A New Method for Modeling Bottom Shear Stress under Irregular Waves

Taufiqur Rachman[1] and Suntoyo[2]

***Abstract*—The bottom shear stress estimation is the most important step to device an input to all the practical sediment transport models. In this paper, the modeling of bottom shear stress in a rough turbulent bottom boundary layer under irregular waves of experimental result is examined by a new calculation method of bottom shear stress based on incorporating velocity and acceleration terms simultaneously. A new acceleration coefficient is proposed to formulate the bottom shear stress under irregular waves. The new formula is further examined with a basic harmonic wave cycle modified with the phase difference and square of the instantaneous friction velocity incorporating the acceleration effect as proposed by the previous researchers. The new method gave the smallest the RMSE value indicating that the new method has the best agreement with the bottom shear stress of experimental results. Therefore, it can effectively be utilized in a beach evolution model by combining it with the irregular wave transformation model.**

Keywords—**bottom shear stress, irregular wave, and turbulent bottom boundary layer**

Abstrak*—*Estimasi tegangan geser dasar merupakan tahapan paling penting yang diperlukan sebagai inputan pada kebanyakan model transportasi sedimen. Dalam makalah ini, pemodelan tegangan geser dasar dari hasil eksperimen turbulent bottom boundary layer melalui dasar kasar untuk gelombang irreguler diuji dengan sebuah metode kalkulasi baru tegangan geser dasar yang didasarkan dengan mengkombinasikan efek kecepatan dan percepatan secara bersamaan. Sebuah koefisien percepatan baru diusulkan untuk merumuskan tegangan geser dasar pada gelombang irreguler. Formula baru ini diuji lebih lanjut dengan harmonic wave cycle yang dimodifikasi dengan beda fasa seperti diusulkan oleh peneliti terdahulu dan kuadrat instantenous friction velocity dengan mengkombinasikan efek percepatan. Metode baru memberikan nilai RMSE terkecil dan menunjukkan bahwa metode baru memiliki persetujuan terbaik dengan tegangan geser dasar hasil eksperimen. Oleh karena itu, metode baru dapat dimanfaatkan secara efektif dalam model evolusi pantai dengan mengkombinasikan terhadap model transformasi gelombang irreguler.

Kata Kunci—***tegangan geser dasar, gelombang irreguler, dan turbulent bottom boundary layer***

I. INTRODUCTION

Investigations into the bottom shear stress under a wave motion have been made by many researchers based on various kinds of turbulence model. The turbulent boundary layer induced by surface waves over a rough bed has received much attention from coastal engineers and oceanographers. Although the thickness of the wave turbulent boundary layers is quite small compared with the water depth, it still plays a very important role in determining the rate of sediment transport, the rate of wave energy dissipation, and the magnitude of bottom shear stress associated with large scale slowly varying currents. Therefore, a quantitative understanding of the mechanism of wave induced bottom boundary layers is of primary importance in predicting coastal or continental shelf processes [1]. Moreover, the bottom boundary layer in water wave propagation is important, because it determines the stress that the water transmits to the bottom, which is important in the near shore morphodynamics and ecosystems, since bottom shear stress is responsible for sediment transport [2-4].

Waves in natural coastal environment are essentially irregular and the properties in the bottom boundary layer are different from those under purely sinusoidal waves. It is therefore, appropriate to study boundary layer behavior under irregular waves to achieve the most representative estimation of bottom shear stress in coastal sediment process.

A number of models have been developed in order to calculate shear stresses under regular waves, most of them assuming the current to be slowly varying over a wave length, see e.g. in [5]. Studies on the effect of the randomness of the wave motion on the bottom friction have been made, among these are in [6-8]. Calculations of shear stresses under irregular waves plus current using Monte Carlo simulations based on parameterized models given in [9]. The boundary layer under random waves alone, as well as under random waves plus current, has been investigated using a dynamic turbulent boundary layer model. This is based upon the linearized boundary layer equations, with horizontally uniform forcing. The turbulence closure is provided by a high Reynolds number $k-\varepsilon$ model [10].

The wave boundary layer and the bottom friction studies associated with sediment movement induced by wave motion for irregular waves is very rarely done, although there, but they are mostly limited to a smooth bed condition for example [11] and [12], which are very different from an actual situation on a sea bottom with roughness bed. Studies on the bottom shear stress on rough bed conditions under irregular waves have been carried out through experimental and proposed a new the estimation method to determine the bottom shear stress [13], but once the results are not so good agreement with

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the experiment. Recently, also has conducted studies of shear stress under irregular waves but more likely to emphasize based on the experimental method with smooth and rough bed conditions [14].

Moreover, for a predictive near-shore morphological model, a more efficient approach to calculate the bottom shear stress is needed for practical applications rather than a more complex approach using a two-phase model [15]. The new calculation method of bottom shear stress in a rough turbulent bottom boundary layer under sawtooth and asymmetric waves have investigated previously through incorporating velocity and acceleration terms provided form the instantaneous wave friction velocity in [16] and [17], respectively. The value of acceleration is obtained from the average value of calculated from experimental results as well as the turbulent boundary layer model results of bottom shear stress.

Bottom shear stress estimation is the most important step as an input to all the practical sediment transport models. Therefore, the estimated bottom shear stress based on the approach used both sinusoidal and non-linear wave should be evaluated involving with the irregularity form effect under irregular wave which is the common flow condition on the seabed for shallow and intermediate water depths, i.e. in coastal zones and on continental shelves. Hereafter it is envisaged that wave boundary layers and bottom shear stress behaviors influenced by the effect of acceleration in irregular wave are different from those in sinusoidal, cnoidal, sawtooth and solitary waves.

The aim of this study is examine the bottom shear stress through experiments in an oscillating wind tunnel over rough bed under irregular waves by means of Laser Doppler Velocimeter (LDV) to measure velocity distribution, as well as turbulent boundary layer numerical model. Furthermore, a new estimation method of the instantaneous bottom shear stress under irregular waves based on incorporating both velocity and an acceleration term is proposed, so it can be obtained a more reliable calculation method to calculate the instantaneous bottom shear stress required as input to sediment transport model.

For the 1-D incompressible unsteady flow the equation of motion within the boundary layer can be expressed as follow,

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau}{\partial z} \tag{1}$$

where u is the instantaneous horizontal velocity, ρ is water density, and p is pressure. At the axis of symmetry or outside boundary layer $u=U$, therefore,

$$\frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + \frac{1}{\rho}\frac{\partial \tau}{\partial z} \tag{2}$$

By introducing the eddy viscosity model, the total shear stress for turbulence flow can be expressed as:

$$\frac{\partial \tau}{\rho} = (v + v_t)\frac{\partial u}{\partial z} \tag{3}$$

where v_t is the eddy viscosity describing the Reynolds stress and v is the kinematic viscosity. Substitution of equation (3) into equation (1) gives the simplified equation for the turbulent flow motion in the bottom boundary layer,

$$\frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial z}\left((v + v_t)\frac{\partial u}{\partial z}\right) \tag{4}$$

For practical computations, turbulent flows are commonly computed by the Navier–Stokes equations in averaged form. However, the averaging process gives rise to the new unknown term representing the transport of mean momentum and heat flux by fluctuating quantities. In order to determine these quantities, turbulence models are required. Two-equation turbulence models are complete turbulence models that fall in the class of eddy viscosity models (models which are based on a turbulent eddy viscosity are called as eddy viscosity models). Two transport equations are derived describing transport of two scalars, for example the turbulent kinetic energy k and its dissipation ε. The Reynolds stress tensor is then computed using an assumption, which relates the Reynolds stress tensor to the velocity gradients and an eddy viscosity. While in one - equation turbulence models (incomplete turbulence model), the transport equation is solved for a turbulent quantity (i.e. the turbulent kinetic energy, k) and a second turbulent quantity is obtained from algebraic expression. In the present paper shear stress transport (SST) k-ω model was used to evaluate the new acceleration coefficient, a_c, and to compare with the experimental data.

Turbulence models can be used to predict the turbulent properties under any wave's motion. The shear stress transport (SST) k-ω model is one of the two-equation turbulence models proposed by [18]. Shear stress transport (SST) k-ω model is a mixed form of the robust formulation of the k-ω model in the near-wall region, with the k-ε model in the outer part of boundary layer. The SST k-ω model is claimed to be more accurate and reliable for wider class of flow than the standard k-ε model as well as the original k-ω model, including the improvement of prediction for adverse pressure gradient flow. In the SST k-ω model the definition of eddy viscosity is modified to account for the transport effects of the principal turbulent shear stress. The SST k-ω model produces slightly lower eddy viscosities than the base line (BSL) k-ω model on flat for zero pressure gradient boundary layers.

The SST k-ω model was used to determine some unknown quantities in equation (4). The SST k-ω model is a two-equation model that gives results similar to the k-ω model of Wilcox in the inner of boundary layer but changes gradually to the Jones-Launder k-ε model towards to the outer boundary layer and the free stream velocity. In order to be able to perform the computations within one set of equations, the Jones-Launder model was first transformed into the k-ω formulation. The blending between the two regions is done by a blending function F_1 changing gradually from one to zero in the desired region.

The functions F_1 and (1- F_1) are multiplied by the original k-ω model of Wilcox and the transformed k-ε model of Launder, respecttively and both are added together. In the near the wall the function F_1 is designed to be one for activating the originnal k-ω model of Wilcox, while in the outer region of boundary layer is to be zero for activating the k-ε model of Jones Launder.

Original $k-\omega$ model:

$$\frac{\partial k}{\partial t}=\frac{\partial}{\partial z}\left\{(\nu+\nu_t\sigma_{k\omega 1})\frac{\partial k}{\partial z}\right\}+\nu_t\left(\frac{\partial u}{\partial z}\right)^2-\beta^*\omega k \tag{5}$$

$$\frac{\partial \omega}{\partial t}=\frac{\partial}{\partial z}\left\{(\nu+\nu_t\sigma_{\omega 1})\frac{\partial \omega}{\partial z}\right\}+\gamma_1\frac{\omega}{k}\left(\frac{\partial u}{\partial z}\right)^2-\beta_1\,\omega^2 \tag{6}$$

Transformed $k-\varepsilon$ model:

$$\frac{\partial k}{\partial t}=\frac{\partial}{\partial z}\left\{(\nu+\nu_t\sigma_{k\omega 2})\frac{\partial k}{\partial z}\right\}+\nu_t\left(\frac{\partial u}{\partial z}\right)^2-\beta^*\omega k \tag{7}$$

$$\frac{\partial \omega}{\partial t}=\frac{\partial}{\partial z}\left\{(\nu+\nu_t\sigma_{\omega 2})\frac{\partial \omega}{\partial z}\right\}+\gamma_2\frac{\omega}{k}\left(\frac{\partial u}{\partial z}\right)^2-\beta_2\,\omega^2+$$

$$2\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial z}\frac{\partial \omega}{\partial z} \tag{8}$$

Both equations (13) and (14) are multiplied by F_1 whereas both Equations (15) and (16) are multiplied by (1- F_1) and then the corresponding equations of each set are added together to give the new model known as the BSL k-ω model. The new governing equations of the transport equation for turbulent kinetic energy k and the dissipation of the turbulent kinetic energy ω from the SST k-ω model as mentioned before are,

$$\frac{\partial k}{\partial t}=\frac{\partial}{\partial z}\left\{(\nu+\nu_t\sigma_{k\omega})\frac{\partial k}{\partial z}\right\}+\nu_t\left(\frac{\partial u}{\partial z}\right)^2-\beta^*\omega k \tag{9}$$

$$\frac{\partial \omega}{\partial t}=\frac{\partial}{\partial z}\left\{(\nu+\nu_t\sigma_{\omega})\frac{\partial \omega}{\partial z}\right\}+\gamma\frac{\nu+\nu_t}{\nu_t}\left(\frac{\partial u}{\partial z}\right)^2-\beta\omega^2+$$

$$2(1-F_1)\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial z}\frac{\partial \omega}{\partial z} \tag{10}$$

$$\nu_t=\frac{k}{\omega} \tag{11}$$

where, $\sigma_{k\omega}$, β^*, σ_ω, γ, and β are model constants, F_1 is a blending function.

In the SST k-ω model the definition of eddy viscosity is modified to account for the transport effects of the principal turbulent shear stress. The new definition of eddy viscosity is as follows,

$$\nu_t=\frac{a_1 k}{\max(a_1\omega, F_1|\partial u/\partial z|)} \tag{12}$$

where F_1 is defined as,

$$F_1=\tanh\left(\arg_2{}^2\right) \tag{13}$$

with,

$$\arg_2=\max\left(\frac{2\sqrt{k}}{0.09\omega z},\frac{500\nu}{y^2\omega}\right) \tag{14}$$

The SST k-ω model produces slightly lower eddy viscosities than the BSL k-ω model on flat plate for zero pressure gradient boundary layers. In order to recover the distribution of the diffusion term constant in the near wall the model constants had to be adjusted for Set 1 i.e. $\sigma_{k\omega 1}$=0.85, $\sigma_{\omega 1}$=0.65, β_1=0.075 and $\gamma_1=\beta_1/\beta\text{-}\sigma_{k\omega 1}\,K^2/\beta^{1/2}$= 0.469. Set 2 constants remain unchanged.

In the numerical method, the non-linear governing equations of the boundary layer for each turbulence models were solved by using a Crank-Nicolson type implicit finite-difference scheme. In order to achieve better accuracy near the wall, the grid spacing was allowed to increase exponentially. In space 100 and in time 7200 steps per wave cycle were used. The convergence was achieved through two stages, the first stage of convergence was based on the dimensionless values of u, k and ω at every time instant during a wave cycle. Second stage of convergence was based on the maximum wall shear stress in a wave cycle. The convergence limit was set to 1.10^{-6} for both the stages. Full description of the numerical technique, boundary conditions and model parameters are provided in [16, 17].

II. METHOD

The experiments were performed by [13] in wind tunnel at Laboratory of Environmental Hydrodynamics Tohoku University Japan which has a length of 5 m and the height and width of the cross section are 20 cm and 10 cm, respectively. The dimension of this cross-section of wind tunnel has been considered in order to the flow velocity was not influence by the sidewall effect. The experiments have been carried out in an oscillating wind tunnel connected with the piston system with air as the working fluid and smoke particles as tracer. This is intended to make an easy treatment if it is compared with water as the working fluid.

A schematic diagram of the experimental arrangement is shown in Figure 1. The experimental system consists of two major components, namely an oscillatory flow generation unit and a flow-measuring unit. The oscillatory flow generation unit was made up of signal control and processing components along with piston mechanism. The piston displacement signal has been fed into the instrument through a PC. The Bretschneider–Mitsuyasu spectrum was used to generate an input signal in this experiment. Input digital signal has been converted to corresponding analog data through a digital-analog (DA) converter. A servomotor, connected through a servomotor driver, was driven by the analog signal. The piston mechanism has been mounted on a screw bar, which was connected to the servomotor. The feed-back on piston displacement, from one instant to the next, has been obtained through a potentiometer that compared the position of the piston at every instant to that of the input signal, and subsequently adjusted the servomotor driver for position at the next instant. The measured flow velocity record was collected by means of an A/D converter with 1/100 s intervals, and the mean velocity profile variation was obtained by averaging over 50 wave cycles. According to [19] at least 50 waves cycles are needed to successfully compute statistical quantities for turbulent condition.

The flow measuring unit comprised of a wind tunnel and one component LDV for flow measurement. Velocity measurements were carried out at 20 points in the vertical direction at the center part of the wind tunnel by means of LDV. The aluminum balls roughness having a diameter of 10 mm (a roughness height, H_r= 10 mm), similar used ideas by [20], was pasted over the bottom surface of the wind tunnel without spacing along the wind tunnel, as shown in Figure 2.

The randomness in ocean waves is due to the presence of numerous component waves of different amplitudes and frequencies those are contained in wave spectrum. The wave spectrum represents the spreading of wave energy over different frequency ranges. In the present analysis the spectral density for irregular wave water surface elevation, $S\eta(f)$ has been computed using Bretschneider-Mitsuyasu spectral density formulation in the following equation (15),

$$S_\eta(f) = 0.257\, H_{1/3}^2\, T_{1/3}\, (T_{1/3} f)^{-5} \exp\{-1.03 (T_{1/3} f)^{-4}\} \tag{15}$$

where, $H_{1/3}$, and $T_{1/3}$ are significant wave height and period respectively, and f is frequency of component waves.

Applying small amplitude wave theory following relationships can be obtained for spectral densities of water surface elevation and free stream velocity, as shown in equations (16) and (17),

$$S_U(f) = H_U^2(f)\, S_\eta(f) \tag{16}$$

$$H_U(f) = \frac{\omega}{\sinh 2\pi h / L} \tag{17}$$

where, S_U (f) is spectral density for free stream velocity, H_U (f) is velocity transfer function, h is water depth, and L and ω ($=2\pi f$) are wave length and angular frequency of component waves respectively.

Equation (17) represents that when the frequency of component wave is increased, the wave length will also increase resulting in a smaller value for velocity transfer function. It means that the velocity spectrum is less influenced by high frequency component waves than that from corresponding water surface elevation.

Obtained velocity spectrum has been used to generate velocity time variation with the approximation that irregular waves can be resolved as a sum of infinite number of regular wavelets with small amplitudes and random phases, as shown in equations (18) and (19),

$$U(t) = \sum_i A_{Ui} \cos(2\pi f_i t + \phi_i) \tag{18}$$

$$A_{Ui} = 2\sqrt{S_U(f)\,\Delta f_i} \tag{19}$$

where, U (t) is instantaneous free stream velocity, A_{Ui} are velocity amplitudes of component waves, f_i are component frequencies, t is time, ϕ_i are component phases and Δf_i are frequency increments between successive wave components.

The definition sketch for irregular wave is given by Holthuijsen as shown in Figure 3 [21]. There are two possible of the wave heights, namely zero down crossing height (measure from a trough to following crest), H_d and zero up crossing height (measure from a crest to following trough), H_u, and wave periods, T_d and T_u. The averages are invariant with respect to the choice of up crossing versus down crossing: $\overline{H_u} = \overline{H_d}$ and $\overline{T_u} = \overline{T_d}$. The average zero crossing periods for a record is often referred to as $\overline{T_z}$ ($=\overline{T_u} = \overline{T_d}$).

Experiments have been carried out only one case under irregular waves. The experimental conditions are given in Table 1.

Reynolds number is calculated using equation (20), to obtain rough bed turbulent flow was set $Re_{1/3}$=5.10^5 to reach a fully turbulent regime, and $T_{1/3}$ = 3.0 s as input wave in this experiment.

$$\mathrm{Re}_{1/3} = \frac{U_{1/3}^2}{\nu \omega_{1/3}} \quad , \; \omega_{1/3} = \frac{2\pi}{T_{1/3}} \tag{20}$$

where, $U_{1/3}$: flow velocity based on parameter of significant wave, $T_{1/3}$: significant wave period, and ν : kinematics viscosity. Moreover, an experiment with R_e=5.10^5 was carried out under sinusoidal wave motion, to investigate the effects of irregularity, where the Reynold number is defined by equation (21).

The condition of the actual experiment is plotted in flow regime proposed by [21], in Figure 4. Here, a horizontal axis is a Reynolds number shown in the following equation (7),

$$R_e = \frac{\hat{U}_w a_m}{\nu} \quad , \; a_m = \frac{U_{1/3}\, T_{1/3}}{2\pi} \tag{21}$$

where, $\hat{U}_w$ is the maximum of the wave-induced velocity just outside the boundary layer, a_m is the excursion length of a water particle under wave motion.

Here, a_m/k_s is the roughness parameter, k_s is the Nikuradse's equivalent roughness defined as $k_s=30z_o$, which is assumed to be equal to the diameter of the roughness element (the aluminum balls diameter of 1 cm), z_o is the roughness height, and S ($=U_{1/3}/(\omega_{1/3}\, y_h)$) is the reciprocal of the Strouhal number, and y_h is the distance from the wall to the axis of symmetry of the measurement section. The diagram is extended to irregular wave motion using the Reynolds number and angular frequency defined by equation (20) as representative quantities. It can be concluded that the condition of the present experiment lies in the rough turbulent regime according to the Reynolds number defined in terms of significant wave.

However, because of the irregularity of the input signal, there are waves with smaller Reynolds numbers. Then, the crest phase or the trough phase of the free stream velocity is regarded as a half cycle of wave motion, and the Reynolds number Re_p is defined by equation (22) for individual waves.

$$\mathrm{Re}_p = \frac{U_p^2}{\nu \omega_p} \; , \quad \omega_p = \frac{\pi}{T_p} \tag{22}$$

where U_p: the maximum velocity during crest or trough phases, and T_p: the period of crest or trough phases. Furthermore, the shape of waves at the free stream velocity, U in this case is shown in Figure 5. Figure 6 shows the time-variation of acceleration.

III. RESULTS AND DISCUSSION

A. Bottom Shear Stress of Experimental Results

Bottom shear stress is estimated by using the logarithmic velocity distribution given in equation (23), as follow,

$$u = \frac{U^*}{\kappa} \ln\left(\frac{z}{z_o}\right) \quad (23)$$

where, u is the flow velocity in the boundary layer, κ is the von Karman's constant (= 0.4), z is the cross-stream distance from theoretical bed level ($z = y + \Delta z$) (Figure 2). For a smooth bottom $z_o = 0$, but for rough bottom, the elevation of theoretical bed level is not a single value above the actual bed surface. The value of z_o for the fully rough turbulent flow is obtained by extrapolation of the logarithmic velocity distribution above the bed to the value of $z=z_o$ where u vanishes. The temporal variations of Δz and z_o are obtained from the extrapolation results of the logarithmic velocity distribution on the fitting a straight line of the logarithmic distribution through a set of velocity profile data at the selected phases angle for each case. These obtained values of Δz and z_o are then averaged to get z_o=0.09 cm. The bottom roughness, k_s can be obtained by applying the Nikuradse's equivalent roughness in which $z_o=k_s/30$. By plotting u against $ln(z/z_0)$, a straight line is drawn through the experimental data, the value of friction velocity, U^* can be obtained from the slope of this line and bottom shear stress, τ_o can then be obtained from equation (24).

$$U^* = \sqrt{\tau_o / \rho} \quad (24)$$

The obtained values of Δz and z_o, as the above mentioned, has a sufficient accuracy for application of logarithmic law in a wide range of velocity profiles near the bottom. Figure 7, showing the logarithmic law, has been approved within the wide range in the near bottom region at the selected phases of velocity profile. Figure 8 shows the time-variation of bottom shear stress under irregular waves.

B. Calculation Method of Bottom Shear Stress under Irregular Waves

In this paper, a new calculation method is proposed to compute the bottom shear stress under irregular waves and the existing calculation method as proposed by [12] and Nielsen [3, 22] are also given.

1. A new calculation method of bottom shear stress under irregular waves

The new calculation method of bottom shear stress under irregular waves is based on incorporating velocity and acceleration terms all at once that is given through the instantaneous friction velocity, $U^*(t)$ as proposed by [16, 17] in equation (25). Both velocity and acceleration terms are adopted from the calculation method proposed by [22]. The phase difference was determined from an empirical formula for practical purposes. The instantaneous friction velocity can be expressed as:

$$U^*(t) = \sqrt{f_w/2}\left\{U\left(t+\frac{\varphi}{\sigma}\right)+\frac{a_c}{\sigma}\frac{\partial U(t)}{\partial t}\right\} \quad (25)$$

The instantaneous bottom shear stress can be calculated proportional to the square of the proposed instantaneous friction velocity, as shown in equation (26).

$$\tau_o(t) = \rho U^*(t)\left|U^*(t)\right| \quad (26)$$

In the new calculation method, a new acceleration coefficient, a_c is determined empirically from both experimental and shear stress transport (SST) $k-\omega$ numerical model results of bottom shear stress using following relationship as shown in equation (27). Here, the value of acceleration coefficient, a_c = 0.485, is obtained from average value of the time variation of acceleration coefficient $a_c(t)$ calculated from experimental result as well as the SST $k-\omega$ numerical model results of bottom shear stress, and is using to expressed irregularity form effect under irregular wave.

$$a_c(t) = \frac{U^*(t)-\sqrt{f_w/2}\ U\left(t+\frac{\varphi}{\sigma}\right)}{\frac{\sqrt{f_w/2}}{\omega}\ \frac{\partial U(t)}{\partial t}} \quad (27)$$

where, f_w: the wave friction coefficient. The friction coefficient proposed by [21] as given in Equation (28) can be used for evaluating in equation (25). $\tau_o(t)$: the instantaneous bottom shear stress, and φ: the phase difference between free stream velocity and bottom shear stress.

$$f_w = \exp\left\{-7.53+8.07\left(\frac{a_m}{z_o}\right)^{-0.100}\right\} \quad (28)$$

The phase difference obtained from measured data under irregular wave, as well as from a sinusoidal wave experiment. The results are shown in Figure 9, in which the triangles indicate the estimation by equation (29) for individual waves.

Although the measurements are slightly lower than equation (29), this difference is negligible. The estimation changes between 20.0 deg. and 25.7 deg. With the mean value of 21.3 deg. whereas the use of the quantities for significant waves yields 20.4 deg. From equation (29), which is very close to the averaged value shown earlier. Thus, it is advisable to use constant phase difference, which can be obtained from significant wave quantities, instead of calculating for individual waves.

$$\varphi = 42.4\, C^{0.153}\frac{1+0.00279\, C^{-0.357}}{1+0.127\, C^{0.563}} \text{ (degree)} \quad (29)$$

$$C = \frac{1}{\kappa\sqrt{\frac{f_w}{2}\frac{a_m}{z_0}}} \text{ for rough bed condition} \quad (30)$$

$$\varphi_1 = \frac{\alpha T}{2}\frac{4\varphi}{T} \text{ (degree)} \quad (31)$$

Figure 10 show the time variation of friction velocity from experimental and the new calculation method results incorporating velocity and acceleration terms as expressed in equation (25).

It can be seen that the contribution of acceleration term have a good agreement with the time variation of friction velocity from experimental.

2. Comparison with existing calculation methods

The new calculation method of bottom shear stress under irregular waves is examined by the existing calculation methods that had been used to examine experimental results. Method 1 is proportional to the square of the time variation of *U(t)*, that of within a basic harmonic wave cycle modified by the phase difference is proposed by [12] in equation (32), as follows:

$$\tau_o\left(t-\frac{\varphi}{\sigma}\right) = \frac{1}{2}\rho\, f_w\, U(t)\left|U(t)\right| \tag{32}$$

Where τ_o (t), the instantaneous bottom shear stress, t, time, σ, the angular frequency, $U(t)$ is the time history of free stream velocity, φ is phase difference between bottom shear stress and free stream velocity and f_w is the wave friction factor where f_w is calculated from equation (28).

Method 2 is proportional to the square of the instantaneous wave friction velocity, $U_*(t)$ incorporating the acceleration effect as proposed by [22] in equations (33) and (34), as follow:

$$U_*(t) = \sqrt{f_w/2}\left\{\cos\varphi\, U(t) + \sin\varphi\,\frac{\partial U(t)}{\partial t}\right\} \tag{33}$$

$$\tau_o(t) = \rho\, U_*(t)\left|U_*(t)\right| \tag{34}$$

This method is based on the assumption that the steady flow component is weak (e.g. in a strong undertow, in a surf zone, etc.).

Phase difference equation given in equation (31) is used for calculating in Method 1 and Method 2. Friction coefficient used in Method 2 is calculated from an equation in equation (35) as proposed by Nielsen [5], as follows:

$$f_w = \exp\left\{5{,}5\left(\frac{a_m}{k_s}\right)^{-0{,}2} - 6{,}3\right\} \tag{35}$$

Correlation between the bottom shear stress of experimental result and the calculation results from three calculation methods are shown in Figure 11. The new method gives the best agreement with the bottom shear stress under irregular waves from experimental results than others method. While, Method 1 and Method 2 gave underestimated value at though part and overestimated value at crest part of bottom shear stress from experimental results, as show in Figure 11.

Comparison among the experimental data, SST k - ω turbulence model and calculation methods for bottom shear stress estimation under irregular waves are given in Figure 12. The new method could predict well the bottom shear stress showing the best agreement with the experimental results along a wave cycle under irregular wave than other methods and SST k - ω turbulence model. Method 2 has given underestimated and overestimated values of the bottom shear stress with the experimental data especially value at trough part and crest part, respectively. While, SST k - ω model and method 1 was not so much in a good agreement with the experimental results along a wave cycle under irregular wave due to was not exclude the velocity and acceleration effect in the calculation of the bottom shear stress.

C. Performance of Calculation Methods of Bottom Shear Stress

The calculation method of bottom shear stress can be evaluated by the root-mean-square error *(RMSE)*, as follows:

$$RSME = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(U_{*cal.} - U_{*\exp.}\right)^2} \tag{36}$$

where, $U_{*cal.}$ is the friction velocity from calculation methods. $U_{*\exp.}$ is the friction velocity from experimental results, N is the total number of data and i is index. If the calculation method is perfect, it can be indicated that the *RMSE* should be zero. It can be concluded that the smaller *RMSE* is better the performance of the calculation methods. The summary of calculation method performance of bottom shear stress is shown in Table 2.

As shown in Table 2 that the new method has highest performance than others methods with RMSE = 1.95. The new Method is better than Method 1 and Method 2. The new method gave the smallest the RMSE value indicating that the new method has the best agreement with the bottom shear stress of experimental results. It can be concluded that the new method can be used to estimate the bottom shear stress under irregular waves and also the phase difference and acceleration coefficient that have been defined in equation (31) and a_c=0.485 were sufficient for this calculation. Therefore, the new method can be used to calculate the bottom shear stress under irregular waves that can be further used to an input sediment transport model under rapid acceleration in practical application.

IV. CONCLUSION

The modeling of bottom shear stress under irregular waves has been investigated. The main results are summarized as follows:

1. The new method of estimating bottom shear stress under irregular waves has shown the best agreement with the experimental data. A new method for calculating the instantaneous bottom shear stress under irregular waves proposed in this study has a sufficient accuracy, so it may be considered as a reliable calculation method which is required as input to sediment transport model under rapid acceleration in a practical application.
2. The phase difference defined based on significant wave is sufficient for this purpose. Furthermore, both the phase difference and the acceleration coefficient defined in the new method were sufficient for this calculation.

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TABLE 1.
EXPERIMENTAL CONDITIONS FOR IRREGULAR WAVES

Exp.	$U_{1/3}$ (cm/s)	$T_{1/3}$ (s)	R_e	a_m/k_s	S
1	392.348	3.0	5.10^5	69.38	18.73

TABLE 2.
THE SUMMARY OF CALCULATION METHOD PERFORMANCE OF BOTTOM SHEAR STRESS

Exp.	The Root-Mean-Square Error *(RMSE)*		
	Method 1	Method 2	New Method
Case 1	8.69	3.07	1.95

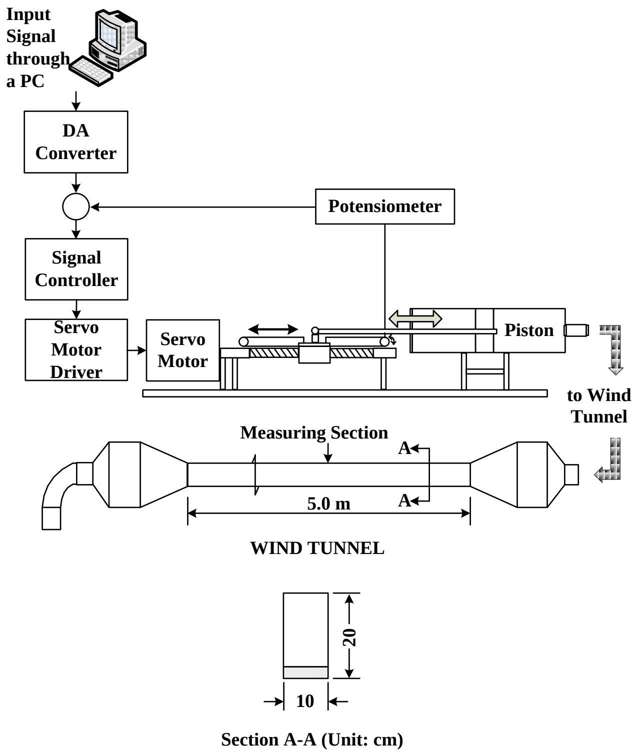

Figure 1. Schematic diagram of experiment system

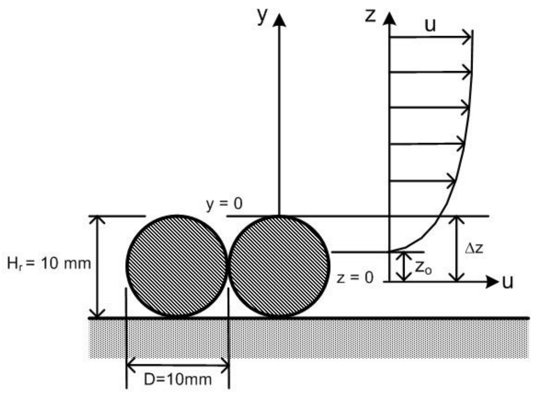

Figure 2. Definition sketch for roughness

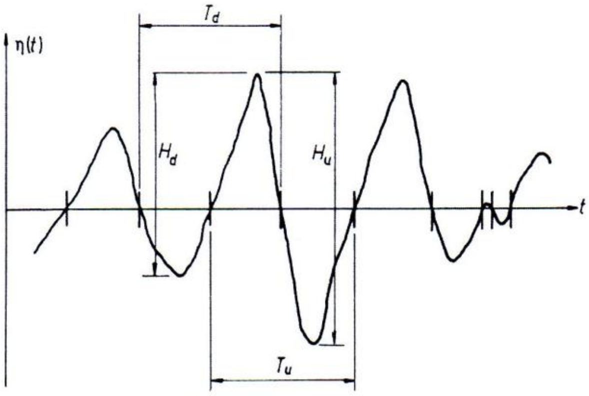

Figure 3. Definision sketch for irreguler wave

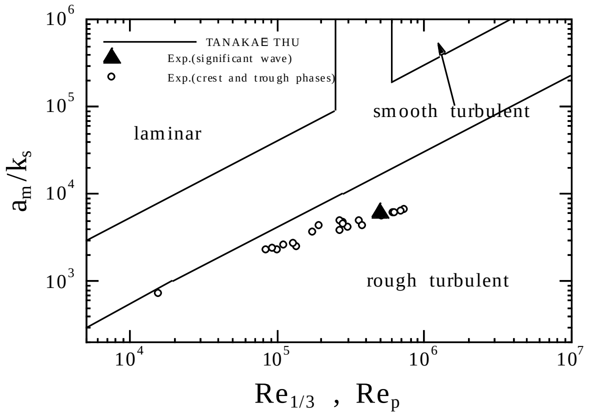

Figure 4. Flow regime

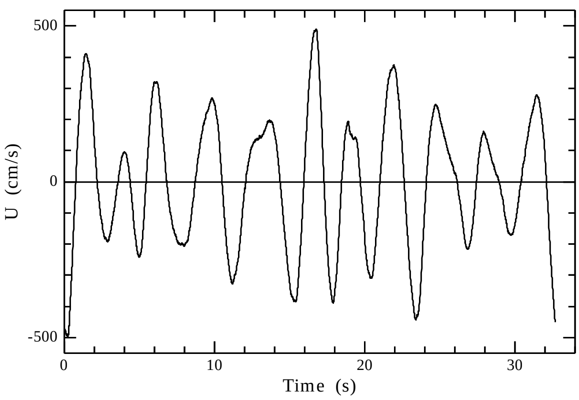

Figure 5. Time-variation of free stream velocity

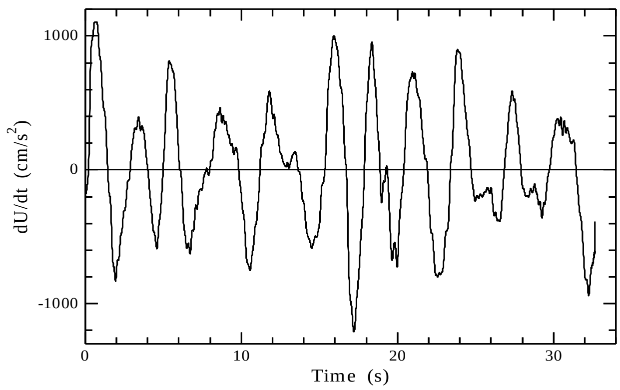

Figure 6. Time-variation of acceleration

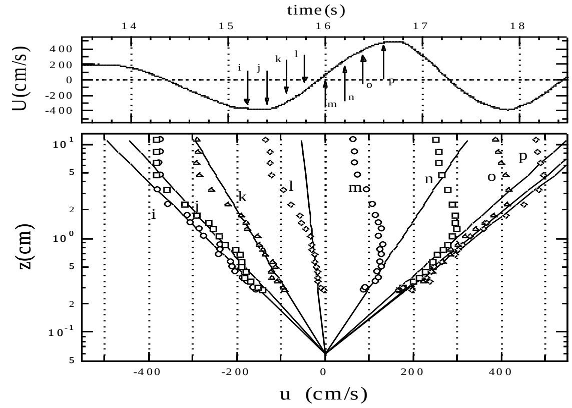

Figure 7. Log-fitting to measured velocity profile

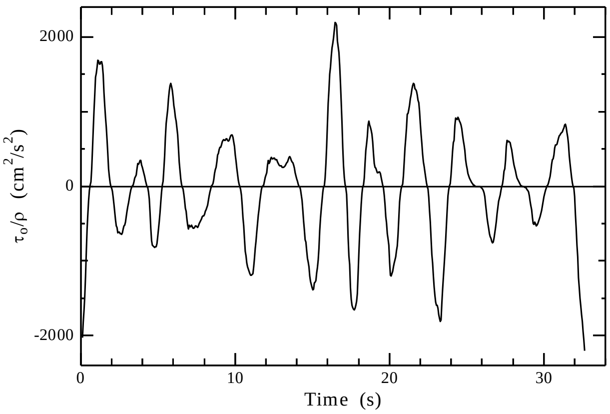

Figure 8. Time-variation of bottom shear stress of experimental results

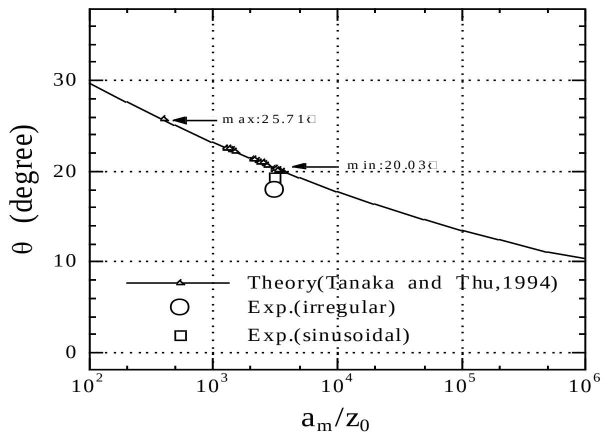

Figure 9. Phase difference

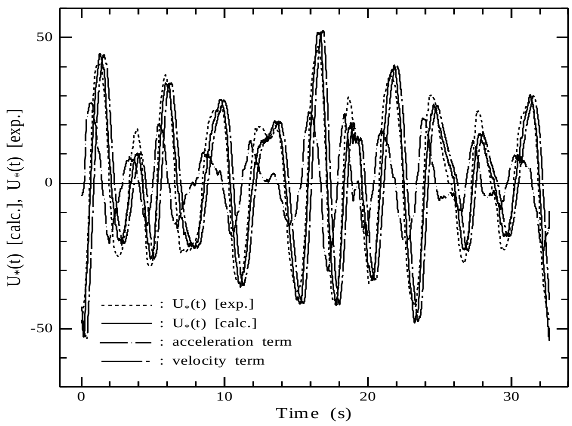

Figure 10. Time-variation of friction velocity

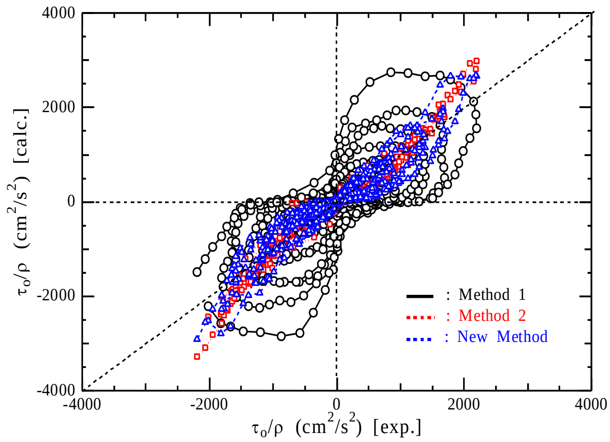

Figure 11. Correlation between experimental and calculation results of bottom shear stress

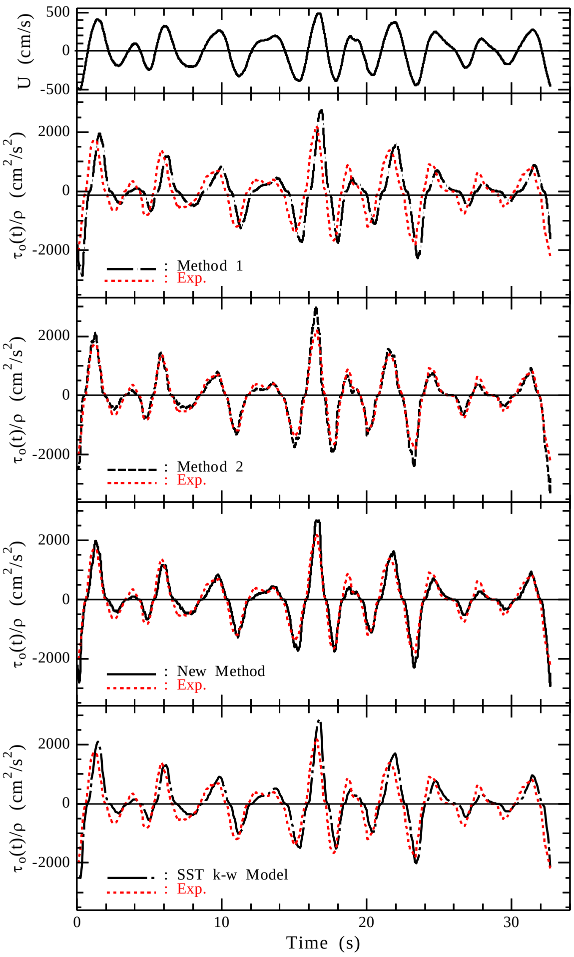

Figure 12. Comparison for bottom shear stress estimation