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A GENERALIZED FINITE DIFFERENCE METHOD FOR TRANSIENT HEAT CONDUCTION ANALYSIS-SHORT COMMUNICATION

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Abstract – This short communication presents a meshless local B-spline basis functions-finite difference (FD) method for transient heat conduction analysis. The method is truly meshless as only scattered nodal distribution is required in the problem domain. It is also simple and efficient to program. As it has the Kronecker delta property, the imposition of boundary conditions can be incorporated efficiently. In the method, any governing equations are discretized by B-spline approximation in the spirit of FD technique using local B-spline collocation. It hence belongs to a generalized FD method, in which any derivative at a point or node is stated as neighbouring nodal values based on the B-spline interpolants. Numerical results show the effectiveness and efficiency of the meshless method for analysis of transient heat conduction in complex domain.

Keywords: A generalized FD; meshless; B-splines; local collocation; transient heat conduction; complex domain.

1. Introduction

Transient heat conduction is an important phenomenon which is commonly encountered in many engineering applications. It is essential to have the precise knowledge of temperature distribution and variation as the temperature varies with respect to time in many engineering or industrial components. Due to the complexity of the component geometry in most practical situations, numerical methods such as finite element (FE) and finite volume (FV) methods are commonly chosen and employed to obtain reasonable results. Nonetheless, due to the fact that mesh generation can be time consuming and the most expensive part in the simulation (Onate *et al*., 1996 ; Griebel. and Schweitzer, 2000), meshless methods have been introduced and developed in the last two decades as new emerging numerical techniques in engineering and science. The interested readers are directed to Liu (2009) and Cottrell *et al*. (2009) for historical development and progression of meshless methods and isogeometric analysis, including references therein.

In this study, a new meshless local B-spline basis functions-finite difference (FD) method is presented for transient heat conduction analysis. The method is truly meshless as only scattered nodal distribution is required in problem domain, hence simple and efficient to program. In the method, any governing equations are discretized by B-spline approximation in the spirit of FD technique using local B-spline collocation i.e. any derivative at a point or node is stated as neighbouring nodal values based on the B-spline interpolants. In comparison to FEM, the method is shown to be computationally efficient.

2. Meshless B-splines based FD method

The i-th univariate B-spline basis function of order k (or degree $p = k-1$), $N_{i,k}(t)$, is defined recursively by the Cox-de Boor recursion formula as (de Boor, 2001):

$$N_{i,1}(t) = \begin{cases} 1 & \text{if } \tau_i < t < \tau_{i+1} \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

$$N_{i,k}(t)=\frac{t-t_i}{t_{i+k-1}-t_i}N_{i,k-1}(t)+\frac{t_{i+k}-t}{t_{i+k}-t_{i+1}}N_{i+1,k-1}(t) \tag{2}$$

$N_{i,1}(t)$ is a step function and the $N_{i,k}(t)$ is a linear combination of two $(k-1)$ order basis functions for $k>1$. In the formula, the convention $0/0=0$ is used for the division calculation. The basis functions of higher dimensional problems are constructed by taking the product of the univariate B-spline basis functions $b=(N_{0,k},...,N_{n,k})$ as the bases for the space C of polynomial functions of degree less than or equal to $k-1$ in an interval $I=[a_1,b_1]=[\tau_0,\tau_{n+k}]$.

Having notation N^a for the B-spline basis functions in general, a function $u(\mathbf{x})$ can then be expressed with B-spline approximation in a form as follows:

$$u(\mathbf{x})\approx\hat{u}(\mathbf{x})=\sum_{i=1}^{N_{\mathrm{B}}}N_i^a(\mathbf{x})\,\alpha_i \tag{3}$$

where: $\mathbf{x}\in R^d$ is vector of spatial variables or point position, $N^a(\mathbf{x})=N_{i,k}(x)\,M_{j,l}(y)$ are the B-spline bases in 2D, α are the coefficients of approximation related to $N^a(\mathbf{x})$ and N_{B} is the number of B-spline bases. The tensor product B-spline bases have the partition of unity property:

$$\sum_{i=0,\,j=0}^{n,m}N_{i,k}(t)\,M_{j,l}(s)=1 \tag{4}$$

The B-spline bases are preferred here to satisfy the Kronecker delta property for convenience of the imposition of boundary conditions. B-spline bases from an open-uniform knot vector with full multiplicity of its end knots (no interior knots) are here chosen. Note that the B-spline basis functions of p degree thus simply reduce into Bezier approximation of the same degree (Piegl and Tiller, 1995; Farin, 2002). Now consider a subset $\chi_i=\{x_i,x_2,...,x_{ns}\}$ in the global set χ containing of the original set of points $x_i,x_2,...,x_{NC}\in\Omega\subset R^d$. The derivative of a function $u(\mathbf{x})$ at x_j of the subset χ_i is obtained as:

$$L\hat{u}(\mathbf{x})=\sum_{i=1}^{N_{\mathrm{B}}}LN_i^a(\mathbf{x})\,\alpha_i \tag{5}$$

$$Lu(x_j)\approx L\hat{u}(x_j)=\sum_{i=1}^{ns}\left[\mathrm{H}(x_j)/\mathbf{B}\right]u(x_i) \tag{6}$$

where: L is the differential operator, $\mathrm{H}(x_j)=LN^a(x_j)$ is the vector containing of the derivatives of B-spline basis functions evaluated at x_j, $\mathbf{B}$ has entries of B-spline bases $b_{i,j}=N_{i,j}^a$, $i=1,...,ns$; $j=1,...,N_B$, $\boldsymbol{\alpha}$ is the vector of $\{\alpha_1,...,\alpha_{N_{\mathrm{B}}}\}^{\mathrm{T}}$ and $\mathbf{u}$ is the nodal values of the subset χ_i.

Weights vector $\mathbf{w}$ and the differentiation matrix $\mathbf{D}_m$ to obtain the integrated solutions $u(\mathbf{x})$ over the problem domain Ω are further stated as:

$$\mathbf{w}=\mathbf{H}(x_i)/\mathbf{B} \tag{7}$$

$$\mathbf{D}_m\mathbf{u}=\mathbf{f} \tag{8}$$

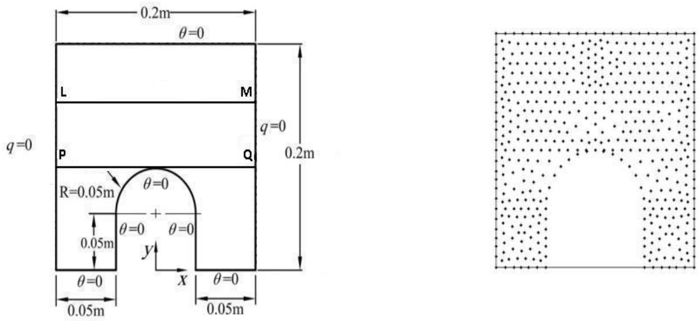
where: $\mathbf{u}$ is the vector of global unknowns and $\mathbf{f}$ is the vector of external forces.

3. Numerical Illustrations

A complex problem domain shown in Fig. 1(a) was considered and subjected to a non-uniform heat source (Hematiyan *et al*., 2011). The material constants are k = 64 W/(m K), c = 434 J/(kg K), ρ = 7850 kg/m^3 and α = 11.7×10^{-6} m^2/s. The problem was also simulated by FEM using ANSYS software with 4126 nodes and 7920 elements for reference or benchmark solutions. Fig. 2(b) shows the meshless discretization. For the transient simulation, the Crank-Nicolson technique was employed in the present study with the time step Δt = 0.1 *s* and the final time t_f was set to be 500 *s*. The heat source is given as :

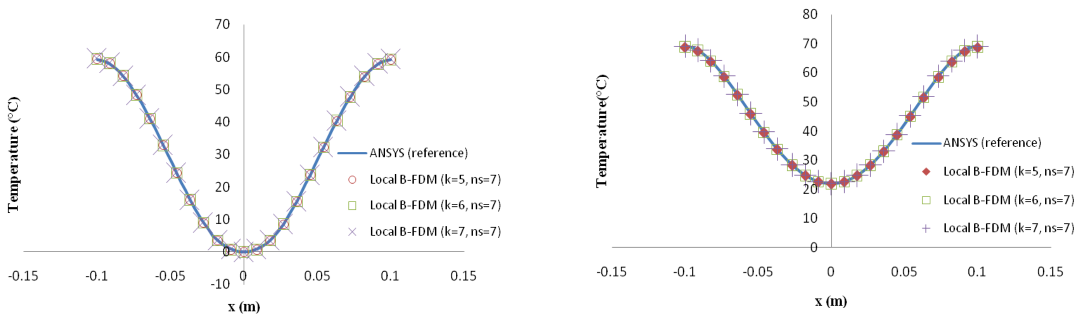
$$\dot{Q}(x,y) = \left(5\times10^{8}\right) xy \sin\left(\frac{\pi x}{0.5}\right) \tag{9}$$

Fig. 2 depicts the comparison of temperatures along the lines P-Q and L-M in the domain by the present local B-FDM and FEM from the transient simulation. The supporting nodes number ns = 7 is fixed and the B-spline order is varied as k = 5, 6 and 7. Fig. 3 shows the temperature contours in the domain obtained by the FE and present local B-FD (k = 5, ns = 12) methods at t_f = 500 *s*. Further, Table 1 presents comparison of simulation times by the local B-FDM (k = 5, ns = 12) and FEM (ANSYS) for the problem. MATLAB was used to program the generalized FD method in HP Compaq with OS Windows XP, processor of Intel Pentium 4 512 MB RAM.



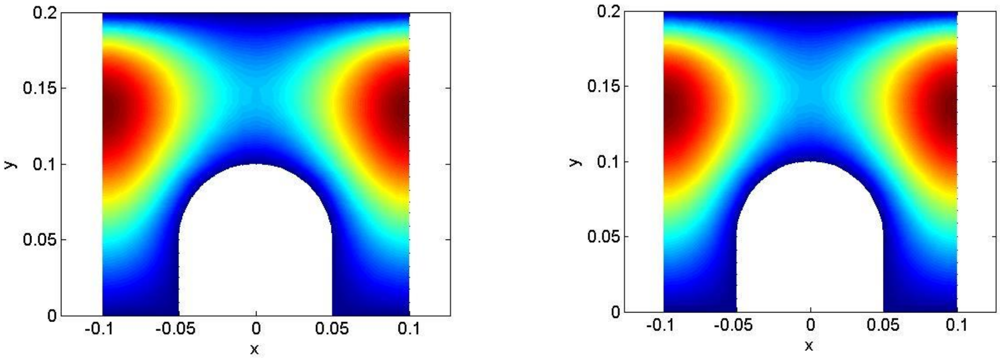
(a) (b)

Fig. 1. (a) The problem geometry and boundary conditions, and (b) 496 nodes for the meshless local B-FDM.



(a) (b)

Fig. 2. Comparison of temperature distributions along the lines (a) P-Q and (b) L-M in the domain by the meshless local B-FD and FE methods.



(a) (b)

Fig. 3. Comparison of temperature contours in the problem domain by the FE and meshless local B-FD methods at $t_f = 500$ *s*.

Table 1. Comparison of simulation times by the FE and meshless local B-FD methods for the transient heat conduction problem

	Δt = 0.1 sec		Δt = 0.4 sec	
496 nodes	FEM	Meshless local B-FDM	FEM	Meshless local B-FDM
Simulation times (sec)	573	129.70	140	35.44
Normalized simulation times	1.1552	0.2615	0.2823	0.0715

4. Conclusions

In this short communication, a new meshless local B-spline basis functions-finite difference (FD) method has been presented for analysis of transient heat conduction. It is truly meshless, hence simple and efficient to program. It belongs to a generalized FD method. Comparison of the simulation results by the method and those by FE method shows its effectiveness and efficiency for transient heat conduction analysis in complex domain.

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