Numerical Procedure for Modeling Crack Closure-Induced Plasticity

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***Abstract*— Numerical procedures are utilized to predict crack closure-induced plasticity on planar surfaces. Skinner's algorithm is presented as an APDL macro command set. Procedures for controlling element size are developed to ensure the continuity of element size gradation. A loading generator is constructed using the *dim parameter, and the Newman model is explored for comparison. The analysis, based on conducted research, yields results lower than 0.05Sy.**

***Keywords*—APDL, Crack closure induced plasticity, Element size gradation control, Finite element method**

I. INTRODUCTION

Crack closure-induced plasticity, a phenomenon crucial for understanding material behaviour under cyclic loading, can be investigated through experimental or numerical approaches. While experimental studies contribute significantly to comprehension, numerical modeling has gained prominence due to the rapid advancements in numerical methods and computational tools. Although computational techniques for crack closure exist, accurate numerical analysis remains indispensable for confidence.

In the realm of numerical research, particularly using finite element methods, a central challenge arises in mapping elements to prevent the creation of discontinuities in element size gradation. This is pivotal for accurate stress predictions, as discontinuities can lead to stress concentration errors and numerical instabilities [1-2].

The cited references underscore the formidable challenge of achieving smooth size gradation in finite element analysis. A classic textbook addresses the intricacies of maintaining consistent element sizes, emphasizing the difficulties in mapping and transitioning between them [3].

Additionally, a paper introduces a formula for determining element size in planar regions, emphasizing the optimization of gradation and shape for precise simulations [4]. Another research contribution introduces an automatic mesh generation technique that prioritizes element size control and gradation, recognizing the challenges in maintaining these aspects during geometry mapping [5].

The complexity of generating meshes with smooth size gradation, especially for intricate geometries, is discussed in another paper. The necessity for robust algorithms to circumvent inaccuracies resulting from discontinuities is emphasized [6].

Choosing an appropriate algorithm for your specific problem geometry and material properties is crucial. Testing and comparing various algorithms tailored to your application is essential for identifying the most effective and efficient solution. Additionally, exploring existing mesh generation tools and libraries with built-in functionality for element size control can simplify development and provide a robust foundation.

In line with these considerations, this research endeavours to develop a straightforward algorithm for controlling element size. The comparison of results with Skinner's algorithm, Newman [7] and Skinner [8] forms a pivotal aspect of this research, aiming to contribute to the ongoing discourse on effective numerical modeling in crack closure-induced plasticity.

II. CRACK CLOSURE

In contrast to most articles related to crack propagation algorithms, which only provide a gradual increase in load (such as the previous research carried out by Berata [9]), the crack closure algorithm simulates repeated loading. This is to determine the effect of the cracked lip valve in the plastic area, which is thought to affect changes in material properties.

Crack closure is the condition where early crack closure occurs under cyclic loading. The crack closes before reaching its minimum load. This condition reduces the crack growth rate by decreasing the effective stress intensity range (see Figure 1). It is assumed that no crack growth occurs when the crack is closed, so the corrected Paris equation replaces the stress intensity range value with its effective value (the difference between maximum stress intensity and closure stress intensity).

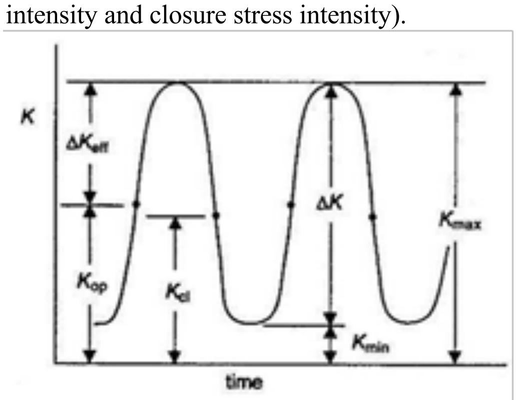

Figure 1. Effective Stress Intensity Factor [10]

Crack closure-induced plasticity results from plastic deformation from residual tensile stress left behind crack propagation or in the plastic wake region (see Figure 2). This phenomenon is interesting because plasticity is the

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only direct cause related to macro-material properties. The elastic-plastic properties of materials, and inherent constants in the material, are then formulated in the constitutive model.

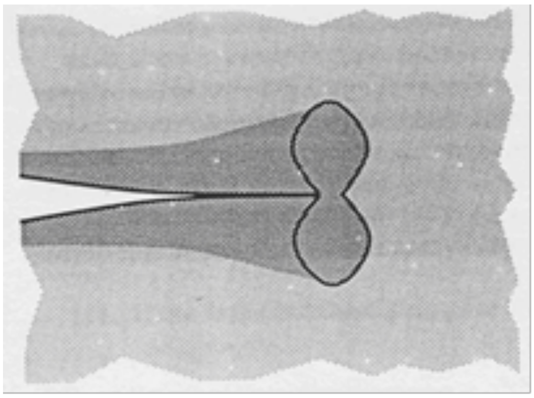

Figure 2. Early Crack Closure Due To Plasticity [10]

The study investigates the retardation effect in plasticity-induced fatigue crack growth due to micro deflections in the crack path. Finite element analysis is employed to model the crack with its kinked tip under various stress intensity factor ranges. [11]

III. CRACK MESHING

Developing a straightforward algorithm for controlling element size in the realm of plasticity-induced fatigue crack closure involves careful consideration of multiple factors. Several critical considerations come to the forefront [12]:

1. Crack geometry and loading: ensure significantly smaller element sizes near the crack tip to accurately capture localized stress and strain fields.
2. Plastic zone size: set element sizes to adequately resolve the plastic zone development and its interaction with the crack closure phenomenon.
3. Mesh transition regions: Establish smooth transitions between fine elements near the crack and coarser elements further away to avoid numerical errors.
4. Computational efficiency: balance accuracy with computational cost, especially in complex simulations with numerous elements.
5. Sharp gradients near the crack tip: implement significantly smaller element sizes near the crack tip to capture highly localized stress and strain fields effectively.
6. Plasticity effects: account for material plasticity and its impact on crack closure and opening behavior.
7. Computational efficiency (again): ensure computational efficiency, particularly for complex geometries and large models.

Potential Approaches for a Simple Algorithm [12]:

1. Distance-Based Gradation: Assign element size based on the distance from the crack tip, ensuring a smooth transition to larger sizes away from the crack. Simple but may lack precision in capturing complex stress distributions.
2. Error-Based Refinement: Begin with a coarse mesh and refine elements based on predetermined error indicators like stress gradient or strain energy density. Adapts to specific problems but may be computationally expensive.
3. Hybrid Methods: Combine distance-based gradation with error-based refinement for a balance between simplicity and accuracy. Requires careful calibration but can offer efficient mesh control.
4. Transition Zone Meshing: Define a transition zone around the crack tip with progressively smaller elements towards the crack. Ensures accuracy around the crack tip while maintaining mesh efficiency elsewhere.
5. Adaptive Mesh Refinement: Implement an adaptive meshing algorithm refining elements near the crack tip based on criteria like stress or strain gradient. More efficient than a pre-defined transition zone.
6. Size Function Based on Plasticity: Use a size function incorporating information about plastic zone size and crack closure behavior. Effective in capturing key features of plasticity-induced fatigue crack closure.

In general, the fundamental algorithm involves refining the mesh size around the crack using an elastic-plastic material model. Load (or displacement) is applied at the far end, cycling between minimum and maximum values. During a cycle, the crack node is released, extending by one element length, and plasticity occurs. The applied load, stress, and displacement at crack surface nodes are monitored, repeating over several elements (or cycles) until a stable crack opening stress is obtained. This iterative process ensures a comprehensive understanding of plasticity-induced fatigue crack closure.

IV. RESEARCH METHODOLOGY

This study commenced with the development of the Skinner algorithm in the Ansys Parametric Design Language (APDL) [13]. The research methodology is illustrated in Figure 3 and the general flow of the Skinner algorithm in Figure 4.

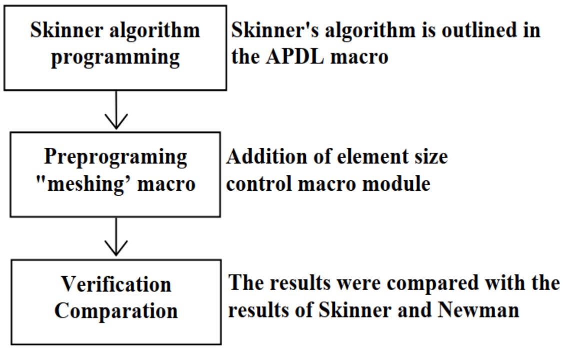

Figure 3. Research Methodology

Loading patterns were constructed using the *dim parameter, with a load ratio (R) of 0 and maximum load (Pmax) set to 150 MPa. A quarter-symmetric geometry of the Centre Crack Tension (CCT) was constructed following the ASTM E647 standard model, with a plate width (W) of 460, and an initial crack length (a) of 23. The material used was 2D Plane Strain, Bilinear Kinematic Hardening with Perfectly Elastic-Plastic behaviour, having an Elastic Modulus (E) of 70,000 MPa, Poisson's ratio of 0.3, and yield stress of 350 MPa.

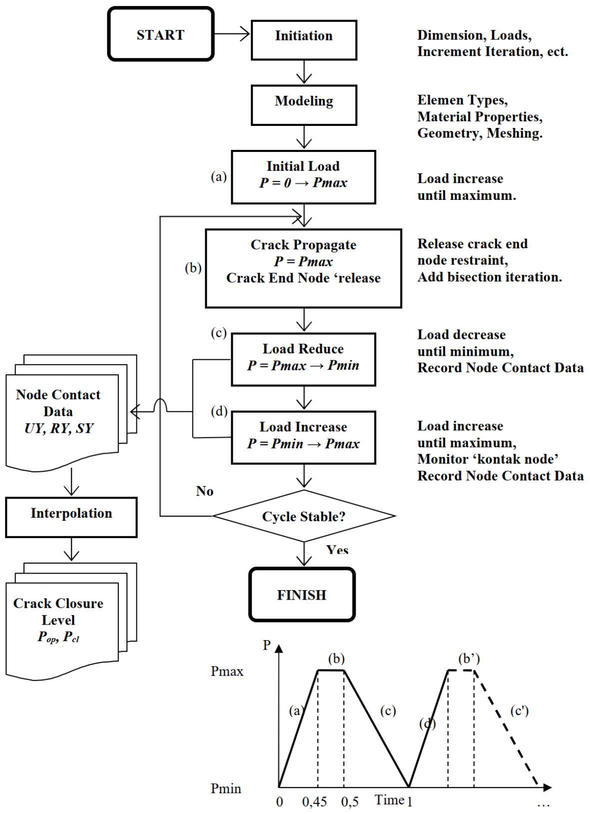

Figure 4. Skinner Program Flow

Afterward, we incorporated a macro module to control the element size. This module transformed input data, including: the number of elements (nn) and multiplication factors (k), into the size of the first (a_{first}) and last elements (a_{last}) on a line. The LESIZE command was employed within this macro to govern the element sizes that partition a given line. For example, if we input number of element nn as 10 and k ratio as 2, the line will be divided into 10 elements with the size of the last element being twice that of the first. Thus, the length of the line is the sum of all element sizes from n=1 to n=nn.

$$L = a + pa + p^2a + \cdots + p^{nn-1}a = \sum_{n=1}^{nn} p^{n-1}a \quad (1)$$

where

$p^{nn-1} = k$ nn: the number of element divider

$p = k^{1/nn-1}$ p: polynomial constant

$k = \frac{a_{last}}{a_{first}}$ k: multiplication factors

a_{first} and a_{last}: the the size of the first and last element

L: the length of the line

To obtain the value of nn, the discrete equation above is approximated with the continuous integral equation from n=0 to n=nn (see Figure 5). Therefore, the form of the integral equation becomes:

$$L = \int_0^{nn} (p^n a)\, dn = a \frac{p^n}{ln(p)}\Big|_0^{nn}$$

$$L = \frac{a}{ln(p)}(p^{nn} - 1) = \frac{a \cdot nn}{ln(k)}(k - 1) \quad (2)$$

Hence, the value of nn can be derived from:

$$nn = \frac{L}{a}\frac{ln(k)}{(k-1)} \quad (3)$$

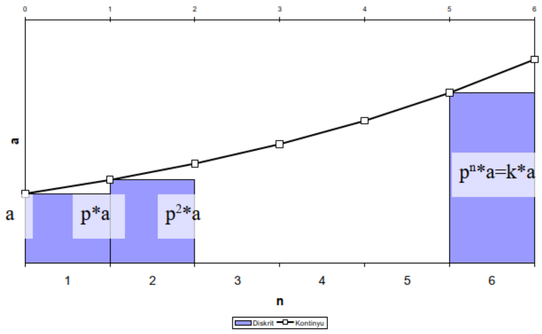

Figure 5. Approximation Of Discrete Equations

This approach exhibits the relative error (the difference between discrete and continuous equations) up to 20% for values of k>>100 and a approaching 1. At moderate values, the relative error is small (<5%). This inaccuracy is not a significant issue since, after determining the required values of k and nn, the variables for the size of the first and last elements are updated to fit the remaining length of the available line. The flowchart of the control module program can be observed in Figure 6.

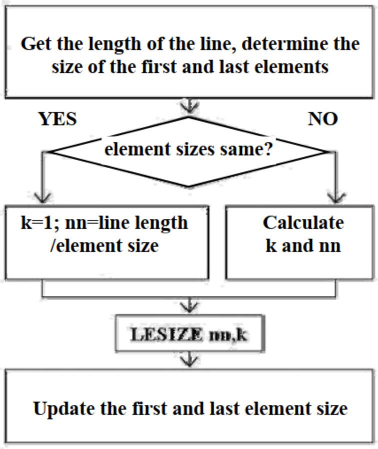

Figure 6. Flow of the Element Sizing Module

The standard specimen model is only a quarter to achieve simulation time efficiency (see Figure 7).

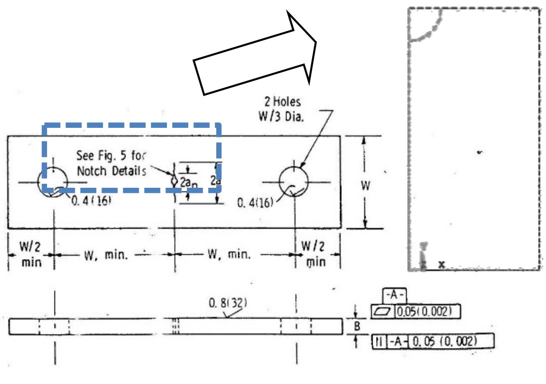

Figure 7. M(T) / CCT Standard Specimen (W<75mm) (ASTM E647)

V. Results and Discussion

The application of the element size control macro module brings about notable improvements in the shape and size of elements, as illustrated in Figure 8. This enhancement is characterized by an approaching element ratio of 1 and a reduction in discontinuous gradation.

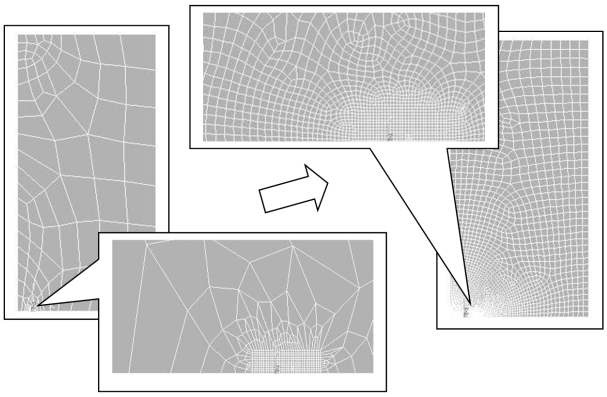

Figure 8. Composition of Elements Before and After Application of Element Size Control Modul

A thorough comparison of the opening analysis results, as illustrated in Figure 9, with the outcomes of Newman and Skinner's research reveals a notable difference in the opening level, amounting to 0.05Sy or 20%. To gain deeper insights, researchers systematically varied several crucial numerical parameters—such as the initial crack length, size of elements surrounding the crack tip, and loading steps—to discern their impact on crack opening and closure. Surprisingly, the differences resulting from these variations did not manifest visibly. This intriguing observation underscores the robustness and independence of the obtained results from the influences of the considered numerical variables

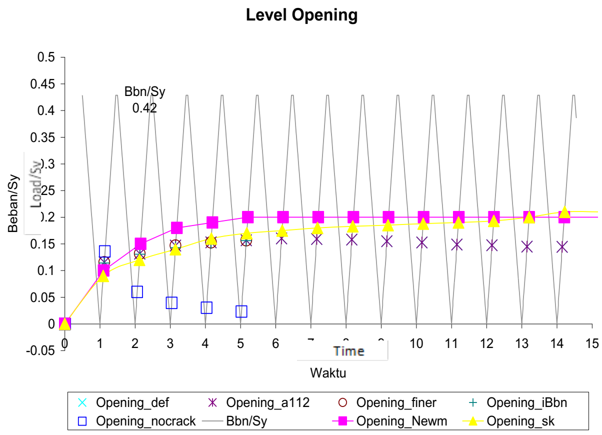

Figure 9. Comparison of Crack Opening Levels

In the context of plane strain conditions, the crack opening level, denoted by the cross symbol, stabilizes at the 10th cycle, reaching a value of 0.14 Sy. Notably, this value is lower by 0.05 Sy compared to the findings of Skinner and Newman's research. In response to this divergence, an attempt was made to refine the element size around the crack tip to 0.06 mm, equivalent to one-third of the previous size, as indicated by the circle symbol. While this refinement led to an improvement in the transient condition of crack opening levels—marked by the absence of spikes observed previously—the stable condition remained unchanged up to the 10th iteration (refer to Figure 9).

It is imperative to underscore the significant influence of the size of crack-tip elements, as it is recommended that their dimensions align with those of the cyclic plastic zone [1]. Furthermore, two critical issues warrant meticulous consideration: the level of mesh refinement and the techniques employed for crack opening value assessment [7]. Both of these factors require careful examination and precise definition to ensure accurate and reliable results.

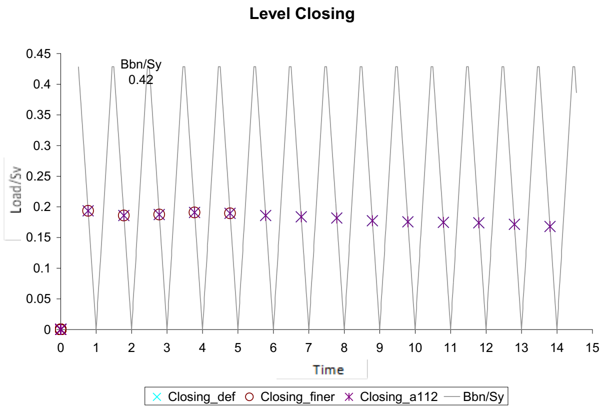

Figure 10. Crack Closure Levels

Furthermore, Figure 10 reveals that the crack opening level stabilizes at 0.15Sy after 12 loading cycles. Simultaneously, the stabilization of the crack closure level at 0.17Sy following 12 loading cycles is evident in the same figure. These findings provide a comprehensive understanding of the dynamic behavior of the crack under various conditions and highlight the intricate interplay of factors influencing crack opening and closure.

VI. Conclusion and Suggestion

The successful development of a straightforward algorithm for modeling crack closure, incorporating the element size control macro module, is a notable outcome. The analysis results, trailing Newman and Skinner by 0.05Sy, warrant further research to pinpoint the contributing factors.

The research on Modeling Crack Closure Induced Plasticity offers valuable insights into the complex dynamics of crack behaviour, shedding light on the influence of various numerical parameters and the significance of crack-tip element size. To enhance the robustness and applicability of this research, consider the following suggestions and future outlook: Refinement of Crack-Tip Element Size; Extended Parametric Studies; Integration of Advanced Meshing Techniques; Validation against Experimental Data; Incorporation of Material Plasticity Models; Development of User-Friendly Tools.

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