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Characterizations of 2-Primal Ternary Semiring using Special Subsets of Ternary Semiring

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Abstract

This research aims to determine the characterizations of 2-primal ternary semiring using special subsets of ternary semiring. We use literature review method to achieve these aims. We define $O'(P)$ and O'_P, the special subsets of ternary semiring S then we determine some properties of them. We also determine the condition for $O(P)$ and O_P in order to the special subsets are ideals of S. The last, the special subsets of S will be used to determine the characterizations of 2-primal ternary semiring. As the results, some the characterizations were S must be a commutative super nilpotent ternary semiring and $O(P) = \overline{O(P)}$ for each prime ideal P of S. Besides that, $O(P) = O_P = N(P)$ and O'_P must has the insertion of factors property or IFP for each prime ideal P of S.

***Keywords**: prime ideal; ternary semiring; 2-primal ternary semiring*

1 Introduction

The concept of semiring was introduced by Vandiver [1], as a generalization of ring. Lister initiated the concept of algebraic system of ternary ring [2], whereas the concept of ternary semiring was introduced by Dutta and Kar [3]. A ternary semiring S is a non-empty set S together with a binary operation, called addition and a ternary multiplication if S is an additive commutative semigroup and satisfy the conditions : $(abc)de = a(bcd)e = ab(cde), (a + b)cd = acd + bcd,$ $a(b + c)d = abd + acd,$ and $ab(c + d) = abc + abd$ for all $a, b, c, d, e \in S$. Ternary semiring S is called commutative if $abc = bac = bca$ for all $a, b, c \in S$. If there exist element $0 \in S$ such that $0 + a = a = a + 0$ and $0ab = a0b = ab0 = 0$ for all $a, b \in S$ then 0 is called the zero element. In this case, we say that S is a ternary semiring with zero. An element $e \in S$ is called the identity element if $eea = eae = aee$ for all $a \in S$ [3]. In this case, we say that S is a ternary semiring with identity. In this paper, S will always denote a ternary semiring with zero and identity.

Many other authors, as well, have worked on ternary semirings, like concepts on ternary semirings [4], special elements of a ternary semiring [5], soft ternary semirings [6], power ternary semirings [7], regular ternary semirings [8], completely regular ternary semiring [9], completely p-regular ternary semiring [10], and weakly special radical class and special radical class of ternary semirings [11]. Besides that, many authors also had research about ideal in ternary semiring like ideals in the ternary semiring of non-positive integers [12], ideal theory in the ternary semiring $\mathbb{Z}_0^-$ [13], singular ideals of ternary semirings [14], weakly prime left ideals and weakly regular ternary semirings [15], fuzzy prime ideals in ternary semirings [16], essential ideals of a ternary semiring [17], subtractive extension of ideals in ternary semirings [18], structure of certain ideals in ternary semirings [19], a-ideals in ternary semirings [20], different types of prime bi-ideals in ternary semirings [21], various tri-ideals in ternary semirings [22], and properties of k-hybrid ideals in ternary semiring [23].

The concept of 2-primal ternary semiring was introduced by Dutta and Mandal [24]. They defined the subsets $N(P), \overline{N(P)}, N_P, \overline{N}_P$ and $O(P), \overline{O(P)}, O_P, \overline{O}_P$ of ternary semiring S for a prime ideal P of S as [25] did in case of rings. They used those subsets to determine characterizations of 2-primal ternary semiring. In this paper, we will define the special subsets $O'(P)$ and O'_P of S then we will determine the characterizastions of 2-primal ternary semiring using them.

2 Literature Review

In this section, we will give some related definitions, propositions, and theorems. The definitions of ideal, prime ideal, and semiprime ideal of ternary semiring S was introduced by Dutta and Mandal in [24]. The definition of ideal of ternary semiring S involved any two elements of S and an element of additive subsemigroup of S. The definition of prime ideal of ternary semiring S involved any three subsets of S, whereas the definition of semiprime ideal of ternary semiring S involved any a subset of S, and can be written mathematically in the following definitions.

Definition 1. [24] *An additive subsemigroup P of a ternary semiring S is called a ideal of S if $s_1s_2a, s_1as_2, as_1s_2 \in P$ for any $a \in P$ and $s_1, s_2 \in S$.*

Definition 2. [24] *A proper ideal P of a ternary semiring S is called a prime ideal of S if $ABC \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$ for any ideals A, B, C of S.*

Proposition 3. [24] *Let S be a ternary semiring and P be a proper ideal of S. Then P is a prime ideal of S if and only if $aSSbSSc \subseteq P$ implies $a \in P$ or $b \in P$ or $c \in P$.*

Definition 4. [24] *A proper ideal P of a ternary semiring S is called a semiprime ideal of S if $A^3 \subseteq P$ implies $A \subseteq P$ for any ideal A of S.*

Definition 5. [24] *A proper ideal P osf a ternary semiring S is called a completely semiprime ideal of S if $a^3 \in P$ implies $a \in P$ for any $a \in S$.*

Dutta and Mandal also defined m-system and IFP of ternary semiring S in [24]. The definitions of them and theorem of m-system were given as follows.

Definition 6. [24] *A non-empty subset T of a ternary semiring S is called an m-system if for each $a, b, c \in T$ there exist $s_1, s_2, s_3, s_4 \in S$ such that $as_1s_2bs_3s_4c \in T$.*

Theorem 7. [24] *Let S be a ternary semiring and P be a proper ideal of S. Then P is a prime ideal of S if and only if P^C is an m-system.*

Definition 8. [24] *An ideal P of a ternary semiring S is called have the insertion of factors property or IFP if $abc \in P$ implies $aSSbSSc \subseteq P$ for any $a, b, c \in S$.*

Now, we will give the definitions of strongly nilpotent and super nilpotent based on [24]. An element a of ternary semiring S is called strongly nilpotent if $\underbrace{aaa \ldots aaa}_{2n+1 \text{ times}} = 0$ and is called super nilpotent if $\underbrace{(aS)(aS) \ldots (aS)(aS)}_{n \text{ times}}\, a = \{0\}$. It can be written mathematically as follows.

Definition 9. [24] *An element a of a ternary semiring S is called strongly nilpotent if there exists a positive integer n such that $a^{2n+1} = 0$. The set of all strongly nilpotent elements of S is denoted by $\mathcal{N}(S)$.*

Definition 10. [24] *An element a of a ternary semiring S is called super nilpotent if there exists a positive integer n such that $(aS)^n a = \{0\}$.*

Definition 11. [24] *A ternary semiring S is called super nilpotent if each $a \in \mathcal{N}(S)$ is super nilpotent.*

Next, the definitions of 2-primal ternary semiring S and spsecial subsets of S based on [24] will be given as follows. We also give a theorem and some propositions about them.

Definition 12. [24] *A ternary semiring S is called 2-primal ternary semiring if $\mathcal{P}(S) = \mathcal{N}(S)$, where $\mathcal{P}(S)$ denotes the intersection of all prime ideals of S.*

Proposition 13. [24] *For any ternary semiring S, $\mathcal{P}(S) \subseteq \mathcal{N}(S)$.*

Theorem 14. [24] *Let S be a 2-primal ternary semiring. Then $\mathcal{P}(S)$ has the IFP.*

Definition 15. [24] *For a prime ideal P of a ternary semiring S, defined*

$$N(P) = \{x \in S : xSSyS \subseteq \mathcal{P}(S) \text{ for some } y \in P^c\},$$

$$\overline{N(P)} = \{x \in S : (xS)^n x \subseteq N(P) \text{ for some positive integer } n\},$$

$$N_P = \{x \in S : xyS \subseteq \mathcal{P}(S) \text{ for some } y \in P^c\},$$

$$\overline{N}_P = \{x \in S : (xS)^n x \subseteq N_P \text{ for some positive integer } n\}.$$

$$O(P) = \{x \in S : xSSyS = \{0\} \text{ for some } y \in P^c\},$$

$$\overline{O(P)} = \{x \in S : (xS)^n x \subseteq O(P) \text{ for some positive integer } n\},$$

$$O_P = \{x \in S : xyS = \{0\} \text{ for some } y \in P^c\},$$
$$\bar{O}_P = \{x \in S : (xS)^n x \subseteq O_P \text{ for some positive integer } n\}.$$

Proposition 16. [24] *Let S be a ternary semiring and P be a prime ideal of S. Then $O(P) \subseteq O_P$.*

Proposition 17. [24] *Let S be a ternary semiring and P_1, P_2 be prime ideals of S such that $P_1 \subseteq P_2$. Then $O(P_2) \subseteq O(P_1)$.*

Proposition 18. [24] *Let S be a ternary semiring and P be a prime ideal of S. Then $O(P)$ is an ideal of S.*

3 Research Method

This research was using deductive proof based on a literature study in the form of books and scientific journals, especially [24] and those related to the ternary semiring theory. We define $O'(P)$ and O'_P, the special subsets of ternary semiring S then we determine some properties of them and their relations with the subsets in Definition 15. We also determine the condition for $O(P)$ and O_P in order to the special subsets are ideals of S. The last, the special subsets of ternary semiring will be used to determine the characterizations of 2-primal ternary semiring.

4 Result and Discussion

4.1 Special Subsets of Ternary Semiring

We will give the definition and some properties of special subsets of ternary semiring and their relations with the subsets in Definition 15.

Definition 19. *Let S be a ternary semiring and P be a prime ideal of S, we define*

$$O'(P) = \{x \in S : xSx \subseteq O(P)\},$$
$$O'_P = \{x \in S : xSx \subseteq O_P\}.$$

Proposition 20. *Let $\boldsymbol{S}$ be a ternary semiring and $\boldsymbol{P}$ be a prime ideal of $\boldsymbol{S}$, then $\boldsymbol{O'(P) \subseteq O'_P}$.*

Proof. Let $x \in O'(P)$, then $xSx \subseteq O(P)$. By Proposition 16, we have $xSx \subseteq O(P) \subseteq O_P$. Therefore, $x \in O'_P$. Hence $O'(P) \subseteq O'_P$. ■

Proposition 21. *Let S be a ternary semiring and P_1, P_2 be prime ideals of S such that $P_1 \subseteq P_2$, then $O'(P_2) \subseteq O'(P_1)$ and ${O'}_{P_2} \subseteq {O'}_{P_1}$.*

Proof. Let $x \in O'(P_2)$, then $xSx \subseteq O(P_2)$. By Proposition 17, we have $O(P_2) \subseteq O(P_1)$. Therefore $xSx \subseteq O(P_1)$ which implies $x \in O'(P_1)$. Hence $O'(P_2) \subseteq O'(P_1)$.

By similar argument we can prove that ${O'}_{P_2} \subseteq {O'}_{P_1}$. ■

Proposition 22. *Let S be a ternary semiring and O_P be an ideal of S for each prime ideal P of S, then $O(P) \subseteq O'(P) \subseteq \overline{O(P)}$ and $O_P \subseteq O'_P \subseteq \bar{O}_P$.*

Proof. Let $x \in O(P)$, then there exists $y \in P^C$ such that $xSSyS = \{0\}$. Hence $(xSx)SSyS = xS(xSSyS) = \{0\}$. Thus $xSx \subseteq O(P)$ which implies $x \in O'(P)$. Therefore $O(P) \subseteq O'(P)$. Now suppose $x \in O'(P)$, then $xSx \subseteq O(P)$. Note that by Proposition 18, $O(P)$ is an ideal of S. Thus $(xS)^n x = xSxS \dots xSxSx \subseteq SSSS \dots SS\big(O(P)\big) \subseteq O(P)$ which implies $x \in \overline{O(P)}$. Therefore $O'(P) \subseteq \overline{O(P)}$. Hence $O(P) \subseteq O'(P) \subseteq \overline{O(P)}$.

Since O_P is ideal of S, by similar argument we can prove that $O_P \subseteq O'_P \subseteq \bar{O}_P$. ∎

Now, we will determine the condition for $N(P)$ and $O(P)$ in order to $O'(P) = O'_P$.

Proposition 23. *Let S be a 2-primal ternary semiring and $O(P) = N(P)$ for each prime ideal P of S, then $O'(P) = O'_P$.*

Proof. By Proposition 20, we have $O'(P) \subseteq O'_P$. Now suppose $x \in O'_P$, then $xSx \subseteq O_P$. Thus there exists $y \in P^c$ such that $(xSx)yS = \{0\} \subseteq \mathcal{P}(S)$. Since S is a 2-primal ternary semiring, then by Theorem 14, $\mathcal{P}(S)$ has the IFP. Therefore, $(xSx)SSySSS \subseteq \mathcal{P}(S)$. Since S contains the identity element, then $(xSx)SSyS = (xSx)SSyS11 \subseteq (xSx)SSySSS \subseteq \mathcal{P}(S)$ which implies $xSx \subseteq N(P) = O(P)$. Therefore $x \in O'(P)$ which implies $O'_P \subseteq O'(P)$. Hence $O'(P) = O'_P$. ∎

Next, we will determine the condition for $O(P)$ and O_P in order to $O'(P)$ and O'_P are ideals of S.

Theorem 24. *Let S be a ternary semiring and P be prime ideal of S. If $O(P)$ is a completely semiprime ideal of S, then $O'(P)$ is an ideal of S.*

Proof. Clearly $O'(P) \subseteq S$ and $0 \in O'(P)$ which implies $O'(P) \neq \emptyset$. Now suppose $x_1, x_2 \in O'(P)$, then $x_1Sx_1 \subseteq O(P)$ and $x_2Sx_2 \subseteq O(P)$. Note that $(x_1)^3 \in x_1Sx_1 \subseteq O(P)$ and $(x_2)^3 \in x_2Sx_2 \subseteq O(P)$. Since $O(P)$ is a completely semiprime ideal of S, then $x_1, x_2 \in O(P)$. Therefore $x_1Sx_2 \subseteq \big(O(P)\big)SS \subseteq O(P)$ and $x_2Sx_1 \subseteq \big(O(P)\big)SS \subseteq O(P)$ which implies $(x_1 + x_2)S(x_1 + x_2) = x_1Sx_1 + x_1Sx_2 + x_2Sx_1 + x_2Sx_2 \subseteq O(P) + O(P) + O(P) + O(P) \subseteq O(P)$. Thus $x_1 + x_2 \in O'(P)$.

Now suppose $x \in O'(P)$ and $s_1, s_2 \in S$, then $xSx \subseteq O(P)$. Note that $x^3 \in xSx \subseteq O(P)$. Since $O(P)$ is a completely semiprime ideal of S, then $x \in O(P)$. Therefore we have

$$(xs_1s_2)S(xs_1s_2) \subseteq \Big(\big(O(P)\big)SS\Big)S(SSS) \subseteq \big(O(P)\big)SS \subseteq O(P),$$

$$(s_1xs_2)S(s_1xs_2) \subseteq \Big(S\big(O(P)\big)S\Big)S(SSS) \subseteq \big(O(P)\big)SS \subseteq O(P),$$

$$(s_1s_2x)S(s_1s_2x) \subseteq \Big(SS\big(O(P)\big)\Big)S(SSS) \subseteq \big(O(P)\big)SS \subseteq O(P).$$

Thus xs_1s_2, s_1xs_2, $s_1s_2x \in O'(P)$. Hence $O'(P)$ is an ideal of S. ∎

By similar argument in Theorem 24, we can also prove that if O_P is a completely semiprime ideal of S, then O'_P is an ideal of S.

4.2 Characterizations of 2-Primal Ternary Semiring

We will give the characterizations of 2-primal ternary semiring using special subsets of ternary semiring.

Theorem 25. *Let S be a commutative super nilpotent ternary semiring, then*

(i) $O'(P) \subseteq P$ for each prime ideal P of S.

(ii) If $O(P) = \overline{O(P)}$, then S is a 2-primal ternary semiring.

Proof. (i). Let P be a prime ideal of S and $x \in O'(P)$, then $xSx \subseteq O(P)$. Thus there exists $y \in P^c$ such that $(xSx)SSyS = \{0\}$. Since S is a commutative ternary semiring, then

$$\begin{aligned} xSSySSx &= xSSy(SSx) = xSSy(xSS) \\ &= xS(Syx)SS = xS(Sxy)SS \\ &= xSSx(ySS) = xSSx(SyS) \\ &= x(SSx)SyS = x(SxS)SyS \\ &= (xSx)SSyS = \{0\} \subseteq \mathcal{P}(S). \end{aligned}$$

Therefore $xSSySSx \subseteq P$ for each prime ideal P of S and by Proposition 3, we have $x \in P$. Hence $O'(P) \subseteq P$ for each prime ideal P of S.

(ii). By Proposition 13, we have $\mathcal{P}(S) \subseteq \mathcal{N}(S)$. Now suppose there exists $x \in \mathcal{N}(S)$ but $x \notin \mathcal{P}(S)$. Thus there exists prime ideal P of S such that $x \notin P$. Since S is super nilpotent ternary semiring, then there exists positive integer n such that $(xS)^n x = \{0\}$. Note that $(xS)^n x = \{0\} \subseteq O(P)$ which implies $x \in \overline{O(P)} = O(P)$. By Proposition 18, we have $O(P)$ is an ideal of S. Therefore $xSx \subseteq \big(O(P)\big)SS \subseteq O(P)$. Hence $x \in O'(P)$. Since by (i) $O'(P) \subseteq P$ for each prime ideal P of S, then we have $x \in P$, a contradiction. So for each $x \in \mathcal{N}(S)$ implies $x \in \mathcal{P}(S)$. Thus $\mathcal{N}(S) \subseteq \mathcal{P}(S)$. Therefore $\mathcal{P}(S) = \mathcal{N}(S)$. Hence S is 2-primal ternary semiring. ■

Theorem 26. *Let S be a commutative ternary semiring and $O(P) = O_P = N(P)$ for each prime ideal P of S. Then S is a 2-primal ternary semiring if and only if O'_P has the IFP for each prime ideal P of S.*

Proof. ($\Rightarrow$). Let P be a prime ideal of S and $x, y, z \in S$ such that $xyz \in O'_P$. Since S is a 2-primal ternary semiring and $O(P) = N(P)$ for each prime ideal P of S, then by Proposition 23, we have $O'(P) = O'_P$. Thus $xyz \in O'(P)$ which implies $(xyz)S(xyz) \subseteq O(P)$. Since S is a commutative ternary semiring and by Proposition 18 we know $O(P)$ is an ideal of S, then we have

$$\begin{aligned} (xSSySSz)S(xSSySSz) &= x(SSy)(SSz)Sx(SSy)(SSz) \\ &= x(ySS)(zSS)Sx(ySS)(zSS) \\ &= xy(SSz)SSSxy(SSz)SS \\ &= xy(zSS)SSSxy(zSS)SS \end{aligned}$$

$$= xyzS(SSSS)(xyz)(SSSS)$$
$$= xyzS(xyz)(SSSS)(SSSS)$$
$$\subseteq (O(P))(SSSS)(SSSS)$$
$$\subseteq O(P).$$

Therefore $xSSySSz \subseteq O'(P) = O'_P$. Hence O'_P has the IFP for each prime ideal P of S.

($\Leftarrow$). By Proposition 13, we have $\mathcal{P}(S) \subseteq \mathcal{N}(S)$. Now suppose there exists $x \in \mathcal{N}(S)$ but $x \notin \mathcal{P}(S)$. Thus there exists prime ideal P of S such that $x \in P^c$. Since P is a prime ideal of S, then by Theorem 7, we have P^c is an m-system. Therefore there exists $s_1, s_2, s_3, s_4 \in S$ such that $xs_1s_2xs_3s_4x \in P^c$. Also since P^c is an m-system, then there exists $s_5, s_6, s_7, s_8 \in S$ such that $(xs_1s_2xs_3s_4x)s_5s_6xs_7s_8x \in P^c$. Repeating above argument, we have $xs_1s_2xs_3s_4x \dots xs_{4n-1}s_{4n}x \in P^c$. Hence $xs_1s_2xs_3s_4x \dots xs_{4n-1}s_{4n}x \notin P$ which implies $xSSxSSx \dots xSSx \nsubseteq P$. Now since $x \in \mathcal{N}(S)$, then there exists positive integer n such that $x^{2n+1} = 0$. Note that $x^{2n+1} = xx(xxx \dots xx) = 0 \in O'_P$. Since O'_P has the IFP for each prime ideal P of S, then $(xSSxSSx)x(x \dots xx) \subseteq O'_P$. Thus $xSSxSSxSSxSSx \dots xx \subseteq O'_P$. Repeating above argument, we have $xSSxSSx \dots xSSx \subseteq O'_P$. Therefore $(xSSx \dots xSSx)S(xSSx \dots xSSx) \subseteq O_P$. Since $O_P = O(P)$, then $(xSSx \dots xSSx)S(xSSx \dots xSSx) \subseteq O(P)$. Thus $xSSxSSx \dots xSSx \subseteq O'(P)$. Since S is a commutative ternary semiring, then by Theorem 25 (i), we have $O'(P) \subseteq P$ for each prime ideal P of S. Hence $xSSxSSx \dots xSSx \subseteq P$, a contradiction. So for each $x \in \mathcal{N}(S)$ implies $x \in \mathcal{P}(S)$. Thus $\mathcal{N}(S) \subseteq \mathcal{P}(S)$. Therefore $\mathcal{P}(S) = \mathcal{N}(S)$. Hence S is a 2-primal ternary semiring. ■

5 Conclusion

Based on discussion in previous part, some the characterizations of 2-primal ternary semiring using special subsets of ternary semiring as follows :

(i) S is a commutative super nilpotent ternary semiring and $O(P) = \overline{O(P)}$ for each prime ideal P of S.

(ii) $O(P) = O_P = N(P)$ and O'_P has the IFP for each prime ideal P of S.

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