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Complex Potential Methods for a Crack and Three-phase Circular Composite in Anti-plane Elasticity

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Abstract

Interaction between an anti-plane crack with a three-phase circular composite by using complex potential methods is considered in this paper. The solution procedures for solving this problem consist of two parts. In the first part, based on complex potential methods in conjunction with analytical continuation theorem and alternating technique, the complex potential functions of a screw dislocation interacting with three-phase circular composites are obtained. The second part consists of the derivation of logarithmic singular integral equations by introducing the complex potential functions of screw dislocation along the crack border together with superposition technique. The logarithmic singular integral equations is then solved numerically by modeling a crack in place of several segments. Linear interpolation formulae with undetermined coefficients are applied to approximate the dislocation distribution along the elements, except at vicinity of crack tip where the dislocation distribution preserves a square-root singularity. The mode-III stress intensity factors are then obtained numerically in terms of the values of the dislocation density functions of the logarithmic singular integral equations.

Keywords: complex potential methods; anti-plane crack; three-phase circular composite; logarithmic singular integral equations; mode-III stress intensity factors

1. Introduction

Complex potential methods is a mathematic tools that has been used widely to solve elasticity problems [1]. Those include a dislocation embedded in any shapes of multilayered composite, such as: circular media [2], elliptic media [3], and plane layered media [4]. A fracture mechanics problems also can be solved using complex potential methods together with logarithmic singular integral equations [5]. The logarithmic singular integrals are established by using dislocation solutions as the Green's function in conjunction with the principle superposition. The merit of this technique is that allows us to easily deal with the interface continuity conditions of multi-layered composites. The studies that has been done using those techiques including an anti-plane crack interacting with reinforced elliptic hole [6], elliptically layered media [7], eccentric circular inclusion [8], and tri-material media [9]. To our knowledge, an anti-plane crack interacting with multi layered circular media has not been recorded in the literature. It is therefore the purpose of this paper to provide mode-III stress intensity factors of a crack in three-phase circular composite using complex potential methods together with logarithmic singular integral equations.

2. Solution Procedures

Consider a three-phase circular composite composed of three number of dissimilar materials bonded along concentric circular interfaces with a crack located in infinite matrix or in core inclusion subjected to a remote uniform shear load as shown on Figure 1. Let S1 denote the infinite matrix, S2 denote the coating layer, and S3 denote the core inclusion, respectively. The boundaries of coating layer are two circles Γ_1 and Γ_2 which are assumed to be perfect, i.e. both tractions and displacements are continuous across the two interfaces. The origin of the Cartesian coordinate system is chosen to be at the center of the inner circle Γ_2 with r unit radius and outer circle Γ_1 with R unit radius. Let the infinite matrix (core inclusion) contain a line crack with length 2a is located in x-axis with distance h from outer circle (inner circle) interface. In addition, the direction of remote uniform shear load is 90° from x-axis. The complex potential method plays an important role in anti-plane elasticity. In the method, the resultant force P and the displacement ω can be described in terms of complex potentials $\theta(z)$ which expresses in Equation 1 and 2.

$$P = \int (\tau_{xy} dy - \tau_{yz}) = -\frac{Im}{2}\left[\theta(z)\right] \qquad (1)$$

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$$\omega = \frac{1}{2\mu} Re\left[\theta(z)\right] \tag{2}$$

where Re and Im denote the real part and imaginary part of the bracketed expression, respectively. The quantities τ_{xy} and τ_{yz} are the components of shear stresses in x and y direction, respectively, (') is designated as the derivative with respect to the associated argument and μ stands for the shear modulus. Once the anti-plane problem is solved, the complex potential $\theta(z)$ is determined.

Complex potential functions for a screw dislocation and three-phase circular composite is derived by using analytical continuation theorem together with alternating techniques. The complex potential functions for a screw dislocation in matrix is described as expression Equation 3 and Equation 4.

$$\theta(z) = \begin{cases} U_{32}\sum_{n=1}^{\infty}\theta_n(z) & z \in \quad S_3 \\ \sum_{n=1}^{\infty}\theta_n(z) + V_{32}\sum_{n=1}^{\infty}\bar{\theta_n}(\frac{r^2}{z}) & z \in \quad S_2 \\ \theta_n(z) + V_{12}\overline{\theta_0}\left(\frac{r^2}{z}\right) + U_{32}V_{32}\sum_{n=1}^{\infty}\overline{\theta_n}\frac{r^2}{z} & z \in \quad S_1 \end{cases} \tag{3}$$

where,

$$\theta_n(z) = \begin{cases} U_{12}\theta_0(z) & n = 1 \\ V_{12}V_{32}\theta_{n-1}\left(\frac{r^2}{R^2}z\right) & n = 2,3,4 \end{cases} \tag{4}$$

$${}_n(z) = \begin{cases} \theta_0(z) + V_{21}\bar{\theta_0^*}\left(\frac{r^2}{z}\right) + V_{23}V_{12}\sum_{n=1}^{\infty}\bar{\theta_n}\left(\frac{r^2}{z-z_a} + \overline{z_A}\right) & z \in S_3 \\ \frac{\mu_2 b_0}{2\pi i} + log \quad z + U_{32}\theta_0^*(z) + V_{32}V_{12}\sum_{n=1}^{\infty}\theta_n\left(\frac{R^2}{\frac{r^2}{z}}\right) + V_{12}\sum_{n=1}^{\infty}\bar{\theta_n}\left(\frac{R^2}{z}\right) & z \in S_2 \\ \frac{\mu_2 b_0}{2\pi i} + log \quad z + U_{21}U_{32}\theta_0^*(z) + U_{21}V_{32}V_{12}\sum_{n=1}^{\infty}\theta_n\left(\frac{R^2}{\frac{r^2}{z}}\right) & z \in S_1 \end{cases} \tag{5}$$

$$\theta_n(z) = \begin{cases} U_{32}\bar{\theta_0^*}(z) & n = 1 \\ V_{32}V_{12}\theta_{n-1}\left(\frac{R^2}{\frac{r^2}{z}}\right) & n = 2,3,4 \end{cases} \tag{6}$$

$$\theta_n(z) = \frac{\mu_3 b_0}{2\pi i}log(z - z_t) \qquad \theta_0^*(z) = \frac{\mu_3 b_0}{2\pi i}log(1 - \frac{z_t}{z}) \tag{7}$$

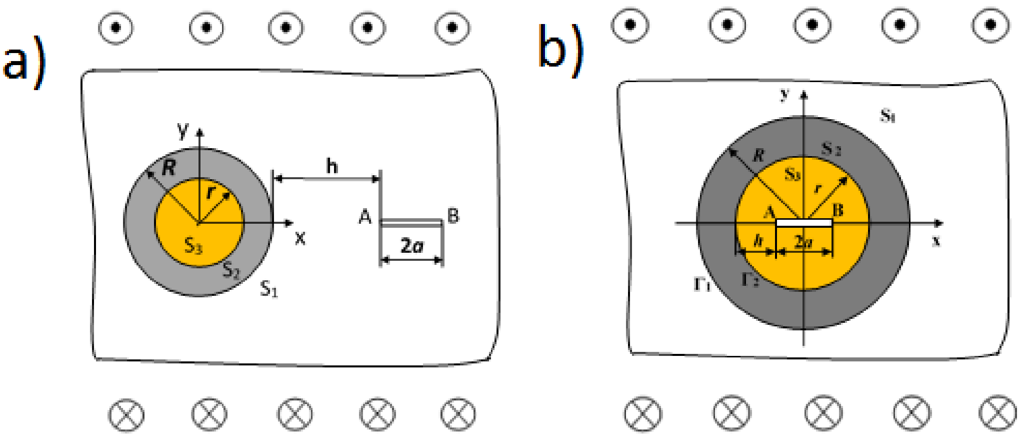

Figure 1. A three-phase circular composite interacting with (a) a crack embedded in matrix, (b) a crack located in core inclusion, under uniform shear load in anti-plane elasticity.

Meanwhile, the complex potential functions of the other case in which the screw dislocation is located in core inclusion can be expressed in Equation 5 where recurrence formula for $\Theta n(z)$ is expressed in Equation 6 and Equation 7.

A crack can be modeled by place a dislocation distribution along the prospective site of the crack. In mathematical expression, it simply makes integration along the crack and change the dislocation to dislocation density function as Equation 8 and :

$$\theta_n(z) = \frac{\mu_1}{2\pi i}\int_L b_0(s)log(z - z_t)ds \tag{8}$$

$$\theta_0^*(z) = \frac{\mu_1}{2\pi i}\int_L b_0(s)log(z - \frac{z_t}{z})ds \quad (9)$$

where $b_0(s)$ indicate the density function and z_t is a point on the crack. By placing a continuous distribution of dislocation density along the crack L and applications of principle of superposition lead to the singular integral equation with logarithmic singular Kernels. In order to solve the logarithmic singular integral, it need to used boundary element technique in which a crack is divided to several segments and performed an appropriate interpolation formula for each segment. After the dislocation density coefficients for left tip and crack tip solved numerically, then mode-III stress intensity factors can be obtained accordingly as expressed in Equation 10 and 11:

$$K_{III}(tip-A) = -\sqrt{\pi}\lim_{s_1\to 0} b_0(s_1)s_1^{\frac{1}{2}} = -\sqrt{\pi d_1 b_{0,1}} \quad (10)$$

$$K_{III}(tip-B) = \sqrt{\pi}\lim_{s_{N+1}\to 2d} b_0(s_{N+1})s_{N+1}^{\frac{1}{2}} = \sqrt{\pi d_N}b_{0,N+1} \quad (11)$$

3. Numerical Results

To prove the suggested technique and to provide more results of the mode-III stress intensity factors, several numerical results are carried out below. The results including two cases: a crack embedded in matrix and a crack located in core inclusion.

From the calculated results in Figure 2 showed that for a crack embedded in matrix, the influence of the coating layer material properties μ_1/μ_2 = 0.7, 0.9, 2, 3) are not the same. In the softer coating layer case (μ_2/μ_1 = 0.7, 0.9), the results should be expected, mode-III stress intensity factors always increase when approaching outer circular interface layer. On the other hand, in the stiffer coating layer (μ_2/μ_1 = 2, 3), the mode-III stress intensity factors should decrease when a crack approaching outer circular interface. This tendency happen because the stiffer materials tends to be a barrier for crack propagation.

In the another case for a crack located in core inclusion, Figure 3 showed that the behavior of the studied problem does not appear much different from the case of a crack embedded in matrix. Only in some particular cases, for example μ_2/μ_2 = 2 the mode-III srtress intensity factors is increasing when a crack approaching inner circular interface, apparently because softer matrix give bigger effect rather than stiffer coating layer. However, it is not happen forμ_2/μ_3 = 3 in which stiffer coating layer make the mode-III stress intensity factors decrease when a crack approaching inner circular interface.

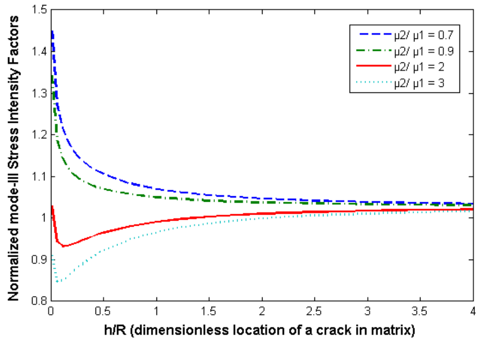

Figure 2. Normalized mode-III stress intensity factor versus dimensionless location of a crack in matrix with different $\mu2/\mu1$ for $\mu3/\mu1$ = 0.5 and r/R= 0.9.

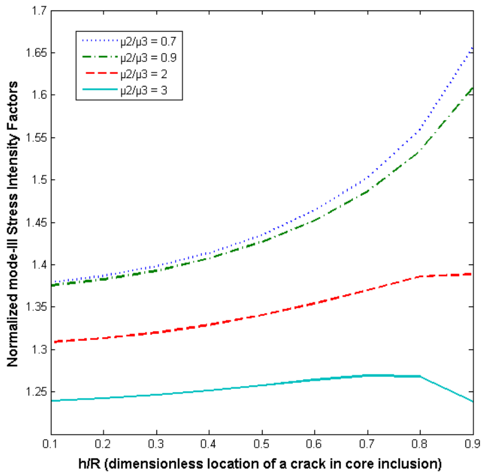

Figure 3. Normalized mode-III stress intensity factor versus dimensionless location of a crack in core inclusion with different $\mu2/\mu1$ for $\mu3/\mu1$ = 0.5 and r/R= 0.9.

4. Conlusions

Complex potential methods together with logarithmic singular integral equations has been used to solve interaction between a crack with three-phase circular composite under a remote uniform shear load. Numerical calculations are performed to investigate the effect of material properties combinations on mode-III stress intensity factors, either for a crack embedded in matrix or a crack located in core inclusion.

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