 = 8.98 \times 10^{12} GeV \approx 10^{13} GeV \]
\[ M_2 = 1.07 \times 10^{14} GeV \approx 10^{14} GeV \] (16)

for normal hierarchy, and

\[ M_1 = 9.12 \times 10^{12} GeV \approx 10^{13} GeV \]
\[ M_2 = 9.07 \times 10^{13} GeV \approx 10^{14} GeV \] (17)

for inverse hierarchy.

**IV. LEPTON ASYMMETRY**

When the Majorana right-handed neutrinos decay into leptons and Higgs scalars, they violate the lepton number since right-handed neutrino fermionic lines do not have any preferred arrow

\[ N_R \rightarrow \ell + H^* \]
\[ N_R \rightarrow \bar{\ell} + H \] (18)

The interference between the tree-level decay amplitude and the absorptive part of the one-loop vertex leads to a lepton asymmetry

\[ \epsilon = \sum_{i=1,2} \frac{\Gamma(N_i \rightarrow \ell H^*) - \Gamma(N_i \rightarrow \bar{\ell} H)}{\Gamma(N_i \rightarrow \ell H^*) + \Gamma(N_i \rightarrow \bar{\ell} H)} \] (19)

We assume a hierarchical mass pattern of the heavy neutrinos \( M_1 << M_2 \). In this case, the interactions of \( N_1 \) can be in thermal equilibrium when \( N_2 \) decay is washed-out by the lepton number violating processes with \( \ell \). Thus only the decays of \( N_1 \) are relevant for generation of the final lepton asymmetry \( \epsilon \approx \epsilon_1 \). In this case, the CP asymmetry parameter in the
TABLE II: Texture Dirac mass matrices and their light neutrino mass matrices

<table>
<thead>
<tr>
<th>$m_D$</th>
<th>$m_{\nu}$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{pmatrix} x &amp; x &amp; x \ y &amp; y &amp; y \end{pmatrix}$</td>
<td>$\begin{pmatrix} 0 &amp; 0 &amp; 0 \ 0 &amp; 3y^2/M_2^2 + 4y^2/M_1^2 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>two zeros</td>
</tr>
<tr>
<td>$\begin{pmatrix} -2x &amp; x &amp; x \ -2y &amp; y &amp; y \end{pmatrix}$</td>
<td>$\begin{pmatrix} 6x^2/M_2^2 + 6x^2/M_1^2 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>two zeros</td>
</tr>
<tr>
<td>$\begin{pmatrix} 0 &amp; x &amp; -x \ 0 &amp; y &amp; -y \end{pmatrix}$</td>
<td>$\begin{pmatrix} 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \ 0 &amp; 2y^2/M_2^2 + 2y^2/M_1^2 \end{pmatrix}$</td>
<td>two zeros</td>
</tr>
<tr>
<td>$\begin{pmatrix} 0 &amp; x &amp; -x \ y &amp; y &amp; y \end{pmatrix}$</td>
<td>$\begin{pmatrix} 0 &amp; 3y^2/M_2^2 &amp; 0 \ 0 &amp; 2y^2/M_2^2 + 2y^2/M_1^2 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>NH</td>
</tr>
<tr>
<td>$\begin{pmatrix} x &amp; x &amp; x \ 0 &amp; y &amp; -y \end{pmatrix}$</td>
<td>$\begin{pmatrix} 6y^2/M_2^2 &amp; 0 &amp; 0 \ 0 &amp; 3y^2/M_2^2 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>IH</td>
</tr>
<tr>
<td>$\begin{pmatrix} -2x &amp; x &amp; x \ y &amp; y &amp; y \end{pmatrix}$</td>
<td>$\begin{pmatrix} 6y^2/M_2^2 &amp; 0 &amp; 0 \ 0 &amp; 3y^2/M_2^2 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>IH</td>
</tr>
<tr>
<td>$\begin{pmatrix} 0 &amp; x &amp; x \ y &amp; z &amp; z \end{pmatrix}$</td>
<td>$\begin{pmatrix} y(y-z)/M_1^2 &amp; 0 &amp; 0 \ 0 &amp; y(y+z)/M_2^2 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>IH, x*</td>
</tr>
<tr>
<td>$\begin{pmatrix} t &amp; x &amp; x \ 0 &amp; y &amp; y \end{pmatrix}$</td>
<td>$\begin{pmatrix} t(t+z)/M_1^2 &amp; 0 &amp; 0 \ 0 &amp; t(t+z)/M_2^2 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>IH, y**</td>
</tr>
</tbody>
</table>

In this paper we analyze the texture zeros in the neutrino Yukawa Coupling matrix $m_D$ and the heavy Majorana neutrino mass matrix $M_R$ in the context of the minimal seesaw model including 2 heavy right-handed neutrinos. We illustrate which textures are compatible with the present neutrino oscillation data and discuss their implications for the future neutrino experiments. We do not make the assumption that $M_R$ is diagonal. We examined the minimal seesaw mechanism of $3 \times 2$ Dirac matrix by starting our analysis with the masses of light neutrinos with tri/theta-maximal mixing in the basis where the charged-lepton Yukawa matrix and heavy Majorana neutrino mass matrix are diagonal. We found all possible Dirac mass textures which contain one zero entry or two in the matrix.

\[ \epsilon = \frac{1}{8\pi v^2} \ln \left[ \frac{m_D^2}{M_D^2} \right] f \left( \frac{M_D^2}{M_D^2} \right) \]  

where

\[ f(x) = \sqrt{x} \left( 1 - \frac{1}{x} \right) \ln \left( \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} \right) \]  

In the next section, we will discuss the implications of the results for future neutrino experiments. We will see how the texture zeros can be used to predict the mass hierarchy and mixing angles of the light neutrinos.

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