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Dynamics Analysis of Narrow Tilting Three-Wheeled Vehicle with LQG Control

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Abstract

The need for compact vehicles will increase in urban areas in the future. This paper analyses the dynamics of the actively controlled narrow tilting three-wheel vehicle (NTTWV). The vehicle has 3 wheels, 2 front wheels as steering wheels and the rear wheel as traction wheel. Active tilting system is used to give desired roll angel φ that can resist the centrifugal force to maintain the vehicle stability. The simulation result showed that the NTTWV rolls only 76% of ideal motorcycle rolling angle at same velocity and turning radius. The improvement of critical speed compared to non-tilting three-wheeled vehicle at $\delta = 5°$ and $\delta = 10°$ are 193% and 171%.

Keywords: Narrow, Tilting, Automotive, Mechanical, Modeling, Simulation, Control.

1. Introduction

The biggest problem of urban road is the increase in vehicle number. Cars have a more significant effect than the motorcycle in congestion problems. The urban population of the world is projected to be doubled from 3.3 billion in 2007 into 6.4 billion in 2050 [1]. With its larger dimension, car needs wider road area than 2-wheel vehicles. With a low ratio of passengers per car, this will also cause larger pollution per passenger.

The main advantage of a motorcycle is its small dimension that can be effectively used in crowded city road. The disadvantage of motorcycle is its safety level. With only 2 wheels, motorcycle only makes use of gyroscopic force to maintain its stability. This weakness caused motorcycle to be one of the most significant factors of accidents on the road.

There are researches in combining car and motorcycle that realized in narrow tilting vehicle (NTTWV). This vehicle is a three-wheeled vehicle than can tilt as motorcycle to resist centrifugal force when turning. With lower dimension and weight than the car, NTTWV has better efficiency while carry 2 passengers. Small dimension makes this vehicle needs smaller road area. NTTWV has better safety and comfort than the motorcycle because of 3 wheels used. With those advantages described, this vehicle is supposed to be the best compromise as urban vehicle that has some advantages such as safety, comfort and efficiency [2].

Based on wheel configuration, there are two types of three-wheel vehicle. The first type has 2 wheels in the front and 1 rear wheel as studied in [2, 3]. This configuration has the advantage in braking when cornering. The second type has 1 front wheel and 2 wheels in the front [4]. This has better stability during acceleration when the vehicle turns. The comparison of both wheel configuration is studied by comparing the critical velocity in constant steer angle [5]. Another classification of three-wheel vehicle is based on its suspension configuration. The common mechanisms are A-arm and trailing arm suspension. The A-arm has better swing motion, but it needs more joint. In other hand, the trailing arm needs simpler mechanism, but its swing motion can vary the wheel base length [6].

Narrow tilting three-wheeled vehicle required an active tilting mechanism and control to get better stability due to its small track width. Some tilting mechanisms have been developed to aim simplicity and effectiveness. There are two general types of tilting mechanism: cabin tilting and chassis tilting [6]. Chassis tilting mechanism can give more significant tilting effect than cabin mechanism. Consequently, it needs more energy and more sophisticated mechanism.

Vehicle dynamic is nonlinear dynamic that need to be simplified for simulation and control. Some researchers used Magic formula for tire modeling[2, 3, 7]. This tire model used some parameter to estimate tire characteristic. A simpler model called Dugoff model is an alternative to Magic formula model [8]. It modeled forces under

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longitudinal and lateral tire force generation. Uniform vertical pressure needs to be assumed to satisfy this model. External disturbance can also be included to make the simulation analysis more realistic such as road configuration effect[9].

Some control methods for tilting torque are used in some previous researches such as LQR control [7, 10]. The tilting system needs to ensure the vehicle to have a desired roll angle based on lateral and longitudinal acceleration. A more robust control method such as sliding mode control is used in [2, 3, 11, 12]. The goal of the control is to optimize the performance and energy efficiency.

Most studies on active control of narrow tilting vehicle designed a control system to generate ideal roll angle of the pure inverted pendulum model as in motorcycle. In fact, ideal roll is very difficult to be applied in the tilting mechanism of narrow track vehicle due to joint angle limitations. This study proposes a new control system that considers a feasible roll angle instead of the ideal roll angle. The mathematical model of the vehicle used linear tire model that is generated by estimating the nonlinear Dugoff model. LQG control is then used to control the tilting torque to aim the desired roll angle. Simulation is performed by varying roll gain K_φ to get several roll angle references $\varphi_{desired}$. The effects of this variation for the vehicle behavior are also analyzed.

2. Dynamic model

Figure 1 shows the model of the narrow tilting three-wheeled vehicle studied. This vehicle carries 2 passengers including the driver. The tilting mechanism used in this vehicle is active tilting system to ensure the vehicle has desired roll angle to resist the centrifugal force. The detailed front suspension and tilting system are shown in Figure 2 . This mechanism has some advantages. It only needs 1 input motor to actuate 2 front wheels. Another advantage is that this mechanism can ensure both right and left front wheel to have a parallel chamber angle with the body roll angle and it can maintain the suspension to be nearly independent of roll angle so the equivalent stiffness K_{eq} can be more controllable.

Figure 3 shows the simplified bicycle model to analyze the planar dynamics of the vehicle. The planar dynamic includes lateral and yaw dynamic that are modeled in eq.1 and Figure 2. In this bicycle model, m is total vehicle mass while K_f and K_r are cornering stiffness coefficients of front and rear tire. The vehicle longitudinal velocity V and the steer angle δ produce side slip angle β and roll angle φ and cause the vehicle to rotate at certain yaw rate r. Eq. 3 and eq.4 are used for determining the roll effect on tire lateral forces. K_{cf} and K_{cr} are cornering thrust stiffness coefficients that caused by suspension geometry during rolling.

$$mV\frac{\beta}{dt} + 2\left(K_f + K_r\right)\beta - \left(mV + \frac{2\left(L_fK_f + L_rK_r\right)}{V}\right)r + 2Y_\varphi\varphi = -2K_f\delta \tag{1}$$

$$-2\left(-L_fK_f + L_rK_r\right)\beta + I_z\frac{dr}{dt} + \frac{2\left(L_f^2K_f + L_r^2K_r\right)}{V}r - 2N_\varphi\varphi = 2L_fK_f\delta \tag{2}$$

$$Y_\varphi = \left(\frac{\partial\alpha_f}{\partial\varphi}K_f + \frac{\partial\alpha_r}{\partial\varphi}K_r\right) - \left(\frac{\partial\varphi_f}{\partial\varphi}K_cf + \frac{\partial\varphi_r}{\partial\varphi}K_cr\right) \tag{3}$$

$$N_\varphi = \left(\frac{\partial\alpha_f}{\partial\varphi}L_fK_f + \frac{\partial\alpha_r}{\partial\varphi}L_rK_r\right) - \left(\frac{\partial\varphi_f}{\partial\varphi}L_fK_cf + \frac{\partial\varphi_r}{\partial\varphi}L_rK_cr\right) \tag{4}$$

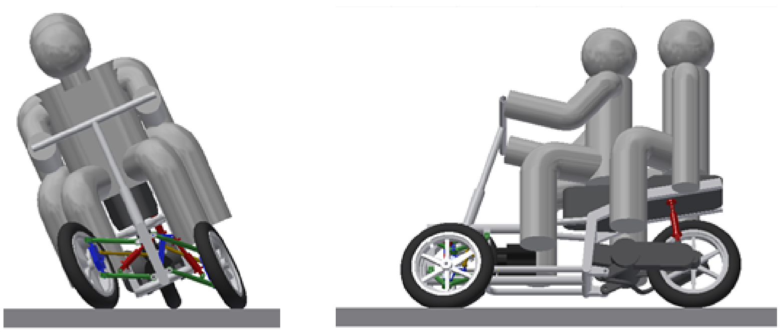


Figure 1. 3D model of narrow tilting three-wheeled vehicle (NTTWV).

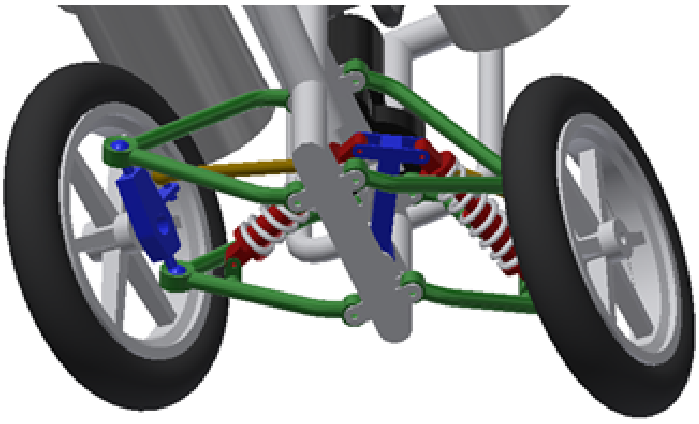


Figure 2. Front suspension and tilting mechanism.

Roll dynamic of this vehicle is modeled independently from lateral dynamic. This independence is to make the mathematical model can be formed in the standard state-space model. This roll dynamic is modeled as a rigid body mechanism in Figure 4. Eq.5 is the roll dynamic equation, where m_φ is the total mass of rolling bodies that center is placed in height h_φ . These rolling bodies include all bodies except some suspension parts. The actuator motor is added to this equation as a tilting torque $T_{tilting}$.

$$-m_\varphi h_\varphi V r + I_x \frac{d^2\varphi}{dt^2} - m_\varphi g h_\varphi \varphi = T_{tilting} \tag{5}$$

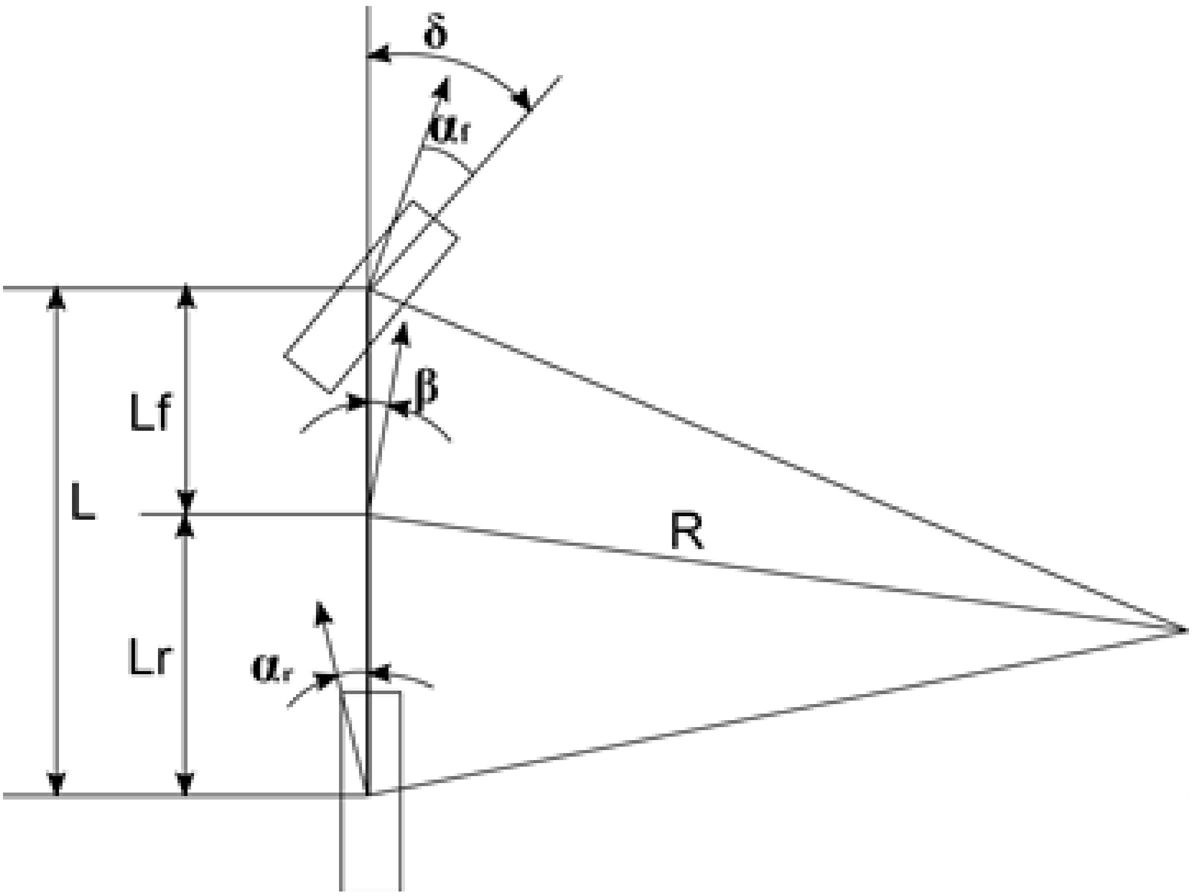


Figure 3. Bicycle model of the NTTWV.

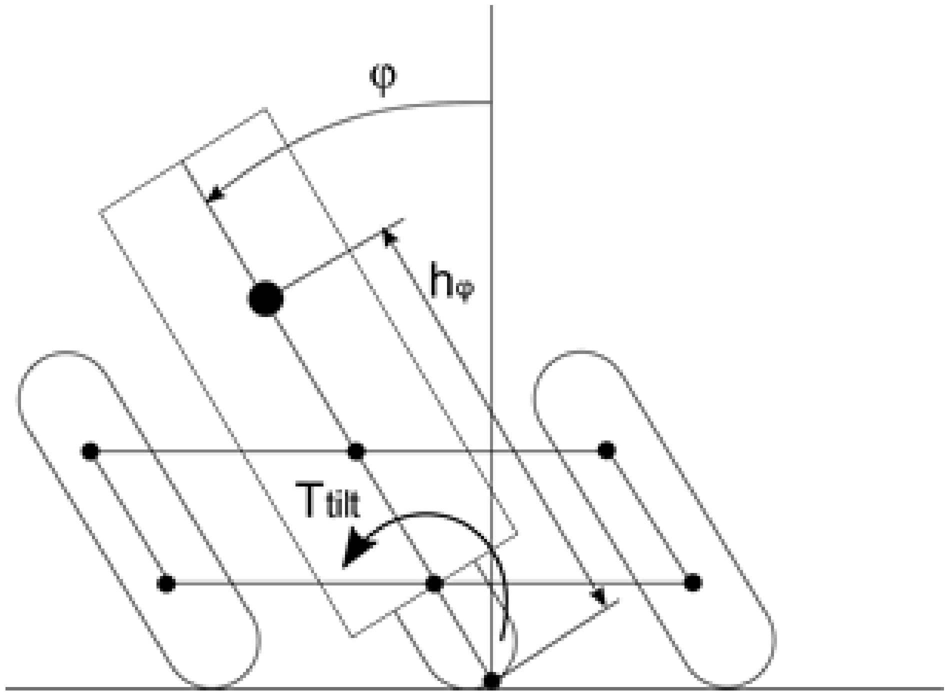


Figure 4. Roll dynamic model.

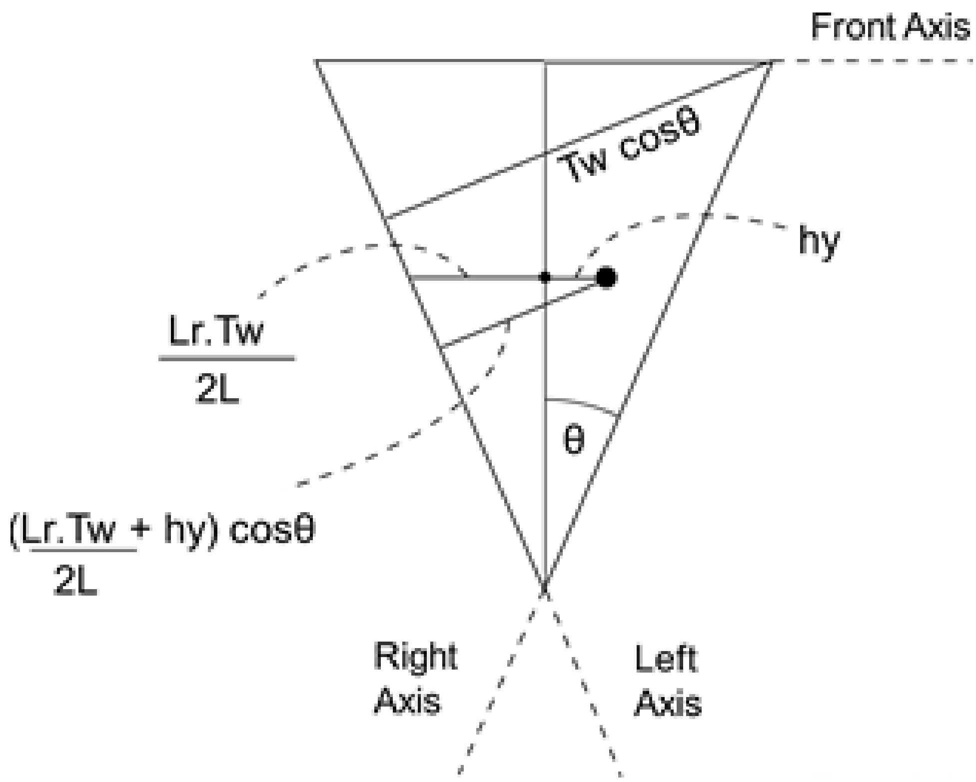


Figure 5. Vertical dynamic of the NTTWV.

3. Tire normal force

Three-wheel vehicle has unique behavior in term of generating normal force for each tire. As shown in Figure 5, moment equilibrium need to be analyzed in 3 axes. If the vehicle turns into right direction, the active-controlled vehicle will roll into right to resist the centrifugal force occur caused by turning motion. As in Eq.6, the roll angle will change the center of gravity (COG) of the vehicle to right direction calculated as h_y. In vertical direction, the height of COG changes from h into h_z. Eq.7 and eq.9 is calculated from front, left and right axes. Because the vehicle turns right, the critical vertical force F_z must be in the front-right tire F_{zfr}.

$$h_y = hsin\varphi, h_z = hcos\varphi \tag{6}$$

$$m_v g L_f + F_x h_z - F_z r L = 0$$

$$\frac{m_v g L_f + F_x h_z}{L} \tag{7}$$

$$m_v g \left(\frac{L_r T_w}{2L} + h_y\right) \cos\theta - \left(F_y \cos\theta + F_x \sin\theta\right) h_z - F_z f r T_w \cos\theta = 0$$

$$F_{zfr} = \left(\frac{m_v g \left(\frac{L_r T_w}{2L} + h_y\right) \cos\theta - \left(F_y \cos\theta + F_x \sin\theta\right) h_z}{T_w \cos\theta}\right) \tag{8}$$

$$-m_v g \left(\frac{L_r T_w}{2L} - h_y\right) \cos\theta - \left(F_y \cos\theta - F_x \sin\theta\right) h_z + F_{zfr} T_w \cos\theta = 0$$

$$F_{zfr} = \left(\frac{m_v g \left(\frac{L_r T_w}{2L} + h_y\right) \cos\theta - \left(F_y \cos\theta + F_x \sin\theta\right) h_z}{T_w \cos\theta}\right) \tag{9}$$

4. Estimating cornering stiffness from nonlinear tire model

Cornering stiffness K_f and K_r need to be estimated based on nonlinear tire model. K_f is total cornering stiffness of two font wheels while K_r is single rear tire cornering stiffness. Tire dynamic is ideally modeled in nonlinear formula to get more precision estimate. Because of its nonlinearity, it is difficult to make this nonlinear tire function in state space form. For that reason, a linear model is chosen as constant K_f and K_r based on Dugoff model. Dugoff model is the analytical approximation of tire forces [13]. This tire model formula used to model the tire forces as in eq.10 and eq.12. To calculate longitudinal force F_{xi} and lateral force F_{yi},This formula requires some variables: vertical force (F_{zi}), longitudinal stiffness (C_σ), cornering stiffness (C_α), slip ratio (σ), tire side slip (α), and friction coefficient (μ) [14, 15]. Those vertical forces are calculated from eq.7 and eq.9 that are derived from moment equation in each axis shown in Figure 5.

Figure 6 shows that the cornering stiffness coefficient K_f and K_r are estimated by averaging the nonlinear cornering stiffness coefficient from Dugoff model. Constant K_f and K_r need to be generated because the state space requires linear equations. This linearity is also important to provide controllable model. Lateral force comparison between linear and nonlinear Dugoff model is shown in Figure 7.

$$F_{xi} = C_\sigma \left(\frac{\sigma}{1+\sigma}\right) f(k) \tag{10}$$

$$F_{yi} = C_\alpha \left(\frac{\tan\alpha}{1+\sigma}\right) f(k) \tag{11}$$

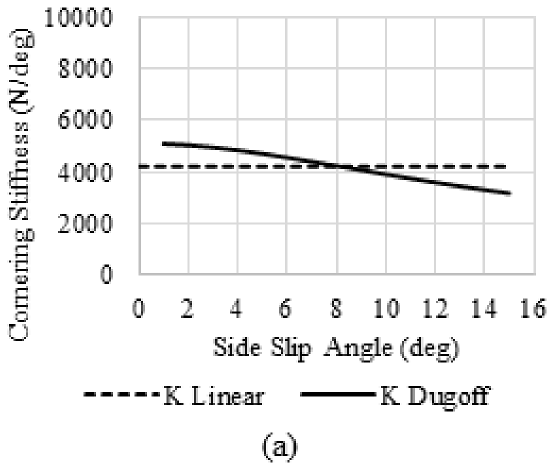
$$k = \frac{\mu F_{zi}(1+\sigma)}{2\sqrt{\left(C_\sigma \sigma\right)^2 + \left(C_\alpha \tan\alpha\right)^2}} \tag{12}$$

where

$$f(k) = (2-k)k \quad if \quad k < 1$$

and

$$f(k) = 1 \quad if \quad k \geq 1$$



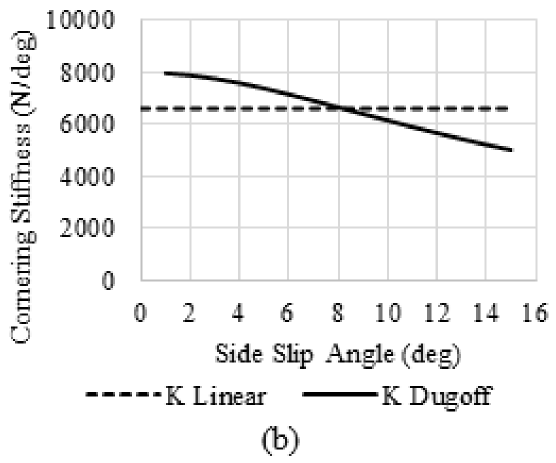
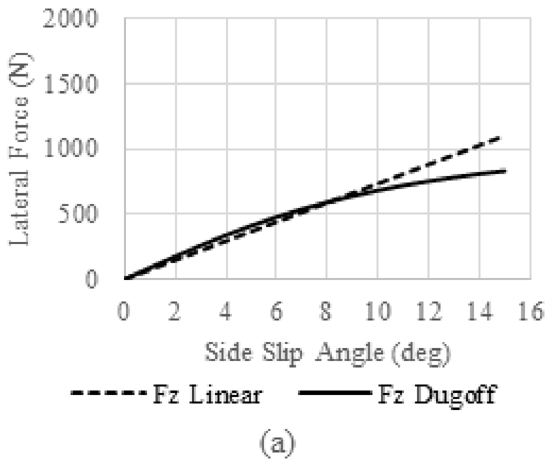


Figure 6. Cornering Stiffness of linear and nonlinear Dugoff model: (a) Front Tires; (b) Rear Tire.



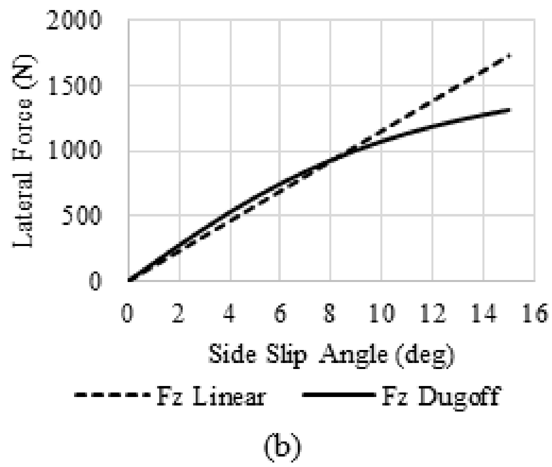


Figure 7. Lateral Force of linear and nonlinear Dugoff model: (a) Front Tires; (b) Rear Tire.

5. Controller Design

Linear Quadratic Gaussian (LQG) is used in this study to control the tilting torque. LQG combines linear quadratic estimator (LQE) or Kalman filter and Linear quadratic regulator (LQR) as in eq.13. The output sensed for control is only roll angle. The remaining output need to be estimated using LQE. All estimated outputs are regulated by LQR to determine the controlled torque for the tilting mechanism

$$\frac{d}{dt}\begin{bmatrix} x \\ \varepsilon \end{bmatrix} = \begin{bmatrix} (A - BK_{LQR}) & BK_{LQR} \\ 0 & (A - CK_{LQE}) \end{bmatrix} \begin{bmatrix} x \\ \varepsilon \end{bmatrix} + \begin{bmatrix} I & 0 \\ I & -K_{LQE} \end{bmatrix} \begin{bmatrix} w_d \\ w_n \end{bmatrix} \tag{13}$$

$$\epsilon = \chi - \hat{\chi}$$

The sensed output, roll angle, is compared to desired roll angle calculated in eq.14 and eq.15. The roll gain K_ϕ is to change ideal roll angle ϕ_{ideal} into desired roll angle $\phi_{desired}$. Ideal rolls angle is used by pure inverted pendulum vehicle such as motorcycle. The NTTWV has track width that allows it to have smaller roll angle, so the desired roll angle $\phi_{desired}$ is ideal roll angle ϕ_{ideal} multiplied by roll gain K_ϕ. The complete control diagram can be seen in Figure 8. This gain will also produce the resultant acceleration that is felt by the passenger during cornering[16]. The resultant acceleration of the vehicle is calculated in eq.16.

$$\varphi_{ideal} = \tan^{-}1\left(\frac{Vr}{g}\right) \tag{14}$$

$$\varphi_{desired} = K_{\varphi}\varphi_{ideal} \tag{15}$$

$$a_{res} = V_r \cos\phi - g\sin\phi \tag{16}$$

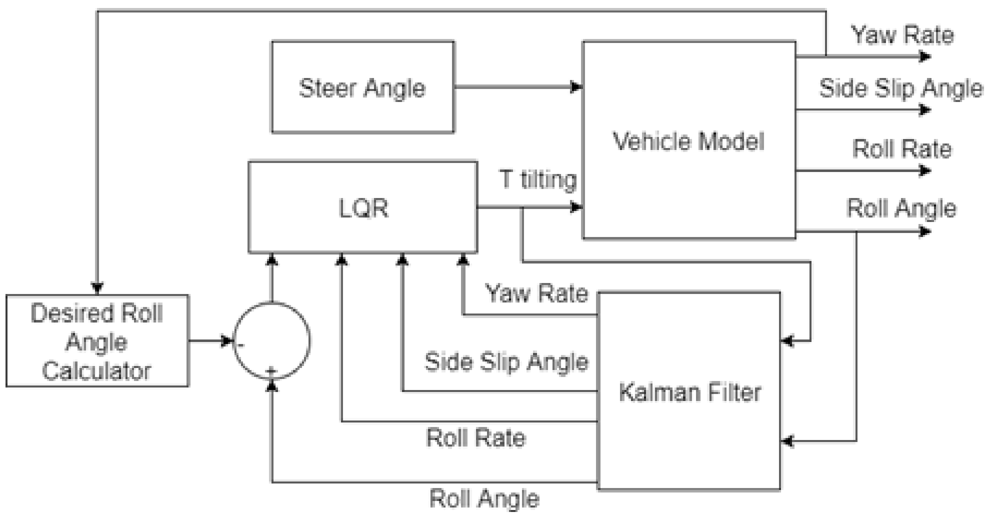


Figure 8. LQG Control Diagram for NTTWV.

6. Simulation Result

Q matrix on the LQG controller is varied during the simulation. The diagonal matrix Q has $Q_{yawrate}$, $Q_{sideslipangle}$, Q_{roll} and $Q_{rollrate}$. To obtain the effect of the variation, only Q_{roll} is varied on the Q matrix and the remaining Q is equal to 1 as shown in eq.17. The values of the variation are Q_{roll}=10; 100 and 1000. The simulation result shows that the rise time of the response is proportional with the Q_{roll} as in Figure 9.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & Q_{roll} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{17}$$

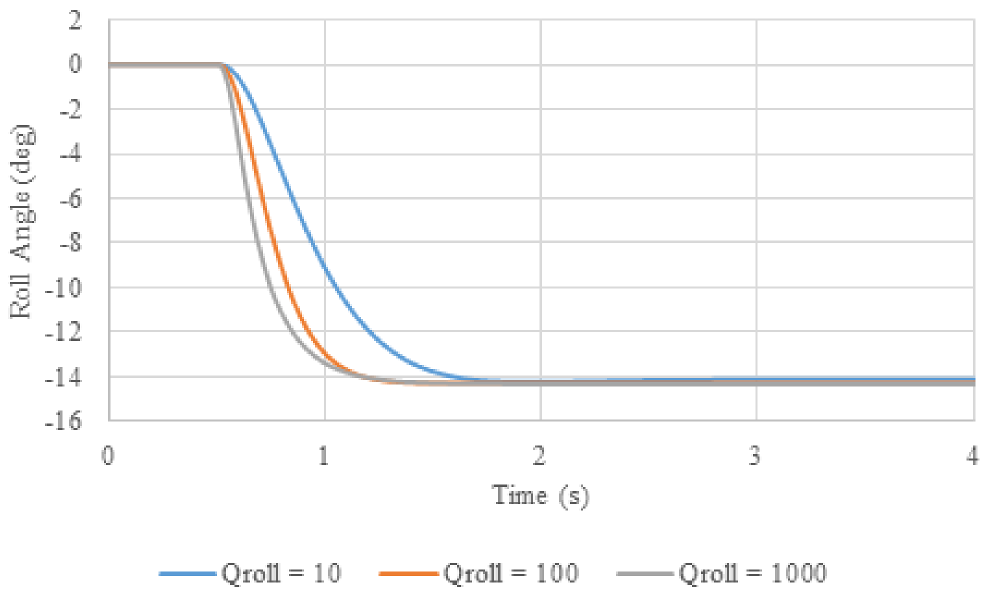


Figure 9. Roll angle response in Q_{roll} variation.

Determination of the roll gain is based on 2 parameters: roll angle limitation on A-arm joint and the ability to prevent the inner tire to be lifted from the road surface. The main force that can cause the vehicle to roll out is the lateral or centrifugal force. This force is proportional with the centrifugal acceleration. The roll angle that is needed to prevent roll out is also proportional with centrifugal or lateral acceleration. The effect of lateral acceleration and roll gain to the roll angle is shown on Figure 10. The blue area shows the allowed values of roll because this area is below 25°. The orange area shows roll angle values more than 25°.

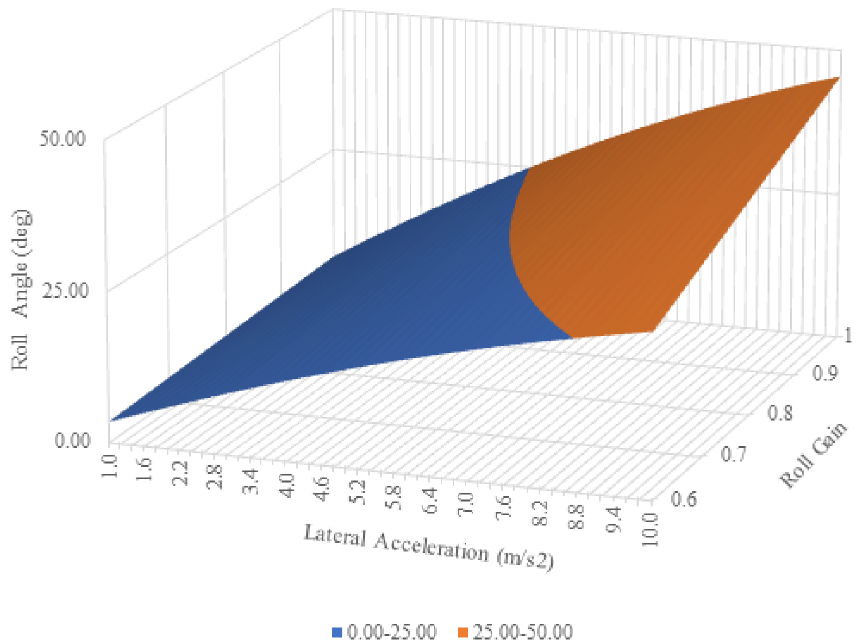


Figure 10. The effect of lateral acceleration and roll gain on roll angle

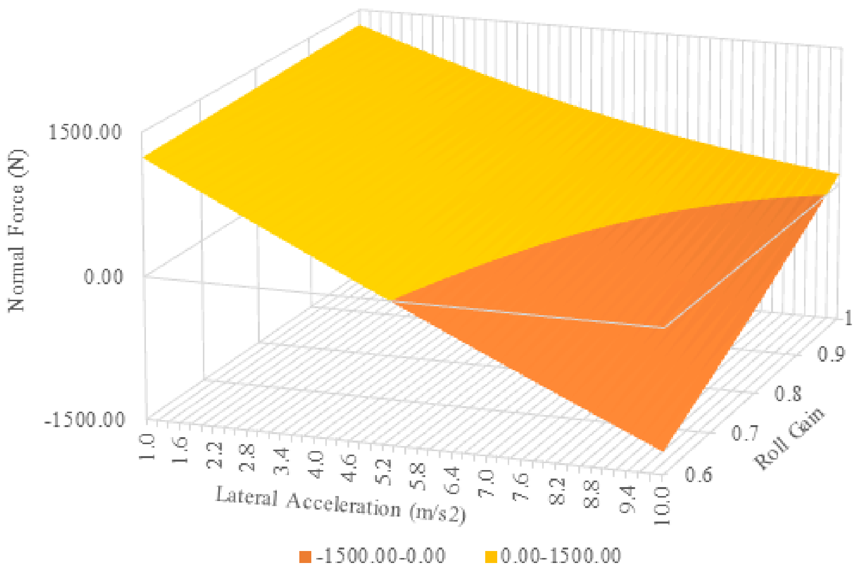


Figure 11. The effect of lateral acceleration and roll gain on normal force if the inner tire

The second parameter that is used to determine the roll gain is that the normal force of the inner tire need to be always positive. Positive value indicates that the tire is still in contact with the road surface. If the normal force is negative, it indicates that the tire is lifted and can cause the vehicle to roll out.

The yellow area on Figure 11 shows the positive values of normal force and the orange area shows the negative values of normal force. Normal force of the inner tire is inversely proportional to lateral acceleration while it is proportional to the roll gain.

The roll gain value is determined by evaluating combining to limits on Figure 10 and Figure 11. The obtained roll gain is 0.76. this value is obtained at 6.2 m/s^2 of lateral acceleration and 25° of the roll angle. At this state, roll gain 0.76 still have positive value of inner tire normal force. Hereinafter, the roll gain 0.76 is used in the remaining simulation.

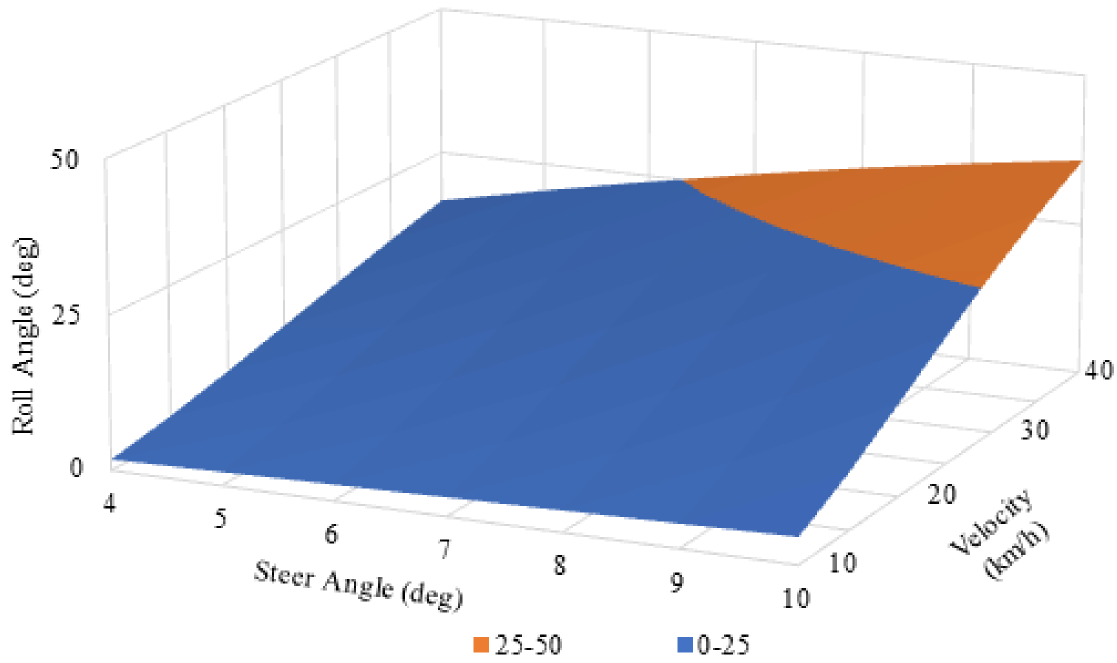


Figure 12. The effect of the steering angle and longitudinal velocity to roll angle at roll gain 0.76

The effect of the steering angle and longitudinal velocity to roll angle at roll gain 0.76 is shown in Figure 12. The roll angle is proportional to steering angle because the turning radius of the vehicle is inversely proportional to steering angle. The same relation is also found in longitudinal velocity. The roll angle of the vehicle is proportional to longitudinal velocity. This occurs because the lateral force is directly proportional to square of longitudinal velocity.

The blue area shows the allowed values of roll because this area is below 25°. The orange area shows roll angle values more than 25°. The limits of longitudinal velocity 40 km/h and steering angle 10° are chosen because these values are assumed can represent the real condition of riding in urban road.

Turning radius is the important parameter to evaluate the turning ability of a vehicle. Turning radius is not only affected by steering angle but also affected by longitudinal velocity. This correlation is caused by proportional relation between lateral force and longitudinal velocity. The lateral force directly affects the front and rear tire side slip angles.

Figure 13 shows the effect of steering angle and longitudinal velocity to turning radius at roll gain 0.76. the turning radius is inversely proportional to steering angle but the relation between these 2 parameters is not linear. On the other hand, the turning radius is proportional to longitudinal velocity. This relation indicates that the NT-TWV sustains understeer during cornering. This turning characteristic is occurred because the front side slip angle is larger than rear side slip angle.

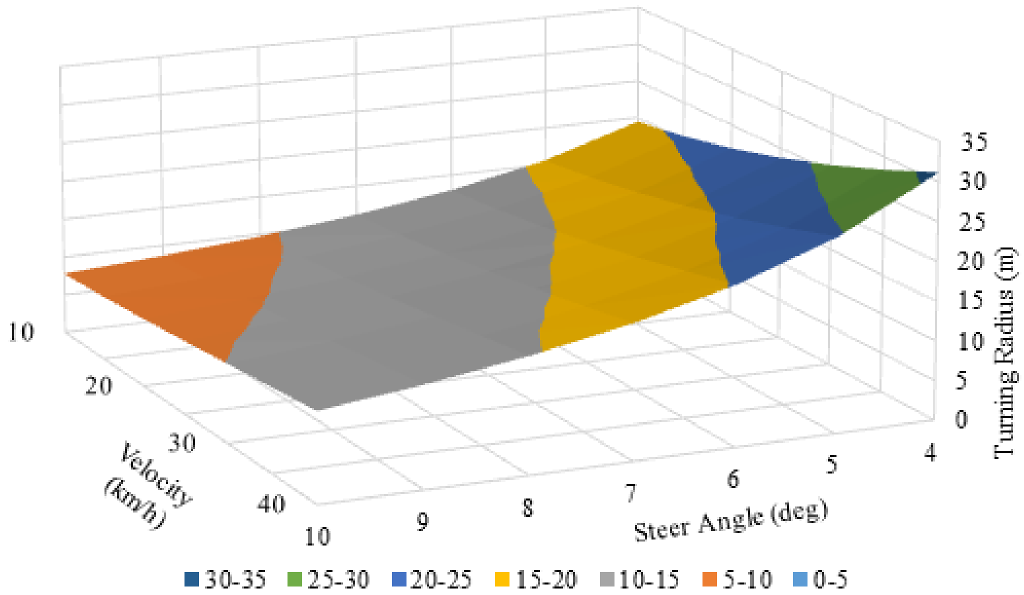


Figure 13. The effect of the steering angle and longitudinal velocity to turning radius at roll gain 0.76

The comfort aspect of the NTTWV is evaluated using resultant acceleration as shown in Figure 14. The resultant acceleration is felt by the passenger during cornering. Because of active roll angle mechanism, resultant acceleration that is felt by passenger in NTTWV is lower than car's.

The resultant acceleration is caused by the implementing of roll gain on the control system. The pure inverted pendulum vehicle such as motorcycle has zero resultant acceleration. The direction of this acceleration is perpendicular with vertical axis of roll bodies. the resultant acceleration is proportional to longitudinal velocity and steering angle. This proportional relation occurs because basically the resultant acceleration is the sum of lateral acceleration and gravity.

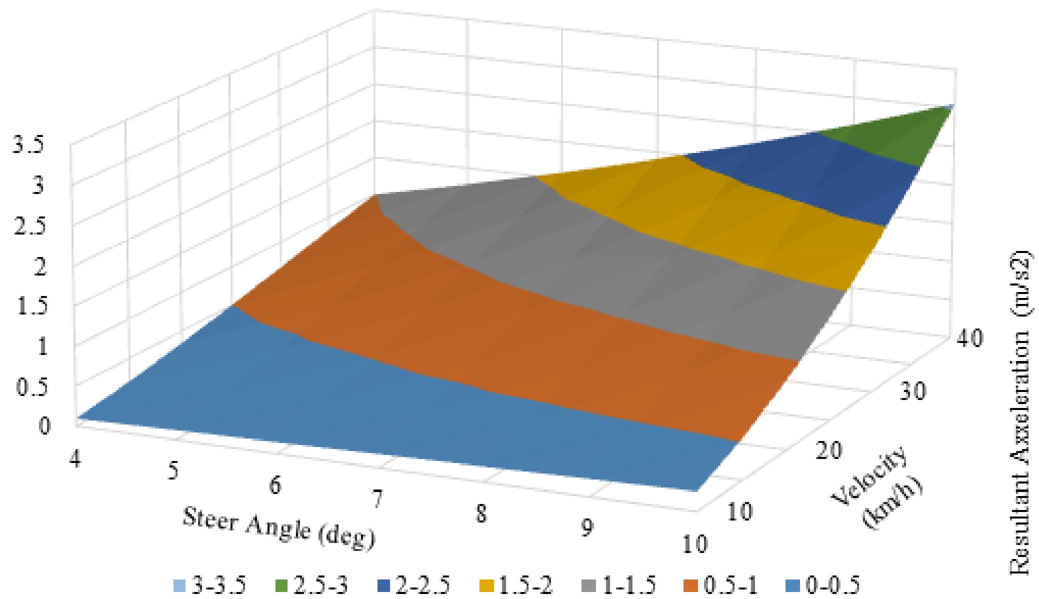


Figure 14. The effect of the steering angle and longitudinal velocity to resultant acceleration at roll gain 0.76

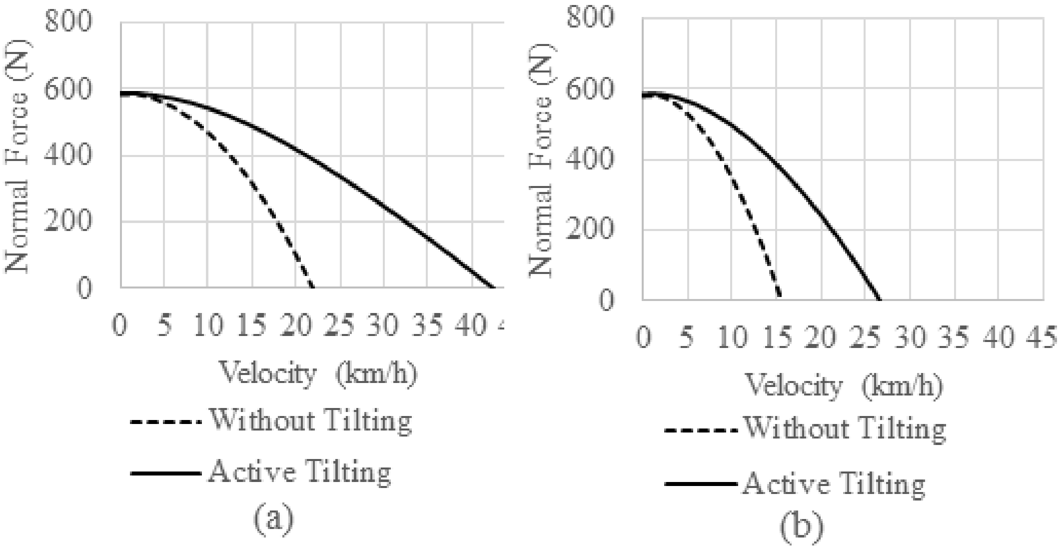


Figure 15. Inner tire normal force of non-tilting and active tilting 3-wheeled vehicle: (a) δ = 5 deg; (b) δ = 10 deg

7. Conlusions

The linear tire model is estimated from nonlinear Dugoff model to be used in this mathematical model of NTTWV. This vehicle is also validated using Autodesk Inventor Dynamic Simulation. the validation result shows that both simulation have identical curve of vehicle position. LQG control is designed to control the tilting input in tilting mechanism. The active tilting simulation showed that gain 0.76 is the best gain that optimally compromise between roll angle needed and positive normal force to ensure the inner tire (right tire) is always in contact with road surfaces. Roll gain 0.76 means that the NTTWV will roll only 76% of motorcycle rolling angle at same velocity and turning radius. The improvement of critical speed compared to non-tilting three-wheeled vehicle at $\delta = 5^\circ$ and $\delta = 10^\circ$ are 193% and 171%.

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